Optimal Sales Strategy for Seasonal Demand with Product Life Cycle considering Markdown Sales and Price Sensitivity

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Abstract. For sales of seasonal products such as mobile phones, PCs and apparels in the primary market, it is necessary to consider handling of the unsold products and seasonal demands with product life cycle. This paper presents the optimal sales strategy for a retailer who sells a single product with seasonal demand in the primary market with a three-phase product life cycle: the growth phase, the mature phase and the decline phase in a single selling period. Seasonal demand in each phase is time-varying. This paper discusses incorporation of markdown sales into the decline phase as sales effort by the retailer. This paper determines the optimal sales strategy regarding the length of seasonal interval and the markdown ratio in the decline phase with price sensitivity so as to maximize the retailer's profit. Using numerical examples, the benefit of markdown sales in the decline phase is shown by comparing the retailer's profit with and without markdown sales in the decline phase. The sensitivity analysis is conducted to illustrate how degree of price sensitivity in demand for markdown ratio, demand pattern in the decline phase, the unit purchasing cost and the unit inventory holding cost per time affect the optimal sales strategy.

Keywords: seasonal demand, product life cycle, markdown sales, price sensitivity in demand, optimal sales strategy

1. INTRODUCTION

For sales of seasonal products such as mobile phones, PCs and apparels in the primary market, it is necessary to consider handling of the unsold products and seasonal demands with product life cycle. There are some previous papers regarding handling the unsold products in a primary market. Lee (2009) and Kusukawa (2014) proposed the optimal sales strategy for order quantity and markdown ratio considering a primary market and a markdown market. However, in these previous papers, the optimal sales strategy for seasonal products with product life cycle were not discussed. Regarding this topic, Chen and Cheng (2006) reported that demand of seasonal product had mainly a three-phase product life cycle which consisted of the growth phase, the mature phase and the decline phase and the length of each phase depends on each product.

It is necessary for manufacturers and retailers to consider change in demand of seasonal products with the product life cycle and determine the optimal sales strategy for seasonal products with product life cycle.

Regarding the optimal sales strategy for seasonal products with the product life cycle, there are some previous papers (Chen and Cheng, C. T., 2006; Chen et al., 2007a; Chen et al., 2007b; Chen et al., 2007c; Banerjee and Sharma, 2010). Banerjee and Sharma (2010) proposed the optimal inventory-sales strategy in consideration of a three-phase product life cycle and price sensitivity of demand of seasonal products. In this previous paper, sales of seasonal products in a primary market and a markdown market during multiple sales periods were considered. The optimal sales strategy for sales epoch and sales price for seasonal products in the primary market were determined under a given sales price and a given markdown ratio in a markdown market in each sales period. However, in this previous paper, seasonal products were sold until the end of the decline phase and sales price of seasonal products in the decline phase is as same as that in grow phase and mature phase. Also, markdown sales of product were made in a markdown market, not in a primary market. In this markdown sales, markdown ratio was given exogenously.

The motivation of this paper is to incorporate product life cycle and markdown sales of seasonal products into modelling and the optimal sales strategy for sales termination time and markdown ratio in the decline phase in a primary market. Concretely, this paper presents the optimal sales strategy for a retailer who sells a single product with seasonal demand in a primary market with a three-phase product life cycle: the growth phase, the mature phase and the decline phase in a single selling period. Seasonal demands in each phase is time-varying. This paper discusses incorporation of markdown sales into the decline phase as sales effort by the retailer. This paper determines the optimal sales strategy for sales termination time and markdown ratio in the decline phase with price sensitivity so as to maximize the retailer's profit. Using numerical examples, the benefit of markdown sales in the decline phase is shown by comparing the retailer's profit with and without markdown sales in the decline phase. The sensitivity analysis is conducted to illustrate how degree of price sensitivity in demand for markdown ratio, demand pattern in the decline phase, the unit purchasing cost and the unit inventory holding cost per time affect the optimal sales strategy. The contribution of this paper provides the following managerial insights:

- Modelling of seasonal demands with product life cycle consisting of the growth phase, the mature phase, the decline phase in a single selling period
- Effect of markdown sales on seasonal demands in the decline phase
- Derivation of the initial inventory of product before the beginning of a single selling period when seasonal demands have product life cycle in a single selling period
- The theoretical analysis and the numerical search to determine the optimal sales strategy for the sales termination time and markdown ratio in the decline phase in a single selling period.

2. NOTATION

$\phi(\in 1,2,3)$: subscript to identify phase ϕ in product life cycle ($\phi=1$: the growth phase, $\phi=2$: the mature phase, $\phi=3$: the decline phase)

T_0 : starting time of product sales in a primary market

T : length of selling period in a primary market

T_1 : ending time of the growth phase (phase 1) $(T_0 \le t \le T_1)$

T_2 : ending time of the mature phase (phase 2) $(T_1 < t \le T_2)$

T_3 : ending time of the decline phase (phase 3) $(T_2 < t \le T,\ T = T_1 + T_2 + T_3)$

p : the unit sale price of product

w : the unit purchasing cost of product.

h : the unit inventory holding cost of a single type of products per unit of time

$f_\phi(t)(T_{\phi-1} \le t < T_\phi)$: demand function at time t in phase $\phi(=1,2,3)$

d_c : markdown ratio of the unit sales price in the decline phase (phase 3) $(0 \le d_c \le 1)$

T_L : sales termination time in the decline phase (phase 3) $(T_2 \le T_L \le T_3,\ T_3 = T)$

$T_L(d_c)$: the provisional sales termination time in the decline phase under markdown ratio d_c $(T_2 \le T_L(d_c) \le T_3,\ T_3 = T, 0 \le d_c \le 1)$

$g(d_c)$: the degree of price sensitivity in demand in the decline phase for markdown ratio $d_c (0 \le d_c \le 1)$

$f_3^{d_c}(t, d_c)$: demand function at time t in the decline phase for markdown ratio $d_c (0 \le d_c \le 1)$

$I[T_L \mid d_c]$: initial inventory level at T_0 for T_L in the decline phase under markdown ratio d_c

$\pi[T_L \mid d_c]$: the retailer's profit for T_L in the decline phase under markdown ratio d_c

d_c^* : the optimal markdown ratio in the decline phase

T_L^* : the optimal sales termination time in the decline phase

3. MODEL DESCRIPTIONS

(1) The seasonal demand with a three-phase product life cycle, the growth phase (phase 1: $T_0 < t \le T_1$), the mature phase(phase 2: $T_1 < t \le T_2$), the decline phase (phase 3: $T_2 < t \le T$, $T = T_1 + T_2 + T_3$) is considered. Seasonal demand in each phase is time-varying. This paper considers the sales termination time T_L in phase 3. Figure 1 shows seasonal demand in phase $\phi(\in 1,2,3)$ for time t and sales termination time in phase 3.

(2) A retailer purchases a single type of seasonal product with the unit purchasing cost w and sells the product with the unit sales price p at a primary market. The retailer incurs the unit inventory holding cost h including the maintenance cost per unit per unit time during the growth phase (phase 1, $T_0 < t \le T_1$), the mature phase (phase 2, $T_1 < t \le T_2$), the decline phase (phase 3, $T_2 < t \le T$, $T = T_1 + T_2 + T$) of a selling period.

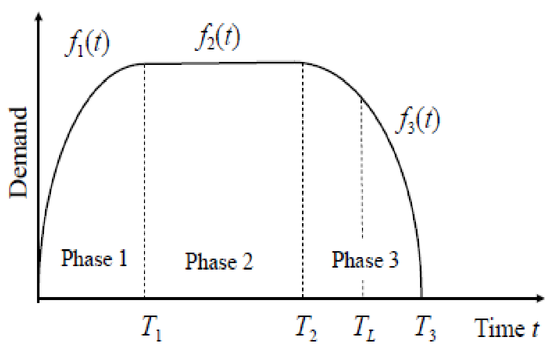

Figure 1: seasonal demand in phase $\phi(\in 1,2,3)$ in terms of time and sales termination time in phase 3

(3) A retailer has the initial inventory $I[T_L \mid d_c]$ of product at T_0 for T_L in phase 3 under markdown ratio d_c. Product sales are finished once the initial inventory $I[T_L \mid d_c]$ of product is 0 even during the selling season.

(4) The unit sales price of product is marked down at markdown ratio d_c $(0 \le d_c \le 1)$ for the unit sales price p in the decline phase (phase 3) of the selling period in order to make product sales promotion. Note that d_c=0 indicates no markdown sales.

4. MODEL ASSUMPTIONS

(1) A single type of seasonal product including appliance such as mobile phone and PC is considered. A sales strategy for a single seasonal product during a single selling period in a primary market is discussed.

(2) This paper discusses incorporation of markdown sales into the decline phase (phase 3) as sales effort by a retailer. In this case, it is assumed that the seasonal demand in the decline phase (phase 3) is sensitive in markdown ratio d_c $(0 \le d_c \le 1)$. Using the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio $d_c (0 \le d_c \le 1)$, the demand function in the decline phase (phase 3) is expressed as

$$f_3^{d_c}(t, d_c) = f_3(t)\{1 + g(d_c)\} . \tag{1}$$

Figure 2 shows the effect of markdown ratio on the demand in the decline phase (phase 3). Figure 2 indicates that as d_c becomes higher, the demand in phase 3 $f_3^{d_c}(t, d_c)$ tends to increase.

(3) Characteristics of demand function at time t in phase ϕ $(\in 1, 2, 3)$ is assumed as follows:

(i) Characteristic of demand function in phase 1:

The demand in phase 1 (the growth phase) increases with time t in the rage where $(0 \le t \le T_1)$.

$$df_1(t)/dt > 0 . \tag{2}$$

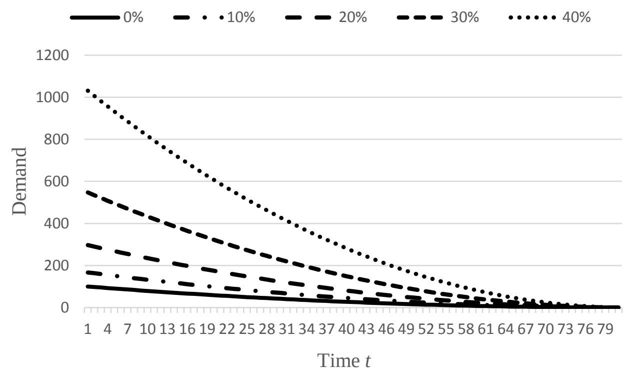

Figure 2: effect of markdown ratio on the demand in the decline phase (phase 3)

(ii) Characteristic of demand function in phase 2:

The demand in phase 2 (the mature phase) is constant with time t in the range where $(T_1 < t \le T_2)$.

$$df_2(t)/dt = 0 . \tag{3}$$

(iii) Characteristic of demand function in phase 3:

The demand in phase 3 (the decline phase) decreases with time t in the rage where $(T_2 < t \le T, T = T_1 + T_2 + T_3)$ under markdown ratio d_c.

$$\partial f_3^{d_c}(t, d_c)/\partial t = \partial f_3(t)\{1 + g(d_c)\}/\partial t < 0 . \tag{4}$$

Also, from model assumption (2) and Figure 2, the demand in phase 3 increases with markdown ratio d_c $(0 \le d_c \le 1)$ under time t in the rage where $(T_2 < t \le T, T = T_1 + T_2 + T_3)$. It is caused by the effect of the degree of price sensitivity $g(d_c)$ in demand in phase 3 for markdown ratio $d_c (0 \le d_c \le 1)$, $d\,g(d_c)/d\,d_c > 0$. From model assumption (2) and Figure 2,

$$\partial f_3^{d_c}(t, d_c)/\partial d_c = d\{ f_3(t)\{1 + g(d_c)\}\}/d\,d_c > 0 . \tag{5}$$

5. MODEL FORMULATIONS

In this section, the revenues, the costs and the retailer's profit during a single selling period are formulated as the following procedures.

From 4. model assumptions (2)-(3), the total demand d_ϕ in phase $\phi(\in 1, 2, 3)$ under markdown ratio d_c $(0 \le d_c \le 1)$ in phase 3 (the decline phase) can be obtained as

$$d_1 = \int_{T_0}^{T_1} f_1(t)dt \tag{6}$$

$$d_2 = \int_{T_1}^{T_2} f_2(t)dt \tag{7}$$

$$d_3 = \int_{T_2}^{T_3} f_3^{d_c}(t, d_c)dt . \tag{8}$$

The initial inventory level $I[T_L, d_c]$ of a retailer at T_0 for sales termination time T_L and markdown ratio d_c in the decline phase (phase 3) is obtained as

$$I[T_L, d_c] = d_1 + d_2 + \int_{T_2}^{T_L} f_3^{d_c}(t, d_c)dt . \tag{9}$$

First, the retailer's revenue during a single selling period is discussed. The retailer's revenues consists of the product sales including markdown sales of product.

The $RE(T_L, d_c)$ for sales termination time T_L and markdown ratio d_c in the decline phase (phase 3) is obtained as

$$RE(T_L, d_c) = p(d_1 + d_2) + (1 - d_c)p\int_{T_2}^{T_L} f_3^{d_c}(t, d_c)dt . \tag{10}$$

Next, the retailer's costs during a single selling period are discussed. The retailer's costs consists of the purchasing cost and the inventory holding cost including the maintenance cost.

The purchasing cost $W\left(T_L,d_c\right)$ for sales termination time T_L and markdown ratio d_c in the decline phase (phase 3) is obtained as

$$W(T_L,d_c)=wI(T_L,d_c)=w(d_1+d_2+\int_{T_2}^{T_L} f_3^{d_c}(t,d_c)dt) \ . \tag{11}$$

The inventory holding cost $IH\left[T_L,d_c\right]$ for sales termination time T_L and markdown ratio d_c in the decline phase (phase 3) is obtained as

$$IH(T_L,d_c)=h\int_{T_0}^{T_1}(t-T_0)\, f_1(t)dt+h\int_{T_1}^{T_2}(t-T_1)\, f_2(t)dt$$
$$+h\int_{T_2}^{T_L}(t-T_2)\, f_3^{d_c}(t,d_c)dt \ . \tag{12}$$

Finally, the retailer's profit during a single selling period is discussed. The retailer's profit is calculated as the revenue (the product sales) minus the total cost (the purchasing cost + the inventory holding cost). Therefore, the retailer's profit $\pi(T_L,d_c)$ for sales termination time T_L and markdown ratio d_c in the decline phase(phase 3) is obtained as

$$\pi(T_L,d_c)=RE(T_L,d_c)-W(T_L,d_c)-IH(T_L,d_c) \ . \tag{13}$$

6. OPTIMAL SALES STRATEGY

The decision procedures for the optimal sales termination time $T_L^*\ (T_2\le T_L^*\le T)$ and the optimal markdown ratio $d_c^*(0\le d_c^*\le 1)$ in the decline phase (phase 3) is discussed.

6.1 Provisional Decision for Sales Termination Time in Decline Phase (Phase 3) under Markdown Ratio d_c

It is investigated if the retailer's profit $\pi\left(T_L\middle|d_c\right)$ is a concave function in terms of sales termination time T_L in the decline phase (phase 3) under markdown ratio d_c.

The first-order differential equation of the retailer's profit in Eq. (13) in terms of sales termination time T_L in phase 3 under markdown ratio d_c is obtained as

$$\frac{d\pi\left(T_L\mid d_c\right)}{dT_L}=f_3^{d_c}(T_L,d_c)\left((1-d_c)p-w-h(T_L-T_2)\right) . \tag{14}$$

In Eq. (14), the demand in phase 3 $f_3^{d_c}(T_L,d_c)$ is always positive in the range $T_2\le T_L\le T_3$. Also, the term $(1-d_c)p-w-h(T_L-T_2)$ in Eq. (14) is a decreasing function in terms of sales termination time T_L. In this case, sales termination time T_L which satisfies with the following condition:

$$(1-d_c)p-w-h(T_L-T_2)=0 \tag{15}$$

maximizes the retailer's profit in Eq. (13). Therefore, the provisional sales termination time in phase 3 under markdown ratio $d_c(0\le d_c\le 1)$ $T_L\left(d_c\right)$ is determined as sales termination time T_L which satisfies with Eq. (15). Therefore, $T_L\left(d_c\right)$ is

$$T_L\left(d_c\right)=\frac{((1-d_c)-w)}{h}+T_2 \ . \tag{16}$$

If $T_L\left(d_c\right)$ obtained in Eq. (16) is not in the range where $T_2\le T_L\left(d_c\right)\le T_3$, $T_L\left(d_c\right)$ is determined as either T_2 or T_3 which maximizes the retailer's profit in Eq. (13).

6.2 Optimal Decision for Sales Termination Time in Decline Phase (Phase 3) and Markdown Ratio

The decision procedures for the optimal sales termination time T_L^* and markdown ratio d_c^* in the decline phase (phase 3) are shown as follows.

[Step 1] Substitute the provisional sales termination $T_L\left(d_c\right)$ under markdown ratio d_c in Eq. (16) into the retailer's profit in Eq. (13).

[Step 2] Calculate the retailer's profit $\pi(d_c\middle|T_L^*)$ obtained in [Step 1] by varying markdown ratio d_c in the range where $(0\le d_c\le 1)$, using the numerical computing.

[Step 3] Determine the combination: $\left(T_L\left(d_c\right),\, d_c\right)$ to maximize the retailer's profit $\pi(d_c\middle|T_L^*)$ calculated in [Step 2] as the optimal sales strategy (T_L^*,d_c^*) through numerical search.

7. NUMERICAL ANALYSIS

Using numerical examples and the decision procedures in section 6, the results of the optimal sales strategy for the optimal sales termination time and the optimal markdown ratio in the decline phase (phase 3) are shown. From the numerical analysis, the benefit of markdown sales in the decline phase (phase 3) is verified by comparing the retailer's profit with and without markdown sales in the decline phase (phase 3). Also, the sensitivity analysis is conducted to illustrates how (i) the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio, (ii) the demand pattern in the decline phase (phase 3), (iii) the unit purchasing cost, (iv) the unit inventory holding cost per time affect the optimal sales strategy and the retailer's profit.

Numerical examples in base case are set as T_1=30, T_2=70, T_3=150, p=1000, w=100, h=10. The demand functions in the growth phase (phase 1), the mature phase (phase 2) and the decline phase (phase 3) of base case are set as

$$f_1(t)=(-t^2+100t)/21 \tag{17}$$

$$f_2(t)=100 \tag{18}$$

$$f_3(t)=0.015t^2-4.55t+345 , \tag{19}$$

satisfying the property of the related function in Eqs. (2)-(4).

The degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio $d_c(0\le d_c\le 1)$ of base case is set as

$$g(d_c)=0.72e^{6.585d_c}+0.28 \tag{20}$$

satisfying the property of function in Eq. (5).

Here, the maximum of demand with product life cycle in a single selling period is set as 100.

Table 1: Results of optimal sales strategy in base case of numerical examples

d_c (%)	$T_L(d_c)$	Profit of a retailer	Demand in phase 3	Initial inventory I	Product Sales	Purchasing cost	Inventory cost
0	150	5904048	2720	8434	8434286	843429	1686810
10	150	6719901	4545	10259	9804802	1025930	2058970
20	140	8015060	8071	13763	12153087	1376279	2761748
30	130	9919784	14882	20323	15940260	2032282	3988193
40	120	12440593	28041	32131	21564605	3213148	5910864
50	110	15227304	53462	52101	28907468	5210065	8470099
d_c^*=60	T_L^*=100	17204807	102574	82362	36373228	8236164	10932257
70	90	16303031	197451	118233	39469762	11823254	11343477
80	80	10455618	380745	129246	30420723	12924647	7040457
90	70	4010714	734849	5714	5714286	571429	1132143

7.1 Base Cases of Numerical Examples

Table 1 shows the optimal sales strategy in base case of numerical examples. The optimal sales termination time is T_L^* and the optimal markdown ratio d_c^* can be obtained from the decision procedures in 6.2, using Eq. (16) in 6.1, the numerical calculations and the numerical search. From Table 1, it can be seen that the provisional sales termination time $T_L(d_c)$ tends to reduce, as the markdown ratio d_c increases. This is caused by the analysis result in Eq. (16). From Table 1, the optimal markdown ratio and the optimal sales termination time to maximize the retailer's profit are $d_c^* = 60(\%)$ and $T_L^* = 100$. The retailer's maximal profit $\pi(T_L^*, d_c^*)$ =17204807. This result indicates that markdown sales in the decline phase (phase 3) can enhances the retailer's profit.

7.2 Sensitivity Analysis

The sensitivity analysis on the optimal sales strategy and the retailer's profit is conducted by changing each of four items: (i) the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio, (ii) the demand pattern in the decline phase (phase 3), (iii) the unit purchasing cost, (iv) the unit inventory holding cost per time, from base case of numerical examples one by one.

7.2.1 Effect of Degree of Price Sensitivity in Demand in Decline Phase (Phase 3) for Markdown Ratio on Optimal Sales Strategy

It is illustrated how the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio d_c affect the optimal sales strategy and the retailer's profit. Figure 4 shows four cases of the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio $d_c (0 \le d_c \le 1)$:

Case 1 : Base case (numerical examples) in Eq. (20)

Case 2 : the situation where the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio d_c is more rapid than that in base case

$$g(d_c) = 473.55 d_c^{\,2} \tag{21}$$

Case 3 : the situation where the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio d_c is more fluent than that in base case

$$g(d_c) = 20 d_c^{\,2} + 8 d_c \tag{22}$$

Case 4 : the situation where the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio d_c converges as d_c becomes either smaller or larger.

$$g(d_c) = 473.55 d_c^2 \qquad (d_c \le 0.3) \tag{23}$$

$$g(d_c) = -56.34 d_c^2 + 93.24 d_c + 19.72 \ \ (d_c > 0.3)\,. \tag{24}$$

Tables 2-4 show the results of the optimal sales strategy in Cases 2-4 of the degree of price sensitivity $g(d_c)$ in demand. From Table 2, the optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit in Case 2 of the degree of price sensitivity $g(d_c)$ are $d_c^* = 50(\%)$, $T_L^* = 110$, and $\pi(T_L^*, d_c^*)$ =72141181. From Table 3, the optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit in Case 2 of the degree of price sensitivity $g(d_c)$ are $d_c^* = 40(\%)$, $T_L^* = 120$ and $\pi(T_L^*, d_c^*)$ = 10061756. From Table 4, the optimal markdown

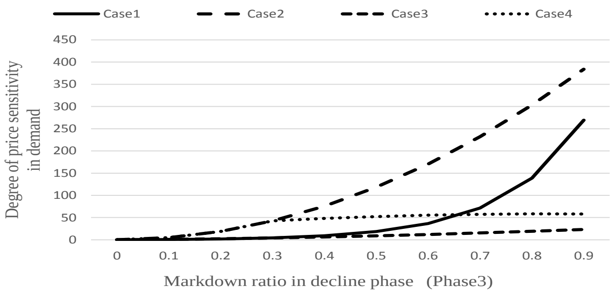

Figure 4 : four cases of the degree of price sensitivity $g(d_c)$ in demand in the decline phase (phase 3) for markdown ratio $d_c (0 \le d_c \le 1)$

Table 2: Results of optimal sales strategy in Case 2 of degree of price sensitivity $g(d_c)$ in the decline phase for markdown ratio

d_c (%)	$T_L(d_c)$	Profit of a retailer	Demand in phase 3	Initial inventory I	Product Sales	Purchasing cost	Inventory cost
0	150	5904048	2720	8434	8434286	843429	1686810
10	150	13309872	15601	21315	19754790	2131485	4313434
20	140	30923274	54242	59807	48988426	5980696	12084455
30	130	51119774	118645	122178	87239131	12217835	23901522
40	120	66784548	208809	202432	123745086	20243229	36717310
d_c^*=50	T_L^*=110	72141181	324734	287469	146591536	28746879	45703476
60	100	64006580	466420	354243	145125900	35424332	45694988
70	90	43472070	633867	366926	114077653	36692551	33913033
80	80	18010696	827076	274058	59382994	27405783	13966515
90	70	4010714	1046045	5714	5714286	571429	1132143

Table 3: Results of optimal sales strategy in Case 3 of degree of price sensitivity in the decline phase for markdown ratio

d_c (%)	$T_L(d_c)$	Profit of a retailer	Demand in phase 3	Initial inventory I	Product Sales	Purchasing cost	Inventory cost
0	150	5904048	2720	8434	8434286	843429	1686810
10	150	7253381	5440	11154	10610286	1115429	2241476
20	140	8599156	9248	14937	13092286	1493679	2999451
30	130	9626714	14144	19598	15433086	1959829	3846543
d_c^*=40	T_L^*=120	10061756	20128	24677	17091786	2467679	4562351
50	110	9717381	27200	29314	17514286	2931429	4865476
60	100	8559089	35360	32137	16283286	3213679	4510518
70	90	6787781	44608	31134	13340286	3113429	3439076
80	80	4940756	54944	23541	9279586	2354079	1984751
90	70	4010714	66368	5714	5714286	571429	1132143

Table 4: Results of optimal sales strategy in Case 4 of degree of price sensitivity in the decline phase for markdown ratio

d_c (%)	$T_L(d_c)$	Profit of a retailer	Demand in phase 3	Initial inventory I	Product Sales	Purchasing cost	Inventory cost
0	150	5904048	2720	8434	8434286	843429	1686810
10	150	13309872	15601	21315	19754790	2131485	4313434
20	140	30923274	54242	59807	48988426	5980696	12084455
d_c^*=30	T_L^*=130	51119774	118645	122178	87239131	12217835	23901522
40	120	44079731	133284	131281	81054246	13128089	23846426
50	110	34401568	144854	131396	68555186	13139609	21014010
60	100	23737227	153358	120310	51552527	12030989	15784311
70	90	13896631	158797	96205	32861637	9620546	9344460

80	80	6738886	161172	58006	16172687	5800629	3633172
90	70	4010714	160482	5714	5714286	571429	1132143

ratio, the optimal sales termination time and the retailer's maximal profit in Case 2 of the degree of price sensitivity $g(d_c)$ are $d_c^* = 30(\%)$, $T_L^* = 130$ and $\pi(T_L^*, d_c^*) = 51119774$. Here, in base case, $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =17204807.

From Tables 1-4, the following results can be seen:

- The values of d_c^* in Cases 2, 3 and 4 of the degree of price sensitivity $g(d_c)$ in demand are smaller than that in Case 1. From the analysis result in Eqs. (15) and (16), the values of T_L^* in Cases 2, 3 and 4 increase due to the decreases of d_c^* in Cases 2, 3 and 4 of $g(d_c)$.
- d_c^* in Case 4 is determined as the value of markdown ratio where the increasing degree of the demand in the decline phase (phase 3) begins to gentle.

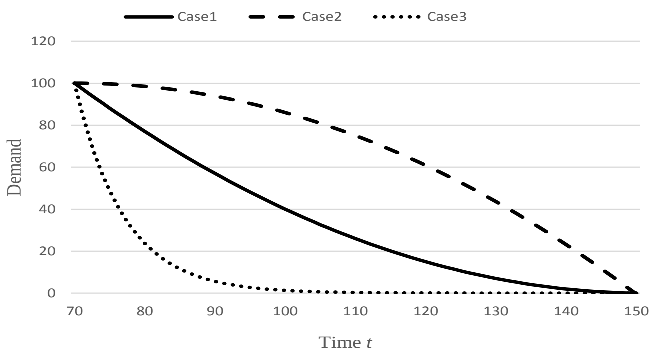

Figure 5 Three cases of the demand pattern $f_3(t)$ in the decline phase (phase 3) for time t

Table 5 Effect of demand pattern $f_3(t)$ in the decline phase (phase 3) on optimal sales strategy

demand pattern in the decline phase (phase 3)	d_c^* (%)	T_L^*	Maximal profit of a retailer
Case 1 (Base case)	60	100	17204807
Case 2	60	100	20595392
Case 3	60	100	4180413

- The decreasing degree of the retailer's profit due to the increase of markdown ratio is larger than the increasing ratio of the retailer's profit due to the increase of the demand in the decline phase (phase 3) for markdown ratio. This result implies that markdown sales should moderate (refrain) in the situation where the increasing degree of the demand in the decline phase (phase 3) for markdown ratio is small.

7.2.2 Effect of Demand Pattern in Decline Phase (Phase 3) on Optimal Sales Strategy

It is illustrated how the demand pattern $f_3(t)$ in the decline phase (phase 3) affect the optimal sales strategy and the retailer's profit. Figure 5 shows three cases of the demand pattern $f_3(t)$ in the decline phase (phase 3) for time t:

Case 1 : Base case (numerical examples) in Eq. (18)(19)

Case 2 : the situation where the decreasing degree of demand in the decline phase (phase 3) for time t is more fluent than that in base case

$$f_3(t) = -0.0158t^2 + 2.22t + 22.04 \tag{25}$$

Case 3 : the situation where the decreasing degree of demand in the decline phase (phase 3) for time t is more rapid than that in base case

$$f_3(t) = 10^{-\frac{1}{16}(t-102)}. \tag{26}$$

Table 5 shows the effect of the demand pattern $f_3(t)$ in the decline phase (phase 3) on optimal sales strategy. From Table 5, the optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit in Case 2 of the decline phase (phase 3) are $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =20595392. The optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit in Case 3 of the decline phase (phase 3) are $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =4180413. Here, in base case, $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =17204807. From Eqs. (16) and (18), it can be seen that the optimal sales termination time and the optimal markdown ratio are unaffected by the demand pattern $f_3(t)$.

Table 6 Effect of purchasing cost on optimal sales strategy

purchasing cost w	d_c^* (%)	T_L^*	Maximal profit of a retailer
100 (Base case)	60	100	17204807
200	50	100	10316163
300	40	100	6474771

Table 7 Effect of inventory holding cost on optimal sales strategy

Inventory holding Cost h	d_c^* (%)	T_L^*	Maximal profit of a retailer
10 (Base case)	60	100	17204807
30	60	80	6955243
50	60	76	2712871

7.2.3 Effect of Purchasing Cost on Optimal Sales Strategy

It is illustrated how the unit purchasing cost affect the optimal sales strategy and the retailer's profit. The following results are obtained when the purchasing cost changed to w = 200, 300 from w = 100. Table 6 shows the effect of purchasing cost on optimal sales strategy. From Table 6, the optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit when w = 200 are $d_c^* = 50(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ = 10316163. The optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit when w = 300 are $d_c^* = 40(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =6474771. Here, in base case, $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =17204807. Thus, it can be seen that the optimal markdown ratio tends to decrease as purchasing cost increases and this leads to the reduction of the retailer's profits.

7.2.4 Effect of Inventory Holding Cost on Optimal Sales Strategy

It is illustrated how the unit inventory holding cost per time affect the optimal sales strategy and the retailer's profit. The following results are obtained when the inventory holding cost changed to h = 30, 50 from h = 10. Table 7 shows the effect of inventory holding cost on optimal sales strategy. From Table 7, the optimal markdown ratio, the optimal sales termination time and the retailer's maximal profit when h = 30 are $d_c^* = 60(\%)$, $T_L^* = 80$ and $\pi(T_L^*, d_c^*)$ = 6955243, meanwhile there results when h = 50 are $d_c^* = 60(\%)$, $T_L^* = 76$ and $\pi(T_L^*, d_c^*)$ = 2712871. Here, in base case, $d_c^* = 60(\%)$, $T_L^* = 100$ and $\pi(T_L^*, d_c^*)$ =17204807. Therefore, the following finding are obtained:

- As the unit inventory holding cost per unit of time increases, the optimal sales termination time tends to reduce from the analysis result in Eq. (16).
- As the unit inventory holding cost per unit of time increases, the retailer's profit tends to reduce due to decrease of the optimal sales termination time.
- The optimal markdown ratio is unaffected by the change of the unit inventory holding cost per unit of time so much.
- The optimal sales termination time is more affected by the unit inventory holding cost per unit of time than the purchasing cost. This is verified from theoretical result in Eqs. (15) and (16).

8. CONCLUSIONS

This paper presented the optimal sales strategy for a retailer who sold a single product with seasonal demand in a primary market with a three-phase product life cycle: the growth phase, the mature phase and the decline phase in a single selling period. Seasonal demand in each phase was time-varying. This paper discussed incorporation of markdown sales into the decline phase as sales effort by the retailer. This paper determined the optimal sales strategy regarding the sales termination time and the markdown ratio in the decline phase with price sensitivity in a primary market so as to maximize the retailer's profit in a selling period. Using numerical examples, the benefit of markdown sales in the decline phase was shown by comparing the retailer's profit with and without markdown sales in the decline phase. The sensitivity analysis was conducted to illustrate how degree of price sensitivity in demand for markdown ratio, demand pattern in the decline phase, the unit purchasing cost and the unit inventory holding cost per time affect the optimal sales strategy.

This paper can provide managerial insights from outcomes of the sales strategy with a three-phase product life cycles regarding the sales termination time and the markdown ratio obtained from both the theoretical research and the numerical search to not only academic researchers, but also real-world policymakers.

Here, this study considered the followings:

- Demands in the growth phase, the mature phase, the decline phase in a single selling period could be estimated as specific functions before the beginning of the selling period.
- There was no inventory of product at the sales termination time at the decline phase in a primary market.
- Retail prices in the growth phase, the mature phase, the decline phase in a single selling period were deterministic.

As future researches, it will be necessary to extend the following topics into the theoretical analysis and the numerical search in this model:

- The uncertainty in demand with product life cycle
- The optimal decision for retail price considering product life cycle
- The situation where there is the unsold product at the sales termination time in a primary market with product life cycle
- The situation where the unsold product in the primary market at the sales termination time can be resold in a markdown market

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