Optimal Process Mean Setting Under the Quality and Cost

Chung-Ho Chen
Department of Management and Information Technology
Southern Taiwan University of Science and Technology, Tainan, Taiwan
Tel: (+886) 6-2533131 ext. 4129, Email: chench@stust.edu.tw

Abstract. In this study, the author considers the optimal process mean setting problem with process adjust ment cost and product quality. Assume that the process adjustment cost is proportional to the square of pr ocess mean and reciprocal of variance. Taguchi's asymmetric quadratic quality loss function is adopted for measuring the product quality. The uniform and triangular distributions of quality characteristic are address ed in the formulation of mathematical model. The optimal process mean will be obtained by minimizing t he expected total cost of product. Finally, the numerical example and sensitivity analysis of parameters are provided for illustration.

Keywords: process mean, asymmetric quadratic quality loss function, uniform distribution, triangular distri bution

1. INTRODUCTION

Economic selection of process mean is an important problem for modern statistical process control (SPC). The optimal setting of this value usually has the significant effect on the expected total cost/profit per unit product. Recently, there are many works addressing the above problem with the integration evaluation about quality, production, inventory, and reliability. Taguchi (1986) redefined the product quality as the loss of society when the product is shipped to the customer. If the product quality has the minimum bias and variability, then the product reaches the optimal level. For the modern quality management, one addresses the six-sigma method for quality promotion and improvement.

Taguchi's (1986) quadratic quality loss function has been successfully applied in some topics of quality control. It connects with the methodology of process control and quality improvement. Previous researchers, Chen (2005) Chen and Lai (2007), Darwish (2009, 2013), Darwish and Abdulmalek (2012), Darwish and Duffuaa, (2010), Duffuaa and El-Ga'aly (2013a, 2013b), Feng and Kapur (2006a, 2006b), Jeang (2010, 2011a, 2011b), Jeang and Lin (2014), Kapur (1988), Kapur and Cho (1994, 1996), Kapur and Wang (1987), and Sin et al. (2010) have addressed that some economic settings about specification limits, control chart, process mean, and manufacturing quantity.

The normal quality characteristic with unknown process mean and variance is usually assumed in the SPC. Modern product control and design always addresses the predictive method for determining the optimum product quality. The statistical and economic designs are two available methods for obtaining the optimum product parameters. In this study, the author considers the optimal process mean setting with process adjustment cost and product quality loss. One presents a mathematical model with the uniform/triangular quality characteristics and asymmetric quadratic quality loss of product within the tolerance zone for determining the optimal process mean. The major contribution of this paper is to propose an integrated quality model by considering the optimal process parameters and product quality cost.

2. MATHEMATICAL MODEL

There are two important parameters affecting the product quality, i.e., process mean (μ) and variance (σ^2). Huang (2001) considered one of process adjustment cost, denoted by $C(\mu,\sigma)=\beta_1\mu^2+\dfrac{\beta_2}{\sigma^2}$, where β_1 and β_2 are positive constants. The above equation means that the cost of setting the process mean of quality characteristic is proportional to μ^2, while the cost for controlling the process variance of quality characteristic is proportional to $\dfrac{1}{\sigma^2}$.

2.1 Uniform Quality Characteristic

Consider the product characteristic, *Y*, is uniformed with $U(m-T+x, m+T+x)$, where m is the design target, x is the positive distance of the manufacturing target from the design target and T is the tolerance zone. Hence, the probability density function of Y is $f(y)=\dfrac{1}{2T}, m-T+x\le y\le m+T+x$. In this work, one adopts the asymmetric quadratic quality loss function within the tolerance zone for measuring product quality. Hence, the average quality loss of product per item is

$$
\begin{aligned}
&L(x,T)\\
&=\int_{m-T+x}^{m} f(y)k_1(m-y)^2\,dy+\int_{m}^{m+T+x} f(y)k_2(y-m)^2\,dy\\
&=[k_1(T-x)^3+k_2(T+x)^3]/6T \qquad (1)
\end{aligned}
$$

where k_1 is the quality loss coefficient when $Y\le m$ and k_2 is the quality loss coefficient when $Y>m$.

Hence, the expected total cost of product including the process adjustment cost and quality loss is defined as

$$
\begin{aligned}
&ETC\\
&=C(\mu,\sigma)+L(x,T)\\
&=\beta_1\mu^2+\frac{\beta_2}{\sigma^2}+L(x,T) \qquad (2)\\
&=\beta_1(m+x)^2+\frac{3\beta_2}{T^2}+[k_1(T-x)^3+k_2(T+x)^3]/6T
\end{aligned}
$$

In order to determine the optimal x value, one takes the first and second order derivatives of equation (2) about x. We have $\dfrac{dETC}{dx}=2\beta_1(m+x)+[k_2(T+x)^2-k_1(T-x)^2]/2T$

and $\dfrac{d^2ETC}{dx^2}=2\beta_1+k_1+k_2+x(k_2-k_1)/T$. The second order derivative of equation (2) about x is positive when $0\le x\le T(2\beta_1+k_1+k_2)/(k_1-k_2)$ and $k_1>k_2$.

Let the above first-order derivative be zero. Hence, the optimal x with global minimum ETC is

$$x^*=\frac{T(k_1+k_2+2\beta_1)-\sqrt{T^2(k_1+\beta_1)(k_2+\beta_1)+\beta_1Tm(\beta_1-\beta_2)}}{k_1-k_2}.$$

We have the optimal process mean value $=m+x^*$.

2.2 Triangular Quality Characteristic

The sum of two mutually independent uniform random variables follows a triangular distribution. The probability density function of Y is

$$
f(y)=\begin{cases}\dfrac{2(y-a)}{(b-a)(c-a)}, & a\le y\le b\\[2mm] \dfrac{2(c-y)}{(c-b)(c-a)}, & b\le y\le c\\ 0, & \text{elsewhere}\end{cases} \qquad (3)
$$

where a is the minimum value, b is the mode, and c is the maximum value of *Y*, respectively. It can be shown that the expected value and variance of Y are $\mu=\dfrac{a+b+c}{3}$ and $\sigma^2=\dfrac{a^2+b^2+c^2-ab-ac-bc}{18}$. Let

$a = m\text{-}T+x$, $b = m+x$, and $c = m+T+x$. We have $\mu = m + x$ and $\sigma^2 = \dfrac{T^2}{6}$.

One adopts the asymmetric quadratic quality loss function within the tolerance zone for measuring product quality. Hence, the average quality loss of product per item is

$$\begin{aligned} &L(x,T) \\ &= \int_{m-T+x}^{m} f(y)k_1(m-y)^2\,dy + \int_{m}^{m+T+x} f(y)k_2(y-m)^2\,dy \\ &= [k_1(T-x)^4 + k_2(T+x)^4]/12T \end{aligned} \tag{4}$$

where k_1 is the quality loss coefficient when $Y \le m$ and k_2 is the quality loss coefficient when $Y > m$.

Hence, the expected total cost of product including the process adjustment cost and quality loss is defined as

$$\begin{aligned} &ETC \\ &= C(\mu,\sigma) + L(x,T) \\ &= \beta_1\mu^2 + \frac{\beta_2}{\sigma^2} + L(x,T) \\ &= \beta_1(m+x)^2 + \frac{6\beta_2}{T^2} + [k_1(T-x)^4 + k_2(T+x)^4]/12T \end{aligned} \tag{5}$$

In order to determine the optimal x value, one takes the first and second order derivatives of equation (5) about x. We have

$$\frac{dETC}{dx} = 2\beta_1(m+x) + [k_2(T+x)^3 - k_1(T-x)^3]/3T$$

and $\dfrac{d^2ETC}{dx^2} = 2\beta_1 + [k_1(T-x)^2 + k_2(T+x)^2] > 0$. The second order derivative of equation (5) about x is always positive.

Let the above first-order derivative be zero. Hence, the optimal x with global minimum ETC by solving the following equation:

$$t_1x^3 + t_2x^2 + t_3x + t_4 = 0 \tag{6}$$

where

$t_1 = k_1 + k_2$, $t_2 = 3T(k_2 - k_1)$, $t_3 = 3T(k_1T + k_2T + 2\beta_1)$, and $t_4 = T(k_2T^2 - k_1T^2 + 6\beta_1 m)$.

By adopting Newtown's method, one can obtain the approximate optimal x value. We have the optimal process mean value $= m + x^*$.

3. NUMERICAL EXAMPLE

Assume that some parameters are as follows: $k_1 = 200, k_2 = 20, \beta_1 = 5, \beta_2 = 2, T = 5,$ and $m = 5.$ For the uniform quality characteristic, the optimal x value is $x^* = 4.233.$ Table 1 lists the sensitivity analysis for parameters in the uniform distribution. From Table 1, we have two observations: (1) the optimal x value increases as the values of k_2 and T increase; (2) the optimal x value decreases as the values of β_1 and m increase. This result shows that the positive distance of the manufacturing target from the design target is affected by the quality loss coefficient, tolerance zone, cost coefficient of process mean, and design target.

For the triangular quality characteristic, the optimal x value is $x^* = 1.718.$ Table 2 lists the sensitivity analysis for parameters in the triangular distribution. From Table 2, we have two observations: (1) the optimal x value increases as the values of k_1 and T increase; (2) the optimal x value decreases as the values of $k_2, \beta_1,$ and m increase. This result also shows that the positive distance of the manufacturing target from the design target is affected by the quality loss coefficient, tolerance zone, cost coefficient of process mean, and design target.

From Tables1-2, one also has the following conclusion: the optimal x value of triangular distribution is smaller than that of uniform distribution. This result meets the triangular distribution with smaller variance than that of uniform distribution.

Table 1: The sensitivity analysis of parameters for uniform distribution.

k_1	x^*
400	4.215
300	4.196
200	4.233
100	4.688
k_2	x^*
40	4.684
30	4.424
20	4.233
10	4.120
β_1	x^*
10	4.167
8	4.188
6	4.216
4	4.252
T	x^*
10	8.630
8	6.870
6	5.112
4	3.354
m	x^*
10	4.077
8	4.138
6	4.201
4	4.265

4. CONCLUSIONS

In this paper, the author proposes the uniform/ triangular quality characteristic of product with asymmetric quadratic quality loss function within the tolerance zone for determining the optimal process mean. The optimal process mean will be determined by minimizing the expected total cost of product including the process adjustment cost and quality loss.

The management implication of this mathematical model is that it can provide industry/business application for promoting the product/service quality assurance for the customer. The practical application can be adopted in the personalized product/service for increasing the quality performance and cost reduction. The extension of this method to the integrated production, inventory, quality and reliability model may be left for further study.

Table 2: The sensitivity analysis of parameters for triangular distribution.

k_1	x^*
400	2.219
300	2.017
200	1.718
100	1.169
k_2	x^*
40	1.238
30	1.444
20	1.718
10	2.135
β_1	x^*
10	1.613
8	1.654
6	1.696
4	1.740
T	x^*
10	3.587
8	2.845
6	2.097
4	1.333
m	x^*
10	1.638
8	1.670
6	1.702
4	1.734

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