Electricity Market Risk Measurement using Vine-Copula based Monte Carlo Simulation Model

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Abstract. In this paper we propose a vine copula based Monte Carlo simulation model for estimating Portfolio Value at Risk. The vine copula model is introduced to analyze the complex dependence structure of different regional markets in the typical financial markets. Then we construct the vine copula based Portfolio Value at Risk model, taking into account the identified high dimensional dependence structure. Empirical studies in the Australian electricity markets show the existence of nonlinear dependence structure in Australian electricity markets. We further show that PVaR based on the model would achieve more accurate and reliable estimate.

Keywords: Vine Copula Theory；Electricity Market；Portfolio Value at Risk；Dependence Structure

1. INTRODUCTION

As the economics growing, electricity becomes one of the most essential commodities in the modern economics, and the electricity markets become more competitive and volatility. Thus risk management and accurate forecast becomes the key for electricity price fluctuation. Some typical models as wavelet model, empirical mode decomposition (EMD) method and neural network etc, have been employed to forecast the electricity price and manage the risk. Mandal et al., (2013) used a new approach which mixed wavelet transform, firefly algorithm and fuzzy ARTMAP to forecast the electricity price. Zou et al., (2015) gained empirical result using an entropy optimized BEMD model for risk management of electricity market. Yamin et al.,(2004) employed Artificial Neural Network(ANN) to simulate and forecast the short-term electricity price. While some researches argued that multivariate methods would be popular in analyzing the electricity markets, such as Weron (2014) analyzed different models in forecasting electricity price, and argued that multivariate models was the important one of the forecasting models in the future.

In multivariate models, the model that constructs the complex correlation between different variates has received more and more attention in the current papers, such as vine copula model. Sklar’s theorem (1959) about copula offered an efficient way to solve the problem how to measure the dependence in different firms. But conventional copula limits in dimension, because copula is difficult to compute dependence more than two variables. As an extension of copula, Joe (1997) came up with vine copula .Vine copula has been widely used in analyzing multivariate distribution. Zhang et al.,(2014) used vine copula to research the 10 stock markets and compared C-vine, D-vine, R-vine copula models by analyzing their performance in forecasting VaR and ES, then they argued that the three different vine copulas have different structure and the D-vine copula perform better. Dalu Zhang (2014) analyzed the sovereign debt crisis in 11 contries of European Union by the GARCH based vine copula and the result shown that vine copula had a good performance in forecasting the crisis. Joe et al.,(2010) introduced the asymmetric tail dependence in financial market by vine copula ,and proposed that vine copula could give a better way to solve the problem of tail dependence. Here are a set of articles about risk management using vine copula. Righi and Ceretta (2013) researched the dependence structure in Brazilian by D-vine and C-vine, and thought some sectors had the asymmetric tail dependence, so risk management was important in asset allocation. Low et al., (2013) introduced how used canonical vine copula to estimate portfolio value and deemed vine copula models had better statistic and economic performance in high dimensions. Some other researches focused on forecasting volatility by vine copula models , like Vaz et al ., (2014)proposed a robust estimation for pair-copula to forecast realized volatility of Brazilian stocks and the authors thought C-vine copula models had better performance in robust estimation by comparing with other approaches. All of above expresses that the vine copula has been widely utilized to forecast and manage the risk in finance and economics fileds, but few articles focused on electricity markets using vine copula model. Czado and Min (2011) mixed the Bayesian method, Markov chain Monte Carlo (MCMC) method with D-vine copula to estimate the dependence structure of the Australian electricity markets. Smith(2015) used five different electricity markets to illustrate vine copula models and the research shown that vine copula could propose a new way to estimate the multivariate dependence. And those researches that employed the vine copula in electricity markets only analyze the dependence structure of variates and less involve the risk evaluation of electricity markets, so in this paper we forecast the dependence structure of electricity markets and calculate the Value at Risk(VaR) and Expected Shortfall (ES).

This paper proposes a vine copula model to research the dependence between different electricity markets, then uses Monte Carlo simulation model to estimate value at risk. Vine copula can provide a efficient approach to solve the problem in high dimensions and improve the accuracy of forecast. As the same time, vine copula can estimate the volatility by researching the dependence between markets. Through Monte Carlo simulation , we use the vine copula to manage the risk in electricity markets.

The contribution of this paper is we apply vine copula to estimate the dependence of electricity markets and implement risk management. And we come up with a efficient way to estimate value at risk by combining vine copula with Monte Carlo simulation.

The rest of this paper is organized as follows. Section 2 briefly reviews the Vine Copula Model. Section 3 proposes the Portfolio Value at Risk model, based on the Vine Copula model. Results from the empirical studies are reported and analyzed in Section 4. Section 5 concludes.

2.VINE COPULA MODEL

2.1 Copula Theory

Sklar (1959) proposed a copula model, which is a function with uniformly distributed margins on the [0,1].And the joint distribution can be decomposed into a copula and a set of marginal distributions. For introducing copula theorem, we assume here are two independent variables as x_1,$x_2$and the copula model can be written as

$$F(x_1,x_2)=C\{F_1(x_1),F_2(x_2)\} \tag{1}$$

where $F(x_1, x_2)$ is the joint distribution and $F_1(x_1)$,$F_2(x_2)$ is the marginal distribution of x_1,x_2 respectively. The copula theorem argues that the copula C can be expressed by the inverse distribution functions of the margins such as

$$C(x_1,x_2)=F\{F_1^{-1}(x_1),F_2^{-1}(x_2)\} \tag{2}$$

For an $F(x_1, x_2)$,the joint density function can be expressed as

$$f(x_1,x_2)=c_{12}\bullet f_1 \bullet f_2 \tag{3}$$

where $f(x_1, x_2)$ is the joint density function, c_{12} is the copula density function like $c_{12}\{F_1(x_1),F_2(x_2)\}$. f_1,f_2 is the marginal density function of $F_1(x_1)$ and $F_2(x_2)$ respectively.

2.2 C-Vine Copula and D-Vine Copula

While copula model is limited in dimensions, it is difficult to be applied in more than two variables. So vine copula theorem is introduced to forecast dependence of high dimensions. Aas et al., (2009) researched multivariate distribution and different dimensions of vine copula,they gave two vine copulas as C-vine copula and D-vine copula. In this paper, we choice the one of them to research the electricity markets.

As Aas's theorem (2009) stated that the C-vine copula model can be decomposed to two parts like marginal density functions and a set of copulas. For n-dimensional C-vine, let marginal density

$$f=\sum_{g=1}^{n} fg(x_g) \tag{4}$$

let a set of copula density functions

$$c=\prod_{k=1}^{n-1}\prod_{j=1}^{n-k} c_{k,k+j|1,....,k-1}\{F(x_k|x_1,....,x_{k-1}),F(x_{k+j}|x_1,....,x_{k-1})\} \tag{5}$$

we use equation（4）to multiply equation（5）， then the C-vine density $f(x_1,\ldots,x_n)$can be written as

$$f(x_1,...,x_n)=f \cdot c \tag{6}$$

The detail information of equation(6)can be expressed as

$$f(x_1,...,x_n)=\prod_{g=1}^{n} f_g(x_g)$$
$$\prod_{k=1}^{n-1}\prod_{j=1}^{n-k} c_{k,k+j|1,....,k-1}\{F(x_k|x_1,....,x_{k-1}),F(x_{k+j}|x_1,....,x_{k-1})\} \tag{7}$$

To explain the C-vine copula, the concept of tree is introduced. For a n-dimensional C-vine copula, the number of trees is n-1.Tree T_g has only one node and n-g edges. So in the function（7）k represents the node and j represents the edge.

In this paper, we use C-vine copula to analyze the dependence of five variables, so the dimension is five. Here are four trees ,four nodes and ten edges when the dimension is five. As can be proved ,the first tree has one node and four edges, the second one has one node and three edges ,the third one has one node and two edges , the last tree has one node and one edge. The five-dimensional C-vine copula function can be shown as

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5)=&f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3)\cdot f_4(x_4)\cdot f_5(x_5)\cdot \\ &c_{12}(u_1,u_2)\cdot c_{13}(u_1,u_3)\cdot c_{14}(u_1,u_4)\cdot c_{15}(u_1,u_5)\cdot \\ &c_{23|1}(u_{2|1},u_{3|1})\cdot c_{24|1}(u_{2|1},u_{4|1})\cdot c_{25|1}(u_{2|1},u_{5|1})\cdot \\ &c_{34|12}(u_{3|12},u_{4|12})\cdot c_{35|12}(u_{3|12},u_{5|12})\cdot \\ &c_{45|123}(u_{4|123},u_{5|123}) \end{aligned} \tag{8}$$

Where u_1 expresses the marginal function F_1（x_1）, the $u_{2|1}$ indicates the conditional marginal function $F_{2|1}$（$x_2|x_1$）, and others are similar as $u_{2|1}$。

As similar to the C-Vine copula,the D-Vine copula is also composed of marginal density function and some copulas ,but the significant difference of the two copula model is the each node in D-Vine copula has two edges at the most,while the edge number of one node in C-Vine copula is often more than two.And the function of five-dimensions D-Vine copula can be written as the following.

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5)=&f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3)\cdot f_4(x_4)\cdot f_5(x_5)\cdot \\ &c_{12}(u_1,u_2)\cdot c_{23}(u_2,u_3)\cdot c_{34}(u_3,u_4)\cdot c_{45}(u_4,u_5)\cdot \\ &c_{13|2}(u_{1|2},u_{3|2})\cdot c_{23|4}(u_{2|4},u_{3|4})\cdot c_{34|5}(u_{3|5},u_{4|5})\cdot \\ &c_{14|23}(u_{1|23},u_{4|23})\cdot c_{25|34}(u_{2|34},u_{5|34})\cdot \\ &c_{15|234}(u_{1|234},u_{5|234}) \end{aligned} \tag{9}$$

2.3 Marginal distribution model

In order to obtaining the marginal distribution of each variable , the GARCH(1,1)-Std model is applied in this paper. Through the pseudo-observations , the original returns data can be transformed to the copula data that is the uniform distribution on [0,1], then the dependence structure is selected .Using Monte Carlo simulation, the new data are gained, then by the GARCH(1,1)-Std model the new data are transformed to new simulated returns data. The marginal distribution model can be shown as the follow functions:

$$x_{i,t}=\mu+\sigma_{i,t}\varepsilon_{i,t} \tag{10}$$

$$\sigma_{i,t}^2=\alpha_i+\alpha_{i1}x_{i,t-1}^2+\beta_i\sigma_{i,t-1}^2 \tag{11}$$

$$\varepsilon_{i,t}\sim iid.st(\lambda) \tag{12}$$

3. VINE COPULA BASED MONTE CARLO SIMULATION MODEL FOR ESTIMSTING PORTFOLIO VALUE AT RISK AND EXPECTED SHORTFALL MODEL

In this paper, we use vine copula to estimate portfolio value at risk (VaR) and expected shortfall (ES).For estimating VaR and ES, the Monte Carlo simulation model is used in the paper. The distribution of portfolio's return must be got when estimating VaR and ES, so we estimate the distribution of portfolio's return under same dependence structure at first, then get portfolio's VaR and ES. Before estimating the distribution of portfolio's return, we get the dependence between different markets by vine copula model .We assume that one day holding period and the weight of each asset is equalization and invariant. The steps for estimating portfolio value at risk and expected shortfall by using vine copula model can be described as follow:

Firstly, computing the dependence of electricity markets using vine copula, to get the relationship of different markets.

Secondly, simulating five groups new data U_i (i=1,2,3,4,5) and these new data have same dependence as is computing in first step. Each group has 2896 observations.

Thirdly , converting the U_i to $\varepsilon_{i,t}$ using the inverse function of the Student's t distribution.

Fourthly, using the function (11) to calculate the volatility through the original data.

Finally, choosing a time t,then computing the return as

$$p_t=\sum_{i=1}^{n} w_i x_i \tag{13}$$

Where P_t is the return of portfolio at time t, n=5,w_i represents the weight of asset i and $x_{i,t}$ is the return of asset i at time t. The $x_{i,t}$ will be acquired by function (10).P_t is the variable quantity of the portfolio. So we can compute the distribution of $P_{,t}$ then the VaR and ES can be estimated.

Value at risk is a well-known concept which can be used to estimate the risk and implement the risk management. VaR is the function of time horizon t and quantile level X%, and according to the definition , VaR can be described as maximum loss with a given quantile level at a period of time. It can be written as

$$P(\Delta x_t < VaR)=X\% \tag{14}$$

In the former, the distribution of returns has been computed, so we can get VaR essay.

Expected Shortfall (ES) is a specific concept to estimate the exceed loss of the portfoilo ,comparing with the VaR. It is used to provid a bigger expected loss than VaR .ES is a expectation of the VaR which is the rest part expect the VaR under the quantile level X%. In other words,ES can be forecasted on the basis of the VaR, and in the former,we have got the distribution of the portfoilo and the VaR which the quantile level is X%,so according the theorm of Artzner et al (1997),we can compute the ES as:

$$ES_x=E[\Delta P_k|\Delta P_k<VaR_x] \tag{15}$$

Where we assume that VaR_X is minus, and when we forecast the distribution of the portfoilo, we can get a set of ΔP_k which is the rest 1- X% of the distribution.

4. EMPIRICAL STUDY

4.1 Electricity Price Data and the Descriptive Statistics

Australian electricity market is one of the most competitive energy markets in the world, which has several different regional markets and these markets have very close relationship. So the price of these electricity markets is significantly fluctuating and these markets have great risk exposure. In this paper we use daily average price of five electricity markets like New South Wales (NSW), Queensland (QLD), South Australia (SA), Victoria (VIC) and Tasmania (TAS) ,the time period is from 1 June 2005 to 31 January 2016.Here are 3897 daily price observations for each markets and the data can be obtained from Australian Energy Market Operator(AEMO).For constructing the vine copula model. The returns are obtained by log differenced the original data as r_t=lnp_{t+1}-lnp_t,and the returns series has 3896 observations, in which 2896 observations from 1 June 2005 to 5 May 2013 are used to estimate the model and 1000 observations from 6 May 2013 to 31 January 2016 will be applied to evaluate the effect of the model.

The descriptive statistics of the electricity daily returns in the five regional markets are shown in table1.And the result of JB (Jarque-Bera) text is also contained in table1.

Table 1 ： Descriptive Statistics

Statistic	r_{qld}	r_{nsw}	r_{sa}	r_{tas}	r_{vic}
N	3,896	3,896	3,896	3,896	3,896
Mean	0.0002	0.0001	-0.0001	0.0000	0.0000
St. Dev.	0. 4	0.3	0.5	0. 3	0.4
Min	-4.2	-3.3	-4.0	-4.0	-4.0
Max	4.3	2.7	4.4	3.6	4.1
Skewness	-0.0141	-0.1735	0.3827	-0.2791	-0.1300
Kurtosis	27.1174	38.3726	19.3255	22.2934	32.2629
P_{JB}	0.0000	0.0000	0.0000	0.0000	0.0000

Descriptive statistics of the data in table1 uncover that the P value of JB text is close to zero, which means the distribution of the price in each electricity markets is significantly different from the standard normal distribution .The kurtosis shows that the distribution of each electricity market is different from normal distribution and has fat tail, which reduces the accuracy of estimating the VaR and ES by conventional models. So in this article vine copula model is used to solve the problems caused by abnormal distribution and fat tail.

Before choosing the vine copula model to analyze the dependence of electricity markets, the stationary test and Arch test must be used to deal with the returns data for exploring the data series is stationary or nonstationary and whether it has Arch effect. The table 2 shows the result of stationary test through the ADF test and Arch effect test thought LM test. The result shows the data is stationary.

Table 2: ADF test and Arch test

	r_{qld}	r_{tas}	r_{sa}	r_{vic}	r_{nsw}
LM(5)	645.8 (0.0000)	289.96 (0.0000)	567.22 (0.0000)	406.42 (0.0000)	508.46 (0.0000)
LM(10)	647.58 (0.0000)	352.76 (0.0000)	639 (0.0000)	408.25 (0.0000)	600.03 (0.0000)
LM(15)	649.31 (0.0000)	353.29 (0.0000)	639.63 (0.0000)	408.02 (0.0000)	651.28 (0.0000)
$ADF_{P\text{-}value}$	0.01	0.01	0.01	0.01	0.01

4.2 The vine copula model selection and estimation

In the former section, copula model has one assumption that the distribution of each variables in copula model and vine copula model must be uniform distribution in [0,1],the data series is known as copula data. The pseudo-observations method is used to obtain the copula data.

In this step, the vine copula model is selected and the structure of the vine copula is shown. By the CDVine copula packages in R, the pair copulas of C-Vine copula and D-Vine copula are estimated. In the table 3,it can be found that the most suitable pair copula of C-Vine copula and D-Vine copula is t-copula and the parameter of each pair copula is revealed in table 3. The structure of first tree in both C-Vine copula and D-Vine copula has been uncovered . C-Vine copula and D-Vine copula is different in structure, C-Vine copula has a variable as the key factors in each tree, but the variable is related with two other variables at most. For convenience , the marker NSW,SA,QLD,TAS,VIC is named as from 1 to 5. Using AIC and BIC ,the optimum vine copula model can be chosen , in the table 3,the AIC and BIC of C-Vine copula model is obviously small than D-Vine copula model,so in this paper the C-Vine copula model is selected.

Table 3 :the parameter of C-Vine copula and D-Vine copula

	C-Vine copula			D-Vine copula			
	copula	parameter1	parameter2		copula	parameter1	paramete2
p_{12}	t	0.6188	3.4687	p_{12}	t	0.6188	3.4687
p_{13}	t	0.7934	2.0001	p_{23}	BB8	4.9352	0.4862
p_{14}	t	0.4674	3.8068	p_{34}	BB8	4.4891	0.4085
p_{15}	t	0.862	2.0001	p_{45}	t	0.5866	2.6144
$p_{23\|1}$	BB8(90°)	-1.1208	-0.8647	$p_{13\|2}$	t	0.6876	2.9959
$p_{24\|1}$	BB8(180°)	3.0092	0.5362	$p_{24\|3}$	BB8(180°)	5.7995	0.3883
$p_{25\|1}$	t	0.6327	4.2134	$p_{35\|4}$	Frank	3.8905	0
$p_{34\|12}$	Frank	-0.1124	0	$p_{14\|23}$	Frank	1.3826	0
$p_{35\|12}$	Gaussian	0.0108	0	$p_{25\|34}$	t	0.6691	3.995
$p_{45\|123}$	t	0.2259	7.6515	$p_{15\|234}$	t	0.5286	4.0916
AIC	-14169.8			-12729.07			
BIC	-14056.98			-12616.25			

4.3 The VaR and ES forecasting

In the section, we simulate the returns data series of the five electricity markets using Monte Carlo Simulation with the C-Vine copula which has been constructed and then forecast the Value at Risk(VaR) and Expected Shortfall (ES) of the portfolio. For comparing the performance of different models in forecasting the VaR and the ES, the historical simulation (H-S) method is used to estimate the VaR and the ES of the portfolio. From the table 4,it can be shown that the VaR and the ES using the model in this paper to forecast is less than using H-S method in same confidence level respectively, which means the H-S method may underestimate the risk. And by the research of Kupiec(1995), in order to evaluating the performance of forecasting the VaR, the loss days which respect the loss exceeds the VaR in these days are estimated by the1000 out sample data. The loss days of Vine copula-Monte Carlo model is more than H-S model under the 95% and 97.5% confidence level in the table 4.Which proves the performance of our model is better than the conditional H-S method in forecasting the VaR. In the table 4 the ES of vine copula model is small than the H-S model ,which means the H-S model may underestimate the risk and the model in this paper can more accurately forecast the risk.

Table 4: Value at Risk and Expected Shortfall

	Vine copula-Monte Carlo			H-S		
Confidence level	0.99	0.975	0.95	0.99	0.975	0.95
VaR	-0.8292	-0.5347	-0.3633	-0.8042	-0.5482	-0.3826
Loss days	0	7	37	0	6	31
ES	-1.3973	-0.9396	-0.685	-1.1252	-0.8445	-0.647

5. Conclusion

In this paper we have modeled the complex and time varying dependence structure among different regions in the Australian electricity markets using the introduced Vine Copula model. Based on that, we proposed the Vine Copula based Portfolio Value at Risk model that demonstrates the improved performance during the empirical study's in the Australian electricity markets.

The improved risk estimate accuracy suggests that the nonnormal dependence stricter captured by the Vine Copula model is more appropriate in characterizing the markets data and its multivariate distribution beyond the common elliptical form.

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