Reliability Analysis in the Field Data with Interval Censored

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Abstract. In this study, we consider the field data with interval censored of two models of desktop computers manufactured by one company that both units are more than 3,000,000. To deal with this type of data for reliability analysis, we consider the Weibull model. Several methods such as the maximum likelihood estimation (MLE) via the Differential Evolution (DE) algorithm and the Expectation–Maximization (EM) algorithm, and the Bayesian approach via Lindley's approximation and Markov Chain Monte Carlo (MCMC) are used to estimate the Weibull parameters. Additionally, the confidence intervals for these estimators are obtained. The field data are used to illustrate the applications. The results show that the Bayesian approach with Lindley approximation outperforms the other methods in term of the mean of absolute percentage error in most cases.

Keywords: Weibull model; maximum likelihood estimator; Bayesian with Lindley approximation

1. INTRODUCTION

Predicting failure rate for desktop computers from the field return data is an important task for computer manufacturers. The design capabilities and manufacturing technologies for desktop computers are continuously upgraded, subsequently increasing consumer demands for reliability. However, the calculations of the lifespan and Mean Time Between Failure (MTBF) may differ substantially from actual conditions. This can lead to erroneous estimates of the product maintenance applications, warranty periods, and spare part amounts. The MTBF of a desktop computer is approximately 5 years at normal conditions from an accelerated life test. However, this claim should be verified by the actual life data based on the field return data.

In general, the two-parameter Weibull distribution has been widely used in reliability engineering. Numerous methods such as maximum likelihood estimator (MLE), non-parametric method, and Bayesian approach have been proposed to obtain the estimates of the two-parameter Weibull distribution under complete, type-I censoring, Type-II censoring, multiple censoring, progressive censoring, middle censoring, and interval censored data (Murthy et al., 2004). The MLE method for the Weibull model under interval censored can be found in Flygare et al. (1985). The expectation maximization (EM) algorithm for estimating the Weibull parameters based on the Weibull-to-exponential transformation for interval censored data can be found in Ng and Wang (2009) and Tan (2009). The Bayesian estimation for the Weibull model with interval censored or progressive censoring can be found in Gomez et al. (2004), Kaminskiy and Krivtsov (2005), Kundu (2008). Iyer et al. (2009) presented the analysis of interval/middle censored data for the exponential model. The sample size in above published studies is less than 1000. Based on our best knowledge, there is no work on the reliability analysis of field return data with censored and interval data that the sample size is more than 1 million. This type of data is occurred in big data environment. The volume of data is huge.

In this study, we focus on the field return data with interval censored. We investigate the performance of different methods including the MLE via Differential Evolution (DE) algorithm, EM algorithm and Bayesian estimation to estimate the Weibull model for the field data with censored and interval.

2. WIEBULL MODEL

In this section, the Weibull model is considered for fitting the field return data with interval censored. With r failure terminated data ($t_{1,f}, t_{2,f}, \ldots, t_{r,f}$), m multiple-censored data ($t_{1,s}, t_{2,s}, \ldots, t_{m,s}$), and n failure intervals $[(a_1,b_1),(a_2,b_2),\ldots,(a_n,b_n)]$, the likelihood function is given by

$$L = \prod_{i=1}^{r} f(t_{i,f}) \prod_{j=1}^{m} \left[R(t_{j,s}) \right] \prod_{k=1}^{n} \left[R(a_k) - R(b_k) \right], \qquad (1)$$

where $f(.)$ and $R(.)$ are probability density function and the reliability function, respectively. It should be noted that Equation (1) can cover the analyses for complete data ($m = n = 0$), Type-I censoring ($m \neq 0$ and $n = 0$), Type-II censoring ($m = n = 0$ and a fixed value of r), multiple censoring ($n = 0$), and failure interval ($n \neq 0$).

The reliability function and the probability density functi on of the two-parameter Weibull model are given by

$$R(t;\beta,\theta) = e^{-(t/\theta)^{\beta}} \tag{2}$$

$$f(t;\beta,\theta) = \frac{\beta}{\theta^{\beta}} t^{\beta-1} e^{-(t/\theta)^{\beta}} \tag{3}$$

where $t \geq 0$, $\beta > 0, \theta > 0$. The shape parameter β is also called the "characteristic life". Thus, the failure rate function becomes $h(t;\theta,\beta) = \frac{\beta t^{\beta-1}}{\theta^{\beta}}$. The MTBF of Weibull model is derived by

$$MTBF = \int_0^{\infty} e^{-(\frac{t}{\theta})^{\beta}} dt = \theta\Gamma(1+\frac{1}{\beta}) \tag{4}$$

where $\Gamma(\cdot)$ is a gamma function.

3. PARAMETERS ESTIMATION

3.1 MLE via DE algorithm

Using equation (1), the log-likelihood function is esta blished as:

$$\ln L = r\ln\beta - r\beta\ln\theta + (\beta-1)\sum_{i=1}^{r}\ln t_{i,f} - \frac{\sum_{i=1}^{r} t_{i,f}^{\beta} + \sum_{j=1}^{m} t_{j,s}^{\beta}}{\theta^{\beta}} + \sum_{k=1}^{n}\ln\left(e^{-(a_k/\theta)^{\beta}} - e^{-(b_k/\theta)^{\beta}}\right), \tag{5}$$

The maximum likelihood estimates of β and θ are obtained by setting the first partial derivatives of equation (5) to zero with respect to β and θ, respectively. These simultaneous equations are given by:

$$\frac{\partial \ln L}{\partial\theta} = -\frac{r\beta}{\theta} + \beta\theta^{-\beta-1} \times \left[\left(\sum_{i=1}^{r} t_{i,f}^{\beta} + \sum_{j=1}^{m} t_{j,s}^{\beta}\right) + \sum_{k=1}^{n}\left(\frac{a_k^{\beta} e^{-(a_k/\theta)^{\beta}} - b_k^{\beta} e^{-(b_k/\theta)^{\beta}}}{e^{-(a_k/\theta)^{\beta}} - e^{-(b_k/\theta)^{\beta}}}\right)\right] = 0, \tag{6}$$

$$\frac{\partial \ln L}{\partial\beta} = \frac{r}{\beta} - r\ln\theta + \sum_{i=1}^{r}\ln t_{i,f} + \frac{\ln\theta}{\theta^{\beta}}\left(\sum_{i=1}^{r} t_{i,f}^{\beta} + \sum_{j=1}^{m} t_{j,s}^{\beta}\right) - \frac{1}{\theta^{\beta}}\left(\sum_{i=1}^{r} t_{i,f}^{\beta}\ln t_{i,f} + \sum_{j}^{m} t_{j,s}^{\beta}\ln t_{j,s}\right) - \sum_{k=1}^{n}\left(\frac{(a_k/\theta)^{\beta}\ln(a_k/\theta)e^{-(a_k/\theta)^{\beta}} - (b_k/\theta)^{\beta}\ln(b_k/\theta)b_k^{\beta}e^{-(b_k/\theta)^{\beta}}}{e^{-(a_k/\theta)^{\beta}} - e^{-(b_k/\theta)^{\beta}}}\right) = 0, \tag{7}$$

Newton-Raphson iteration is employed to solve equatio ns (6) and (7). However, the Newton-Raphson algorithm is very sensitive to the initial values of the two parameters. Recently, evolution algorithms such as the particle swarm optimization, cross entropy and DE have been successfully used to estimate the parameters of the Weibull and mixed Weibull model for censored data. The asymptotic variance-covariance matrix of θ and β is obtained by inverting the Fisher information matrix, $I = E\left[-\frac{\partial^2 \ln L}{\partial\theta_i\partial\theta_j}\right], i, j = 1,2$, where $\theta_1 = \beta$ **and** $\theta_2 = \theta$. Thus, we have

$$\begin{bmatrix} Var(\hat{\beta}) & Cov(\hat{\beta},\hat{\theta}) \\ Cov(\hat{\beta},\hat{\theta}) & Var(\hat{\theta}) \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial\beta^2}\Big|_{\hat{\beta},\hat{\theta}} & -\frac{\partial^2 \ln L}{\partial\beta\partial\theta}\Big|_{\hat{\beta},\hat{\theta}} \\ -\frac{\partial^2 \ln L}{\partial\beta\partial\theta}\Big|_{\hat{\beta},\hat{\theta}} & -\frac{\partial^2 \ln L}{\partial\theta^2}\Big|_{\hat{\beta},\hat{\theta}} \end{bmatrix}^{-1}, \tag{8}$$

Thus, an approximate (1-α)100% confidence intervals for θ and β are given by:

$$\hat{\beta} \pm Z_{\alpha/2}\sqrt{Var(\hat{\beta})} \text{ and } \hat{\theta} \pm Z_{\alpha/2}\sqrt{Var(\hat{\theta})} \tag{9}$$

where $Z_{\alpha/2}$ is the 100(1- α/2) percentile of a standard normal distribution.

DE is a search heuristic that was introduced by Storn and Price (1997). It has been successfully applied in a wide variety of fields, from computational physics to operations research. DE belongs to the class of genetic algorithms that use the biology-inspired operations of crossover, mutation, and selection on a population to minimize an objective function over the course of successive generations. DE uses floatingpoint instead of bit-string encoding on population members, and arithmetic instead of logical operations in mutation. It has several advantages such as its simple structure, ease of use, speed, and robustness. The DE procedure is summarized as follows. The variable NP represents the number of parameter vectors in the population. At generation 0, NP guesses the optimal parameter value, and vectors are made using random values between the lower and upper bounds. Each generation involves the creation of a new population from the current population members $x_{i,g}$, where *i* indexes the vectors and *g* indexes the generation. This is accomplished using a differential mutation of the population members. A trial mutant parameter vector $v_{i,g}$ is derived by

$$v_{i,g} = x_{ro} + F \times (x_{r1} - x_{r2}), \qquad (10)$$

where x_{r0}, x_{r1}, and x_{r2} are random vectors, *F* is a positive factor and $F \in (0,1)$. After the first mutation operation, the mutation is continued until either the mutation length has been made or *rand* > *CR*, where *CR* is a crossover probability, where $CR \in [0,1]$. The crossover probability *CR* controls the fraction of the parameter values that are copied from the mutant. The objective function value associated with the children is then determined. If a trial vector has an equal or lower objective function value than the previous vector, it replaces the previous vector in the population; otherwise, the previous vector remains. The choices of *NP*, *F*, and *CR* depend on the specific problem. Price et al. (2006) suggested that the number of parents *NP* should be 10 times the number of parameters. Further, they suggested that *F* = 0.8 and *CR* = 0.9.

3.2 EM algorithm

We propose to use the EM algorithm to compute the MLEs of α and λ. In implementing the EM algorithm, we need to treat this problem as a missing value problem. The EM algorithm has two steps. The first step is the E-step, where the ‘pseudo-likelihood’ function is formed from the likelihood function, by replacing the missing observations with their corresponding expected values. The second step of the EM algorithm is the M-step, where the ‘pseudo- likelihood’ function is maximized to compute the parameters for the next iteration.

1) E-step: Suppose the interval-censored observations are denoted by {z_k; *k* = 1,2,…,*n*}, then the pseudo log-likelihood function is established as

$$\ln L_{complete} = (r+m+n)\ln\beta - (r+m+n)\beta\ln\theta + (\beta-1)\left[\sum_{i=1}^{r}\ln t_{i,f} + \sum_{j=1}^{m}\ln z_{j,s} + \sum_{k=1}^{n}\ln z_k\right] \frac{\sum_{i=1}^{r} t_{i,f}^{\beta} + \sum_{j=1}^{m} z_{j,s}^{\beta} + \sum_{k=1}^{n} z_k^{\beta}}{\theta^{\beta}} \qquad (11)$$

where

$$z_{j,s} = E(t \mid t > t_{j,s}) = \frac{\int_{t_{j,s}}^{\infty} \frac{\beta t^{\beta}}{\theta^{\beta}} e^{-(\frac{t}{\theta})^{\beta}} dt}{e^{-(t_{j,s}/\theta)^{\beta}}}$$

and

$$z_k = E(t \mid a_k < t < b_k) = \frac{\int_{a_k}^{b_k} \frac{\beta t^{\beta}}{\theta^{\beta}} e^{-(\frac{t}{\theta})^{\beta}} dt}{e^{-(a_k/\theta)^{\beta}} - e^{-(b_k/\theta)^{\beta}}}.$$

2) M-step: It involves maximization of pseudo log-likelihood function (11) with respect to β and θ to compute the next iterates.

Let $(\beta^{(l)}, \theta^{(l)})$ be the estimate of (β, θ) at the l-th stage of the EM algorithm, then $(\beta^{(l+1)}, \theta^{(l+1)})$ can be obtained by maximizing Equation (11) with respect to β and θ. For fixed β, the maximum of $\ln L_{complete}$ with respect to θ occurs at $\theta^{(l+1)}(\beta)$, where

$$\theta^{(l+1)}(\beta)=\left(\frac{\sum_{i=1}^{r}t_{i,f}^{\beta}+\sum_{j=1}^{m}z_{j,s}^{\beta}(\beta,\theta^{(l)})+\sum_{k=1}^{n}z_{k}^{\beta}(\beta,\theta^{(l)})}{r+m+n}\right)^{1/\beta}$$

(12)

This result indicates that $\theta^{(l+1)}(\beta)$ is unique and maximizes equation (11) with a given β. Moreover, $\beta^{(l+1)}$ can be obtained by maximizing $\ln L_{complete}$, the 'pseudo-profile log-likelihood function', with respect to β. Using similar argument as the Theorem 2 of Kundu (2008), it can be shown that $\ln L_{complete}(\beta,\theta^{(l+1)}(\beta))$ is an unimodal function of β, with an unique mode. Therefore, if $\beta^{(l+1)}$ maximizes $\ln L_{complete}(\beta,\theta^{(l+1)}(\beta))$, then $\beta^{(l+1)}$ is unique. The maximization of $\ln L_{complete}(\beta,\theta^{(l+1)}(\beta))$ with respect to β can be performed by solving a fixed point type equation $g^{(l)}(\beta)=\beta$, where

$$g^{(l)}(\beta)=(r+m+n)\left[\frac{\sum_{i=1}^{r}t_{i,f}^{\beta}\ln t_{i,f}+\sum_{j=1}^{m}z_{j,s}^{\beta}\ln z_{j,s}(\beta^{(l)},\theta^{(l)})+\sum_{k=1}^{n}z_{k}^{\beta}\ln z_{k}(\beta^{(l)},\theta^{(k)})}{\sum_{i=1}^{r}t_{i,f}^{\beta}+\sum_{j=1}^{m}z_{j,s}^{\beta}(\beta^{(k)},\theta^{(k)})+\sum_{l=1}^{n}z_{l}^{\beta}(\beta^{(k)},\theta^{(k)})}-\left(\sum_{i=1}^{r}\ln t_{i,f}+\sum_{j=1}^{m}\ln z_{j,s}(\beta^{(l)},\theta^{(l)})+\sum_{k=1}^{n}\ln z_{k}(\beta^{(l)},\theta^{(l)})\right)\right]^{-1}$$

(13)

Therefore, simple iterative process can be used to compute $(\beta^{(l+1)},\theta^{(l+1)})$ from $(\beta^{(l)},\theta^{(l)})$.

3.3 Bayesian

Following the approaches of Berger and Sun (1993) and Kundu (2008), when both the parameters are unknown, it is assumed that both β and θ have gamma priors and they are independently distributed.

$$\pi_1(\beta\,|\,a,b)=f_{GA}(\beta;a,b)=\frac{b^a}{\Gamma(a)}\beta^{a-1}e^{-b\beta},\beta>0$$

$$\pi_2(\theta\,|\,c,d)=f_{GA}(\theta;c,d)=\frac{d^c}{\Gamma(c)}\theta^{c-1}e^{-d\theta},\theta>0$$

The joint posterior distribution of θ and β is obtained as

$$\pi^*(\theta,\beta\,|\,t)=\frac{(\theta\beta)^{-1}\prod_{i=1}^{r}\frac{\beta t_{i,f}^{\beta-1}}{\theta^{\beta}}e^{-(\frac{t_{i,f}}{\theta})^{\beta}}\prod_{j=1}^{m}e^{-(\frac{t_{j,s}}{\theta})^{\beta}}\prod_{k=1}^{n}\left(e^{-(a_k/\theta)^{\beta}}-e^{-(b_k/\theta)^{\beta}}\right)}{\int_0^{\infty}\int_0^{\infty}(\theta\beta)^{-1}\prod_{i=1}^{r}\frac{\beta t_{i,f}^{\beta-1}}{\theta^{\beta}}e^{-(\frac{t_{i,f}}{\theta})^{\beta}}\prod_{j=1}^{m}e^{-(\frac{t_{j,s}}{\theta})^{\beta}}\prod_{k=1}^{n}\left(e^{-(a_k/\theta)^{\beta}}-e^{-(b_k/\theta)^{\beta}}\right)d\theta d\beta}.$$

(14)

Using Lindley's approximation, the posterior Bayes estimator of an arbitrary function $u(\theta,\beta)$ is derived as

$$E[u\,|\,t]=u+\frac{1}{2}[u_{11}\sigma_{11}+u_{22}\sigma_{22}]+u_1\rho_1\sigma_{11}+u_2\rho_2\sigma_{22}+\frac{1}{2}\left[L_{30}u_1\sigma_{11}^2+L_{03}u_2\sigma_{22}^2\right]$$

(15)

where

$$u(\theta)=\theta,u_1=\frac{\partial u}{\partial\theta}=1,u_{11}=\frac{\partial^2 u}{\partial\theta^2}=0,u(\beta)=\beta,u_2=\frac{\partial u}{\partial\beta}=1,u_{22}=\frac{\partial^2 u}{\partial\beta^2}=0$$

,

$$\rho(\theta,\beta)=-\ln(\theta)-\ln(\beta),\rho_1=\frac{\partial\rho}{\partial\theta}=-\frac{1}{\theta},\rho_2=\frac{\partial\rho}{\partial\beta}=-\frac{1}{\beta},$$

$$\sigma_{11}=\left(-\frac{\partial^2\ln L}{\partial\theta^2}\right)^{-1},\sigma_{22}=\left(-\frac{\partial^2\ln L}{\partial\beta^2}\right)^{-1},$$

$L_{30}=\frac{\partial^3\ln L}{\partial\theta^3}$, and $L_{03}=\frac{\partial^3\ln L}{\partial\beta^3}$. The second and third partial derivatives of equation (5) with respect to θ and β are given in Appendix 1.

4. EXAMPLES

Example 1: We analyzed the field return data for desktop computers from March 2011 to August 2012. Over the past years, 3,204,827 desktop computers have been sold, and the maintenance provider receives 132,292 of these computers for repair.

As shown in Table 1, the parameter estimates by the MLE via DE algorithm are obtained as $\hat{\beta}$ = 1.0499 and $\hat{\theta}$ = 5442.5. And the 95% confidence intervals of β and are θ are (1.0446, 1.0552) and (5358.54, 5526.47). The parameter estimates by the MLE via EM algorithm are obtained as $\hat{\beta}$ = 1.0499 and $\hat{\theta}$ = 5441.6. And the 95% confidence intervals of β and θ are (1.0446, 1.0552) and (5357.72, 5525.48). The Bayes estimates of β and by Lindley approximation method are $\hat{\beta}$ = 1.0499 and $\hat{\theta}$ = 5442.62. And the 95% confidence intervals of β and θ are (1.0473, 1.0524) and (5422.83, 5462.41). The estimates of β and θ by Markov Chain Monte Carlo method are $\hat{\beta}$ = 1.0499 and $\hat{\theta}$ = 5442.52. And the 95% confidence intervals of β and θ are (1.045, 1.055) and (5356, 5525).

The AIC values of the MLE via DE algorithm, MLE via EM algorithm, Lindley approximation and Markov Chain Monte Carlo are 1678349.845, 1678349.847, 1678349.845, and 1678349.845, respectively. Additionally, the MAPE values of the MLE via DE algorithm, MLE via EM algorithm, Lindley approximation and Markov Chain Monte Carlo are 1.5257%, 1.5262%, 1.5255%, and 1.5258%, respectively. These results indicate that the Lindley approximation method outperforms the other methods.

Example 2: We analyzed the field return data for desktop computers from July 2012 to December 2014. Over the past years, 3,556,433 desktop computers have been sold, and the maintenance provider receives 244,678 of these computers for repair.

As shown in Table 2, the MLE via DE algorithm of the Weibull model are obtained as $\hat{\beta}$ = 1.039909 and $\hat{\theta}$ = 6002.541, the approximate 95% confidence intervals are (1.0360, 1.0438) and (5941.91, 6063.17) with AIC = 3842887.6111; The MLE via EM algorithm are $\hat{\beta}$ = 1.04 and $\hat{\theta}$ = 6000.5, the confidence intervals are (1.0361, 1.0439) and (5939.95, 6061.05) with AIC = 3842887.6172; The Lindley approximation are $\hat{\beta}$ = 1.03991 and $\hat{\theta}$ = 6002.609, the confidence intervals are (1.0386, 1.0412) and (5993.88, 6011.34) with AIC=3842887.6112; The Markov Chain Monte Carlo are $\hat{\beta}$ = 1.03998 and $\hat{\theta}$ = 6002.83, the confidence intervals are (1.036, 1.044) and (5943, 6063) with AIC = 3842887.625 for desktop computers.

In this example, evaluation of MAPE under MLE via DE algorithm, MLE via EM algorithm, Lindley approximation and Markov Chain Monte Carlo are 1.1002%, 1.1009%, 1.1001%, 1.0992%, respectively. This result indicates that the Markov Chain Monte Carlo method performs well when it compares other methods in this study.

Table 1: Comparison results for example 1.

Methods	$\hat{\theta}$	$\hat{\beta}$	AIC	MAPE
MLE via DE algorithm	5442.50 (5358.54, 5526.47)	1.0499 (1.0446, 1.0552)	1678349.8452	1.5257%
MLE via EM algorithm	5441.6 (5357.72, 5525.48)	1.0499 (1.0446, 1.0552)	1678349.847	1.5262%
Lindley approximation	5442.62 (5422.83, 5462.41)	1.0499 (1.0473, 1.0524)	1678349.8453	1.5255%
Markov Chain Monte Carlo	5442.52 (5356, 5525)	1.0499 (1.045, 1.055)	1678349.8454	1.5258%

Table 2: Comparison results for example 2.

Methods	$\hat{\theta}$	$\hat{\beta}$	AIC	MAPE
MLE via DE algorithm	6002.5 (5941.91, 6063.17)	1.0399 (1.0360, 1.0438)	3842887.6111	1.1002%
MLE via EM algorithm	6000.5 (5939.95, 6061.05)	1.04 (1.0361, 1.0439)	3842887.6172	1.1009%
Lindley approximation	6002.6 (5993.88, 6011.34)	1.0399 (1.0386, 1.0412)	3842887.6112	1.1001%
Markov Chain Monte Carlo	6002.8 (5943, 6063)	1.0399 (1.036, 1.044)	3842887.625	1.0992%

5. CONCLUSIONS

This study used the field return data from a desktop product to investigate the reliability of desktop computers. The goodness-of-fit test performed on the field return data showed that Burr XII distribution was the most optimal for the failure probability distributions. In the desktop personal computer market, maintenance providers need determine the spare parts of desktop computers. However, the design capabilities and manufacturing technologies for desktop computers are continuously upgraded. Therefore, it is an important task to predict the failure rate of computers to reduce the costs for repairs. We believe that the results of this study can benefit the important market players, such as consumers, retailers, and manufacturers, regarding computer quality, sales strategy, after-sales warranty services packaging, and manufacturing process improvement.

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