# Optimal Sales Strategies for Dual Channel under Cooperation and Competition considering Customer Purchasing Preference 

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#### Abstract

For product sales under a dual channel(DC) comprising a retail channel(RC) and a direct online channel(DOC), it is necessary to consider customers' purchasing behaviors and preferences. This paper proposes the optimal sales strategy for DC under the situation where customer purchasing preference is unknown. This paper discusses three types of customers: (i) customers who prefer purchasing a single type of products in RC, (ii) customers who prefer purchasing them in DOC, (iii) indecisive customers who purchase in either RC or DOC. A retailer runs RC and determines the optimal retail price. A manufacturer runs DOC and determines the optimal direct online price. This paper assumes that each channel faces price-dependent demand. Two sales strategies are compared: the cooperated sales strategy(COSS) and the competed sales strategy(CMSS). Under COSS, a retailer and a manufacturer determine their prices cooperatively. Under CMSS, they determine their prices competitively. Using numerical examples, how (i) the uncertainty in customer purchasing preference, (ii) the existence ratio of indecisive customers, (iii) the sensitivity of demand by the difference between the retail price and the direct online price and (iv) the decrease ratio of the product demand for the increase in price, affect the optimal decisions under COSS and CMSS.


Keywords: supply chain management, e-commerce, dual-channel, cooperation and competition, customer purchasing preference

## 1. INTRODUCTION

E-commerce has become widespread and made rapid progress because of the commercialization of the Internet, the expansion of sales network. Under this situation, direct online sales where manufacturers sell products directly to customers through online channels have been increasing rapidly. As the sales method of products, a dual channel (DC) comprising a retail channel (RC) run by a retailer and a direct online channel (DOC) run by a manufacturer has become popular (Cai et al., 2009; Khouja et al., 2010; Xu et al., 2012; Huang et al., 2012). It is necessary for agents in DC to consider customer purchasing preference for products sales to operate DC profitably (Khouja et al., 2010).

Regarding this problem, Khouja et al. (2010) assumed
two types of customers: customers who preferred purchasing products through RC and customers who preferred purchasing them through DOC. However, as for customer purchasing preference, Khouja et al. (2010) did not discuss the existence ratio of indecisive customers who purchase the products in either RC or DOC in their analysis modeling. They also didn't incorporate bias for customer purchasing preference into analysis modeling regarding DC.

Differently from previous studies mentioned above, this paper incorporates (i) the existence ratio of indecisive customers and (ii) cooperation and competition between agents in DC into the optimal sales strategy. For product sales under DC comprising RC and DOC, it is necessary to consider customers' purchasing behaviors and preferences. This paper proposes the optimal sales strategy for a DC under the situation where customer purchasing preference is unknown. This paper discusses three types of customers:
(i) customers who prefer purchasing a single type of products in RC, (ii) customers who prefer purchasing them in DOC, (iii) indecisive customers who purchase in either RC or DOC. A retailer runs a RC and determines the optimal retail price. A manufacturer runs a DOC and determines the optimal direct online price. This paper assumes that each channel faces price-dependent demand. Two sales strategies are compared: the cooperated sales strategy (COSS) and the competed sales strategy (CMSS). Under COSS, a retailer and a manufacturer determine their prices cooperatively. Under CMSS, they determine their prices competitively. Using numerical examples, how (i) the uncertainty in customer purchasing preference, (ii) the existence ratio of indecisive customers, (iii) the sensitivity of demand by the difference between the retail price and the direct online price and (iv) the decrease ratio of the product demand for the increase in price, affect the optimal decisions under COSS and CMSS. The contribution of this paper provides managerial insights regarding the optimal sales strategies in DC considering (i) the existence ratio of indecisive customers and (ii) cooperation and competition between agents in DC by theoretical analysis.

## 2. MODEL DESCRIPTIONS

### 2.1 Operational Flows of a Dual Channel (DC)

(1) A manufacturer produces a single type of products with production cost $c$ per product and sells them to a retailer with wholesale price $w$ per product.
(2) The manufacturer sells the same products to customers with direct online price $P_{d}$ per product and then incurs total operational cost $Z_{d}$ in the direct online sale.
(3) A retailer sells the products to customers with retail price $P_{r}$ per product and incurs total operational cost $Z_{r}$ in the retail sale.
(4) After finishing sales of the products in DC, some products are returned to the retailer from RC at ratio $r_{r}$ and to the manufacturer from DOC at ratio $r_{d}$.
(5) The manufacturer buys back the returned products in RC from the retailer with buy-back price $b$ per product. The manufacturer sells all the returned products with disposal price $s$ per product in a second market.

### 2.2 Model Assumptions

(1) The customer purchasing preference $x$ follows a probability distribution. The probability density function of $x$ is $f(x)$. The closer to $0 x$ is, the higher customer purchasing preference through the

DOC is. Also, the closer to $1 x$ is, the higher customer purchasing preference through the RC is.
(2) Using index $t$ of the existence ratio of indecisive customers and the standard deviation $\sigma$ of $x$, the indecisiveness of customers is expressed as $t \sigma$. Therefore, customers with purchasing preferences through DOC are distributed in $0 \leq x \leq 0.5-t \sigma$ and indecisive customers, who purchase in either RC or DOC, are distributed in $0.5-t \sigma \leq x \leq 0.5+t \sigma$, and customers with purchasing preferences through RC are distributed in $0.5+t \sigma \leq x \leq 1$. In this case, the expected ratio of customers $E_{d}$ with purchasing preferences through the DOC, the expected ratio of indecisive customers $Y$, and the expected ratio of customers $E_{r}$ with purchasing preferences through the RC are calculated as

$$
\begin{align*}
& E_{d}=\int_{0}^{0.5-t \sigma} f(x) d x  \tag{1}\\
& Y=\int_{0.5-t \sigma}^{0.5+\sigma} f(x) d x  \tag{2}\\
& E_{r}=\int_{0.5+\sigma}^{1} f(x) d x . \tag{3}
\end{align*}
$$

From Eqs. (1)-(3), demands in both RC and DOC are affected by the probability distribution regarding customer purchasing preference.
(3) The demand of indecisive customers is influenced by sensitivity $\ell$ of demand by the price difference between retail price $P_{r}$ and direct online price $P_{d}$.
(4) Demands in both RC and DOC decrease at the decrease rate $m$ as the individual sales price increases.

## 3. MODEL FORMULATIONS IN DC

### 3.1 Demands of the Products in RC and DOC

Formulations of the product demands in RC and DOC are discussed. Denote $A$ as market volume (potential demand) of the products, $X_{r}$ as $E_{r}+0.5 Y$ and $X_{d}$ as $E_{d}+0.5 Y$.

From subsection 2.2, the product demand $D_{r}$ in RC in terms of the retail price $P_{r}$ and the direct online price $P_{d}$ is formulated as

$$
\begin{equation*}
D_{r}=A X_{r}-A \ell Y\left(P_{r}-P_{d}\right)-m P_{r} . \tag{4}
\end{equation*}
$$

Here, $D_{r}$ in Eq.(4) is obtained as the sum of the product demand in RC not influenced by $P_{r}$ and $P_{d}$ (first term), the product demand of indecisive customers fluctuating as to the price difference between $P_{r}$ and $P_{d}$ (second term), and the product demand in RC decreasing as to $P_{r}$ (third term).

Similarly, the product demand $D_{d}$ in DOC in terms of $P_{r}$ and $P_{d}$ is formulated as
$D_{d}=A X_{d}-A \ell Y\left(P_{d}-P_{r}\right)-m P_{d}$.

## 3.2 the Expected Profits of a Retailer and a Manufacturer

First, the expected profit of a retailer is discussed. From subsection 3.1, the expected profit of the retailer $\Pi_{R}\left(P_{r}, P_{d}\right)$ for $P_{r}$ and $P_{d}$ is calculated as

$$
\begin{equation*}
\Pi_{R}\left(P_{r}, P_{d}\right)=P_{r} D_{r}+b r_{r} D_{r}-w D_{r}-P_{r} r_{r} D_{r}-Z_{r} . \tag{6}
\end{equation*}
$$

Next, the expected profit of a manufacturer is discussed. From subsection 3.1, the expected profit of the manufacturer $\Pi_{M}\left(P_{r}, P_{d}\right)$ for $P_{r}$ and $P_{d}$ is calculated as

$$
\begin{array}{r}
\Pi_{M}\left(P_{r}, P_{d}\right)=P_{d} D_{d}+w D_{r}+s\left(\mathrm{r}_{r} \mathrm{D}_{r}+\mathrm{r}_{d} \mathrm{D}_{d}\right) \\
-b r_{r} D_{r}-P_{d} r_{d} D_{d}-c\left(D_{r}+D_{d}\right)-Z_{d} . \tag{7}
\end{array}
$$

## 4. OPTIMAL SALES STRATEGY IN DC

### 4.1 Optimal Price Decisions under COSS

Under COSS, a retailer and a manufacturer cooperatively determine the optimal retail price and optimal direct online price by the following decision procedures.
[Step 1] The first-second order partial differential equations of the retailer's expected profit in Eq. (6) in terms of the retail price $P_{r}$ under the direct online price $P_{d}$ are derived as
$\frac{\partial \Pi_{R}\left(P_{r}, P_{d}\right)}{\partial P_{r}}=-2(A \ell Y+m)\left(1-r_{r}\right) P_{r}$
$+\left(1-r_{r}\right) X_{r}+\left(w-b r_{r}\right)(A \ell Y+m)+\left(1-r_{r}\right) A \ell Y P_{d}$
$\frac{\partial^{2} \Pi_{R}\left(P_{r}, P_{d}\right)}{\partial P_{r}^{2}}=-2(A l Y+m)\left(1-r_{r}\right)<0$
$\left(\because(A \ell Y+m)>0,\left(1-r_{r}\right)>0\right)$.
Similarly, those of the manufacturer's expected profit in
Eq. (7) in terms of $P_{d}$ under $P_{r}$ are derived as

$$
\begin{align*}
& \frac{\partial \Pi_{M}\left(P_{r}, P_{d}\right)}{\partial P_{d}}=-2(A \ell Y+m)\left(1-r_{d}\right) P_{d}+\left(1-r_{d}\right) X_{d} \\
& -\left(s r_{d}-c\right)(A \ell Y+m)+\left(w-b r_{r}+s r_{r}-s r_{d}\right) A \ell Y \\
& +\left(1-r_{d}\right) A \ell Y P_{r}  \tag{10}\\
& \frac{\partial^{2} \Pi_{M}\left(P_{d}, P_{r}\right)}{\partial P_{d}^{2}}=-2(A \ell Y+m)\left(1-r_{d}\right)<0  \tag{11}\\
& \left(\because(A \ell Y+m)>0,\left(1-r_{d}\right)>0\right) .
\end{align*}
$$

Here, it is verified that Eqs. (9) and (11) are negative from the following conditions: $(A \ell Y+m)>0,\left(1-r_{r}\right)>0$ and $\left(1-r_{d}\right)>0$. Therefore, Eq. (6) is a concave function for $P_{r}$ under $P_{d}$, and Eq. (7) is that for $P_{d}$ under $P_{r}$.
[Step 2] The tentative retail price $P_{r}\left(P_{d}\right)$ under $P_{d}$ is obtained as $P_{r}$, satisfying $\partial \Pi_{R}\left(P_{r}, P_{d}\right) / \partial P_{r}=0$ as

$$
\begin{equation*}
P_{r}\left(P_{d}\right)=\frac{X_{r}}{2(A \ell Y+m)}+\frac{w-b r_{r}}{2\left(1-r_{r}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{d} . \tag{12}
\end{equation*}
$$

Similarly, the tentative direct online price $P_{d}\left(P_{r}\right)$ under $P_{r}$ is obtained as $P_{d}$, satisfying $\partial \Pi_{M}\left(P_{r}, P_{d}\right) / \partial P_{d}=0$ as

$$
\begin{align*}
P_{d}\left(P_{r}\right) & =\frac{X_{d}}{2(A \ell Y+m)}-\frac{s r_{d}-c}{2\left(1-r_{d}\right)} \\
& +\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right) A \ell Y}{2(A \ell Y+m)\left(1-r_{d}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{r} \tag{13}
\end{align*}
$$

[Step 3] The optimal retail price $P_{r}^{\text {coss* }}$ and the optimal direct online price $P_{d}{ }^{\text {coss** }}$ are determined as solutions of simultaneous equations in Eqs. (12) and (13) as

$$
P_{r}^{\text {coss* }^{*}}=\frac{4(A \ell Y+m)^{2}}{3(A \ell Y+m)^{2}+2 m A \ell Y+m^{2}}
$$

$$
\times\left[\begin{array}{l}
\frac{X_{r}}{2(A \ell Y+m)}+\frac{w-b r_{r}}{2\left(1-r_{r}\right)}+\frac{A \ell Y}{2(A \ell Y+m)}  \tag{14}\\
\times\left\{\frac{X_{d}}{2(A \ell Y+m)}-\frac{s r_{d}-c}{2\left(1-r_{d}\right)}+\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right)}{2\left(1-r_{d}\right)(A \ell Y+m)}\right\}
\end{array}\right]
$$

$$
P_{d}^{\text {coss* }}=\frac{4(A \ell Y+m)^{2}}{3(A \ell Y+m)^{2}+2 m A \ell Y+m^{2}}
$$

$$
\times\left[\begin{array}{l}
\frac{X_{d}}{2(A \ell Y+m)}-\frac{s r_{d}-c}{2\left(1-r_{d}\right)}+\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right)}{2\left(1-r_{d}\right)(A \ell Y+m)}  \tag{15}\\
+\frac{A \ell Y}{2(A \ell Y+m)}\left\{\frac{X_{r}}{2(A \ell Y+m)}+\frac{w-b r_{r}}{2\left(1-r_{r}\right)}\right\}
\end{array}\right]
$$

### 4.2 Optimal Price Decisions under CMSS

Under CMSS, a retailer and a manufacturer competitively determine the optimal retail price and the optimal direct online price. this paper adopts the decisionmaking approach in Stackelberg game (Cachon, G.P. and Netessine, S., 2004). In the Stackelberg game, the leader of the decision-making determines the own price optimally so as to maximize the own expected profit. The follower of the decision-making determines the own price optimally so as to maximize the own expected profit under the optimal price determined by the leader. This paper discusses two types of CMSSs: CMSS 1: a retailer is the leader and a manufacturer is the follower and CMSS 2: a manufacturer is the leader and a retailer is the follower.

### 4.2.1 Optimal Price Decisions under CMSS 1

The decision procedures under CMSS 1 is shown below.
[Step 1] A retailer, who is the leader of the decisionmaking, determines the tentative retail price $P_{r}\left(P_{d}\right)$ which maximizes the own expected profit under the direct online price $P_{d} . P_{r}\left(P_{d}\right)$ is obtained as the following solution:
$P_{r}\left(P_{d}\right)=\frac{X_{r}}{2(A \ell Y+m)}+\frac{w-b r_{r}}{2\left(1-r_{r}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{d}$
, satisfying $\partial \Pi_{R}\left(P_{r}, P_{d}\right) / \partial P_{r}=0$ under $P_{d}$.
[Step 2] Substituting the tentative retail price $P_{r}\left(P_{d}\right)$ in Eq. (16) into Eq.(7), the manufacturer's expected profit is rewritten as
$\Pi_{M}\left(P_{r}\left(P_{d}\right), P_{d}\right)=\left(1-r_{d}\right) k_{d 1} P_{d}{ }^{2}$
$+\left\{\left(1-r_{d}\right) n_{d 1}+\left(s r_{d}-c\right) k_{d 1}+\left(w-b r_{r}+s r_{r}-s r_{d}\right) k_{r 1}\right\} P_{d}$
$+\left(s r_{d}-c\right) n_{d 1}+\left(w-b r_{r}+s r_{r}-s r_{d}\right) n_{r 1}-Z_{d}$
$=a_{d} P_{d}{ }^{2}+b_{d} P_{d}+c_{d}$.
Here, $k_{d 1}, n_{d 1}, k_{r 1}, n_{r 1}, a_{d}, b_{d}$ and $c_{d}$ in Eq.(17) are defined as
$k_{d 1}=-(A \ell Y+m)+\frac{(A \ell Y)^{2}}{2(A \ell Y+m)}$
$n_{d 1}=X_{d}+\frac{A \ell Y}{2(A \ell Y+m)} X_{r}+\frac{\left(w-b r_{r}\right) A \ell Y}{2\left(1-r_{r}\right)}$
$k_{r 1}=\frac{1}{2} A \ell Y$
$n_{r 1}=\frac{1}{2} X_{r}-\frac{\left(w-b r_{r}\right)(A \ell Y+m)}{2\left(1-r_{r}\right)}$
$a_{d}=\left(1-r_{d}\right) k_{d 1}$
$b_{d}=\left(1-r_{d}\right) n_{d 1}+\left(s r_{d}-c\right) k_{d 1}+\left(w-b r_{r}+s r_{r}-s r_{d}\right) k_{r 1}$
$c_{d}=\left(s r_{d}-c\right) n_{d 1}+\left(w-b r_{r}+s r_{r}-s r_{d}\right) n_{r 1}-Z_{d}$.
[Step 3] The manufacturer determines the optimal direct online price in CMSS $1 P_{d}^{\text {CMSS } 1^{*}}$ under the tentative retail price $P_{r}\left(P_{d}\right)$ so as to maximize the expected profit in DOC. Eq. (17) is a quadratic function in terms of $P_{d}$. From the characteristic, the optimal direct online price under CMSS $1 P_{d}^{\text {CMSS }}$ * is determined as
$P_{d}^{\text {CMSS }{ }^{*}}=-b_{d} /\left(2 a_{d}\right)$.
[Step 4] Substituting $P_{d}^{\text {CMSS }}{ }^{*}$ in Eq.(18) into Eq.(16), the optimal retail price under CMSS $1 P_{r}^{\text {CMSS }}{ }^{*}$ which maximizes the expected profit in RC is determined as
$P_{r}^{\text {CMSS } 1^{*}}=\frac{X_{r}}{2(A \ell Y+m)}+\frac{w-b r_{r}}{2\left(1-r_{r}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{d}^{\text {CMSS } \text { I* }^{*}}$.

### 4.2.2 Optimal Price Decisions under CMSS 2

The decision procedures under CMSS 2 is shown below.
[Step 1] A manufacturer, who is the leader of the decisionmaking, determines the tentative retail price $P_{d}\left(P_{r}\right)$ which maximizes the own expected profit under the retail price $P_{r} . P_{d}\left(P_{r}\right)$ is obtained as the following solution:

$$
\begin{align*}
P_{d}\left(P_{r}\right) & =\frac{X_{d}}{2(A \ell Y+m)}-\frac{s r_{d}-c}{2\left(1-r_{d}\right)} \\
& +\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right) A \ell Y}{2(A \ell Y+m)\left(1-r_{d}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{r} \tag{20}
\end{align*}
$$

, satisfying $\partial \Pi_{M}\left(P_{r}, P_{d}\right) / \partial P_{d}=0$ under $P_{r}$.
[Step 2] Substituting the tentative retail price $P_{d}\left(P_{r}\right)$ in Eq. (20) into Eq.(6), the retailer's expected profit is rewritten as

$$
\begin{align*}
& \Pi_{R}\left(P_{r}, P_{d}\left(P_{r}\right)\right)=\left(1-r_{r}\right) k_{r 2} P_{r}^{2} \\
& +\left\{\left(1-r_{r}\right) n_{r 2}-\left(w-b r_{r}\right) k_{r 2}\right\} P_{r}-\left(w-b r_{r}\right) n_{r 2}-Z_{r} \\
& \quad=a_{r} P_{d}^{2}+b_{r} P_{d}+c_{r} . \tag{21}
\end{align*}
$$

Here, $k_{r 2}, n_{r 2}, a_{r}, b_{r}$ and $c_{r}$ in Eq.(21) are defined as
$k_{r 2}=-(A \ell Y+m)+\frac{(A \ell Y)^{2}}{2(A \ell Y+m)}$
$n_{r 2}=X_{r}+\frac{A \ell Y}{2(A \ell Y+m)} X_{d}-\frac{\left(s r_{d}-c\right) A \ell Y}{2\left(1-r_{d}\right)}$

$$
+\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right)(A \ell Y)^{2}}{2(A \ell Y+m)\left(1-r_{d}\right)}
$$

$a_{r}=\left(1-r_{r}\right) k_{r 2}$
$b_{r}=\left(1-r_{r}\right) n_{r 2}-\left(w-b r_{r}\right) k_{r 2}$
$c_{r}=-\left(w-b r_{r}\right) n_{r 2}-Z_{r}$.
[Step 3] The retailer determines the optimal retail price in CMSS $2 P_{r}^{\text {CMSS2** }}$ under the tentative direct online price $P_{d}\left(P_{r}\right)$ so as to maximize the expected profit in RC. Eq.(21) is a quadratic function in terms of $P_{r}$. From the characteristic, the optimal retail price under CMSS 2 $P_{r}^{\text {CMSS 2* }}$ is determined as

$$
\begin{equation*}
P_{r}^{\text {CMSS 2* }}=-b_{r} /\left(2 a_{r}\right) . \tag{22}
\end{equation*}
$$

[Step 4] Substituting $P_{r}^{\text {CMSS2* }}$ in Eq.(22) into Eq.(20), the optimal direct online price in CMSS $2 P_{d}^{\text {CMSS2* }}$ which maximizes the expected profit in DOC is determined as
$P_{d}^{\text {CMSS 2* }}=\frac{X_{d}}{2(A \ell Y+m)}-\frac{s r_{d}-c}{2\left(1-r_{d}\right)}$
$+\frac{\left(w-b r_{r}+s r_{r}-s r_{d}\right) A \ell Y}{2(A \ell Y+m)\left(1-r_{d}\right)}+\frac{A \ell Y}{2(A \ell Y+m)} P_{r}^{\text {CMSS2* }}$.

## 5. NUMERICAL SIMULATIONS

The numerical analysis illustrates the results of the optimal decisions under the strategies, COSS and CMSS. The expected profits of a retailer and a manufacturer under COSS are compared with those under CMSS. In addition, the analysis clarifies numerically how (i) the uncertainty in customer purchasing preference, (ii) the existence ratio of indecisive customers, (iii) the sensitivity of demand by the difference between the retail price and the direct online


Figure 1: Distribution of customer purchasing preference $x$ in Case $1 \sim$ Case6
price and (iv) the decrease ratio of the product demand for the increase in price, affect the optimal decisions under COSS and CMSS.

The following system parameters are used as the numerical examples:

$$
\begin{aligned}
& A=100000, w=300, b=50, s=20, c=40, r_{r}=0.01, r_{d}=0.10, \\
& Z_{r}=300000, Z_{d}=800000, t=1.5, \ell=0.01, m=65 .
\end{aligned}
$$

Customer's purchasing preference $x$ is modeled by using the beta distribution with shape parameter $m$ and scale parameter $n$. The probability density function $f(x \mid m, n)$ of $x(0 \leq x \leq 1)$ is given as
$f(x \mid m, n)=(\Gamma(m+n) / \Gamma(m) \Gamma(n)) x^{m-1}(1-x)^{n-1}$
where $\Gamma(\square)$ denotes the gamma function.
In numerical examples, the following six cases of the combination of ( $m, n$ ) regarding $x$ are considered:
Case1: $B(x \mid 1.5,4.5)$ assuming very higher purchasing preference through DOC
Case2: $B(x \mid 3.5,4.5)$ assuming relatively higher purchasing preference through DOC
Case3: $B(x \mid 1,1)$ assuming that $x$ is uniformly distributed within the range where $0 \leq x \leq 1$
Case4: $B(x \mid 5,5)$ assuming that $x$ is distributed like normal distribution, which has no biases for purchasing preference, within the range where $0 \leq x \leq 1$
Case5: $B(x \mid 4.5,3.5)$ assuming relatively higher purchasing preference through RC
Case6: $B(x \mid 4.5,1.5)$ assuming very higher purchasing preference through RC.

Figure 1 shows the distribution of customer's purchasing preference $x$ in Case $1 \sim$ Case6. Figure 1 indicates that the closer to $0 x$ is, the higher customer purchasing preference through the DOC is, meanwhile, the closer to $1 x$ is, the higher customer purchasing preference through the RC is.

By substituting numerical examples into Eqs. (1), (2), and (3), the expected ratios of customers with purchasing preference $E_{d}, Y$, and $E_{r}$ are calculated as

$$
\begin{aligned}
& \text { Case1: } E_{d}=0.5732, Y=0.4219, E_{r}=0.0049 \\
& \text { Case2: } E_{d}=0.1413, Y=0.8256, E_{r}=0.0331 \\
& \text { Case3: } E_{d}=0.0670, Y=0.8660, E_{r}=0.0670 \\
& \text { Case4: } E_{d}=0.0699, Y=0.8601, E_{r}=0.0699 \\
& \text { Case5: } E_{d}=0.0331, Y=0.8256, E_{r}=0.1413 \\
& \text { Case6: } E_{d}=0.0049, Y=0.4219, E_{r}=0.5732 .
\end{aligned}
$$

It can be seen that values of $E_{d}, Y$ and $E_{r}$ in Case 1 are symmetrical about those in Case 6. The combinations of both (Case 2, Case 5) and (Case 3, Case 4) are almost the same relations. Therefore, numerical analysis in aftermentioned 5.1 and 5.2 are conducted by using the expected ratios of customers of Cases 1, 2 and 4. Regarding Tables 1 $\sim 4$ below, the loss of demand in the whole system means the sum of the loss of demand in RC and that in DOC.

### 5.1 Effect of Uncertainty in Customer purchasing preference on the Optimal Price Decisions under Each Sales Strategy and the Expected Profits

Table 1 shows the effect of customer purchasing preference on the optimal price decisions under COSS, CMSS 1 and CMSS 2 and the expected profits in DC.

From Table 1, the following results can be seen: The optimal direct online price under COSS, CMSS 1 and CMSS 2 are the highest in Case 1 which has very higher customer purchasing preference through DOC. This leads to results that the expected profits in DOC under COSS, CMSS 1 and CMSS 2 are the highest in Case 1. In addition, the difference between the optimal retail price and the optimal direct online price in COSS becomes smaller, as the distribution of customer purchasing preference changes from Case 1 through Case 2 to Case 4 . This is because the smaller the bias of the distribution of customer purchasing preference is, not only the more the expected ratio of indecisive customers are, but also the more intensified price competition between a retailer and a manufacturer is.

### 5.2 Comparison of Benefits of Optimal Price Decisions under Each Sales Strategy

Benefits of the optimal price decisions under COSS, CMSS 1 and CMSS 2 are compared.

Table 1 : Influence of customer purchasing preference on the optimal price decisions and the expected profits in DC

| Preference case | Sales strategy | Optimal $P_{r}$ | Optimal $P_{d}$ | Expected profit |  | Loss of demand |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Retailer | Manufacturer | Whole system |
| 1 | COSS | 344 | 394 | 535130 | 15287148 | 47983 |
|  | CMSS 1 | 357 | 399 | 582241 | 15157956 | 49148 |
|  | CMSS 2 | 340 | 385 | 386171 | 15309261 | 47145 |
|  | COSS | 346 | 366 | 1356484 | 15100461 | 46262 |
|  | CMSS 1 | 362 | 373 | 1490544 | 14743706 | 47815 |
|  | CMSS 2 | 340 | 353 | 943162 | 15171917 | 45073 |
| 4 | COSS | 347 | 363 | 1525403 | 15112835 | 46180 |
|  | CMSS 1 | 364 | 371 | 1675541 | 14708289 | 47798 |
|  | CMSS 2 | 341 | 350 | 1057793 | 15195319 | 44922 |

From Table 1, the following results can be seen: Regardless of the distribution of customer purchasing preference, the expected profits of a retailer and a manufacturer have the following magnitude relations between COSS, CMSS 1 and CMSS 2. For a retailer, $($ CMSS 1$)>(\mathrm{COSS})>(\mathrm{CMSS} 2)$, meanwhile, for a manufacturer, (CMSS 2) $>($ COSS $)>$ (CMSS 1). These results lead to the following consequence regarding benefits of the optimal price decisions under COSS and CMSS. (CMSS where the own agent is the leader of the decision-making $)>($ COSS $)>($ CMSS where the own agent is the follower of the decision-making). Therefore, CMSS is the most beneficial sales strategy for a retailer and a manufacturer in the situation where they can determine which agent is the leader of the decision-making. In contrast, COSS is the most profitable sales strategy for a retailer and a manufacturer in the situation where they cannot determine which agent is the leader of the decisionmaking.

### 5.3 Effect of Existence Ratio of Indecisive Customers on the Optimal Price Decision under Each Sales Strategy and the Expected Profits

Table 2 shows the effect of the index $t$ of the existence ratio of indecisive customers on the optimal price decisions and the expected profits in DC under each sales strategy. Here, Case 4 of the distribution of customer purchasing preference is adopted. From Table 2, the following results can be seen:

- The higher $t$ is, the lower the optimal retail price and the optimal direct online price are. This is because the higher $t$ is, the higher the expected ratio of indecisive customers, $Y$, is and then the increase in $Y$ leads to the situation where price competition between the optimal retail price determined by a retailer and the optimal direct online price determined
by a manufacturer is more intensified. From the results on the optimal price decisions, the higher $t$ is, the lower the loss of demand in the whole system is.
- Regardless of $t$, the magnitude relation between the expected profits of the retailer and the manufacturer under each sales strategy is the same as that in 5.2.


### 5.4 Effect of Sensitivity $\ell$ in Price Difference on the Optimal Price Decisions under Each Sales Strategy and the Expected Profits

Table 3 shows the effect of the sensitivity $\ell$ in price difference on the optimal price decisions and the expected profits in DC under each sales strategy. Here, Case 4 of the distribution of customer purchasing preference is adopted. From Table 3, the following results can be seen:

- The higher $\ell$ is, the lower the optimal retail price and the optimal direct online price are. This is because the increase in $\ell$ leads to the situation where price competition between the optimal retail price determined by a retailer and the optimal direct online price determined by a manufacturer is more intensified. From the results on the optimal price decisions, the higher $\ell$ is, the lower the loss of demand in the whole system is.
- Regardless of $\ell$, the magnitude relation between the expected profits of the retailer and the manufacturer under each sales strategy is same as that in 5.2.


### 5.5 Effect of Decrease Ratio $m$ on the Optimal Price Decisions under Each Sales Strategy and the Expected Profits

Table 4 shows the effect of decrease ratio $m$ of demand on the optimal price decisions and the expected profits in DC under each sales strategy. Here, Case 4 of the distribution

Table 2: Effect of index $t$ of existence ratio of indecisive customers on the optimal price decisions and the expected profits

| $t$ | Sales strategy | Optimal $P_{r}$ | Optimal $P_{d}$ | Expected profit |  | Loss of demand |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Manufacturer | Whole system |  |
| 1.5 | COSS | 347 | 363 | 1525403 | 15112835 | 46180 |
|  | CMSS 1 | 364 | 371 | 1675541 | 14708289 | 47798 |
|  | CMSS 2 | 341 | 350 | 1057793 | 15195319 | 44922 |
| 2.0 | COSS | 345 | 362 | 1548684 | 15184300 | 45968 |
|  | CMSS 1 | 362 | 370 | 1707169 | 14759925 | 47558 |
|  | CMSS 2 | 339 | 349 | 1057742 | 15272246 | 44725 |

Table 3 : Effect of the sensitivity $\ell$ in price difference on the optimal price decisions and the expected profits

| $\ell$ | Sales strategy | Optimal $P_{r}$ | Optimal $P_{d}$ | Expected profit |  | Loss of demand |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Manufacturer | Whole system |  |
| 0.010 | COSS | 347 | 363 | 1525403 | 15112835 | 46180 |
|  | CMSS 1 | 364 | 371 | 1675541 | 14708289 | 47798 |
|  | CMSS 2 | 341 | 350 | 1057793 | 15195319 | 44922 |
| 0.015 | COSS | 341 | 359 | 1654277 | 15392308 | 45492 |
|  | CMSS 1 | 357 | 367 | 1837778 | 14901792 | 47010 |
|  | CMSS 2 | 335 | 347 | 1087148 | 15498665 | 44281 |

Table 4 : Effect of decrease ratio $m$ of demand on the optimal price decisions and the expected profits

| $m$ | Sales strategy | Optimal $P_{r}$ | Optimal $P_{d}$ | Expected Profit |  | Loss of demand |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Manufacturer | Whole system |  |
| 20 | COSS | 372 | 393 | 3852305 | 24844452 | 15285 |
|  | CMSS 1 | 403 | 408 | 4305389 | 25326068 | 16225 |
|  | CMSS 2 | 374 | 398 | 4181293 | 24856926 | 15449 |
| 35 | COSS | 363 | 382 | 2926868 | 21372330 | 26081 |
|  | CMSS 1 | 389 | 395 | 3246255 | 21413561 | 27422 |
|  | CMSS 2 | 362 | 381 | 2835924 | 21373708 | 25989 |
| 50 | COSS | 355 | 373 | 2158141 | 18137520 | 36364 |
|  | CMSS 1 | 376 | 382 | 2379586 | 17897181 | 37915 |
|  | CMSS 2 | 351 | 365 | 1817149 | 18166192 | 35778 |
| 65 | COSS | 347 | 363 | 1525403 | 15112835 | 46180 |
|  | CMSS 1 | 364 | 371 | 1675541 | 14708289 | 47798 |
|  | CMSS 2 | 341 | 350 | 1057793 | 15195319 | 44922 |

of customer purchasing preference is adopted. From Table 4, the following results can be seen:
The higher $m$ is, the lower the optimal retail price and the optimal direct online price are. This is because the increase in $m$ means the increase in the loss of demand in the whole system. To avoid it, a retailer and a manufacturer determine their prices lower.

- The higher $m$ is, the higher the loss of the demand in the whole system is. This is because the increase in the loss of demand by increasing $m$ is higher than the increase in the product demand by determining the optimal retail price and the optimal direct online price lower.
- Regarding benefits of optimal price decisions under each sales strategy, the following different results are obtained as to the range of $m$. Two results about the expected profits of the retailer and the manufacturer.
(1) When $m$ is low, the expected profits of the retailer and the manufacturer are the highest under CMSS 1 where the retailer is the leader of the decision-making.
(2) When $m$ is high, the magnitude relation between the expected profits of the retailer and the manufacturer under each sales strategy is the same as that in 5.2.


## 6. CONCLUSIONS

This paper discussed the optimal sales strategies for a dual channel under cooperation and competition in a retail channel (RC) and a direct online channel (DOC), considering customer purchasing preference between both channels.

This paper proposed two sales strategies: the cooperated sales strategy (COSS) between two agents: a retailer and a manufacturer and the competed sales strategy (CMSS) between the two agents. Under COSS, the optimal decisions for retail price and direct online price were made so as to maximize the expected profits of the retailer in RC and the manufacturer in DOC. Under CMSS, this paper adopted the decision-making approach in Stackelberg game. Concretely, this paper discussed two types of CMSSs: CMSS 1: a retailer is the leader of the decision-making and a manufacturer is the follower of the decision-making and CMSS 2: a manufacturer is the leader of the decisionmaking and a retailer is the follower of the decision-making.

The analysis clarified numerically how (i) the uncertainty in customer purchasing preference, (ii) the existence ratio of indecisive customers, (iii) the sensitivity in demand by the difference between retail price and direct online price and (iv) the decrease ratio of the products demand for increase of price, affected the optimal decisions under COSS and CMSS.

Results of theoretical analysis and numerical analysis in this paper verified the following managerial insights:

- When the bias of the customer purchasing preference is large towards either RC or DOC, the agent who has the high bias determines the own optimal price highly.
- The smaller the bias of the customer purchasing preference is, the lower the deference between the optimal retail price and the optimal direct online price is.
- COSS is the most profitable sales strategy for two agents: a retailer and a manufacturer in the situation where they cannot determine which agent is the leader of the decision-making.
- CMSS is the most beneficial sales strategy for two agents: a retailer and a manufacturer in the situation where they can determine which agent is the leader of the decision-making.

As future researches, it will be necessary to incorporate the following topics into the DC in this paper:

- Effect of difference in lead time between a retail channel and a direct online channel
- Effect of advertising in a DC
- Customer purchasing preference considering a brand strength as well as a price difference between agents
- Proposal of the more beneficial cooperated sales strategy
- Time limit for sales considering the product life cycle
- DC model with multiple retailers and multiple manufacturers


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