Verification of Product Performance Management by Process Capability Index

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Abstract. Product performance is usually different each other because a machining and an assembling errors are inevitable. Therefore, machining and assembling processes are monitored and controlled in plant to reduce those errors. Especially in mass production, those processes are controlled statistically, and evaluated or monitored by process capability index or process performance index. The author has been researching possibility of new tolerancing method called statistical tolerance index (STI), which specifies the limitations of process capability indices such as Cp, Cpk, Cc and Cpm on design drawing. Because the STI can control distributions of machined parts sizes, it can be useful tool to control product performance or quality in a mass production process. To evaluate an efficiency of the STI specification, case studies are conducted using cantilever model. The cantilever performance is defined as the maximum tips displacement in this paper, which are depended on sizes of the cantilever. Variation of the sizes is modelled according to its distributions. The distribution is assume to be normal distribution. Specifying STI with tolerance on drawing, it is concluded that product performance should be managed by rather the STI than tolerances.

Keywords: product performance, tolerance, process capability index

1. INTRODUCTION

Up to now, many tolerancing method have been developed. Tolerances of mechanical parts directly effect on product performance and manufacturing cost. Therefore, suitable tolerance should be specified at design stage with consideration of those factors. As you know, lower manufacturing cost and higher product performance are desired. However, those are not compatible with each other.

Product performance of each product is variable because a machining and an assembling errors are inevitable. To keep desired product performance, machining and assembling processes are controlled in plant to reduce those errors. Especially in mass production, those processes are controlled statistically, and evaluated or monitored by process capability index or process performance index. The author has been researching possibility of new tolerancing method called statistical tolerance index (STI), which specifies the limitations of process capability indices such as Cp, Cpk, Cc and Cpm on design drawing. Because the STI can control distributions of machined parts sizes, it can control product performance in a mass production process. To evaluate an efficiency of the STI specification, case study are conducted using cantilever model. The cantilever performance is defined as the maximum tips displacement in this paper, which are depended on size of outlines. Simulation is conducted to calculate each product performance, and variation of the sizes are modelled by changing its parameters within the STI. As a result, product performance should be managed by rather the STI than tolerances.

2. STATISTICAL TOLERANCE INDEX

STI (Statistical Tolerance Index) is tolerance-specification using PCI (Process Capability Index) in a design drawing, as shown in Fig. 1. PCI is the parameter quantified by comparing produced output data with a specified tolerance. Although an actual process does not follow a normal distribution, we assume that the process follows it. There are mainly four parameters, C_p, C_{pk}, C_c, and C_{pm}, which are defined as follows,

$$C_p = \frac{U_i - L_i}{6\sigma_i} \tag{1}$$

$$C_{pk} = \min\left\{\frac{\mu_i - L_i}{3\sigma_i}, \frac{U_i - \mu_i}{3\sigma_i}\right\} \tag{2}$$

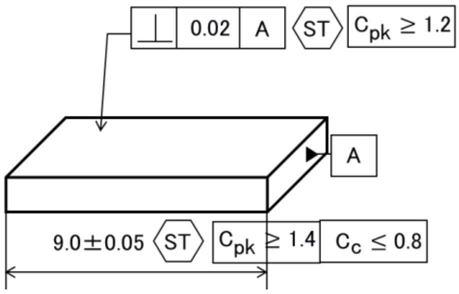

Figure 1. Example of STI specification

$$C_c = \max\left\{\frac{\tau_i - \mu_i}{\tau_i - L_i}, \frac{\mu_i - \tau_i}{U_i - \tau_i}\right\} \tag{3}$$

$$C_{pm} = \frac{U_i - L_i}{6\sqrt{\sigma_i^2 + (\mu_i - \tau_i)^2}} \tag{4}$$

where i is the identifier of parts comprising a product, U and L are the upper and lower limits of size, respectively. σ and μ are the standard deviation and mean of the machined parts, respectively, and τ is the target dimension which exists in tolerance range. When manufacturing process is under statistical control, process capability of the process can be evaluated using these parameters.

STI is to specify these parameters with tolerance on design drawing as shown in Fig. 1. Tolerance controls only limit of size, but STI controls limit of size distribution. Therefore, STI can control product performance more preciously. Only the defect of STI will be cost higher because it is an additional specification.

3. PRODUCT PEFROMANCE

It is assumed that tolerances are already calculated by considering functional limit. Even if machined parts satisfying tolerance limit, x_i is variable between U_i and L_i, performance is variable. Therefore, performance of mass production should be considered statistically. In this study, each x_i is assumed to follow normal distribution $N(\mu_i , \sigma_i)$. Where, μ_i and σ_i are means and standard deviations of x_i. Product performance is depended on each part dimension as follow.

$$y = f(x_1, x_2 \ldots x_n) \tag{5}$$

Where, y is product performance of each part and f is a function related to performance and parts dimensions. Please note that, it the function f is not linear, the performance does not follow a normal distribution. If the product is mass produced, the performance will be controlled lot by lot. Now, product performance of each lot

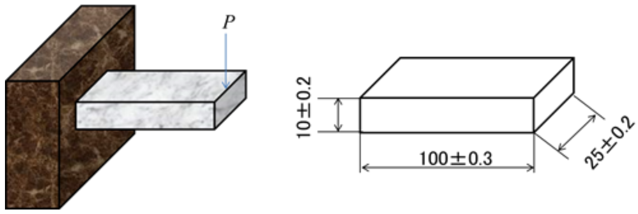

Figure 2. Cantilever as a product model

is defined as a mean value as follows,

$$\bar{y} = \frac{1}{l}\sum_{j=1}^{l} y_j \tag{6}$$

where j is the identifier of lots, and l is a number of lot. In this paper, the value determined by Eq. 6 is called as performance.

The author have studied new tolerancing method using STI and tolerance. The study shows that specifying tolerances is not enough to control the performance at design stage, especially in mass production. Although there are some useful statistical tolerance decision method, the tolerance only specify limit of parts size. If designers specify the distribution of the size, the performance will be suitably controlled. In this study, effectiveness of the STI is evaluated through easy product model.

4. CASE STUDY

4.1 Product Model

In this study, a cantilever is used as a product model, and is used in many products, i.e, building constructs and atomic force microscope. One of performance of the cantilever is a displacement value when its tip is loaded with a force. In this study, the simplest cantilever model as shown in Fig. 2 is used in case study. Tip Displacement of the cantilever D [mm] is theoretically calculated as follows,

$$D = \frac{Px_1^3}{3EI} = \frac{4Px_1^3}{x_2 x_3^3 E} \tag{7}$$

where, P [N] is loaded force and E [N/mm^2] is the modulus of longitudinal elasticity of the material, x_1 [mm] and x_2 [mm] and x_3 [mm] is length and width and height of the cantilever. In this paper, it is assumed that is regarded as product performance. Nominal values and tolerances of them are assumed to be determined as shown in Fig. 2, based on required displacement limit. Nominal value of x_1, x_2 and x_3 is set to 100, 25 and 10 respectively. Tolerance of x_1, x_2 and x_3 is set to plus or minus 0.3, 0.2 and 0.2, respectively.

The loaded force P is set to 100 [N]. The material is assumed to be a steel, so that the modulus of longitudinal elasticity E is set to 200,000. If those conditions are given,

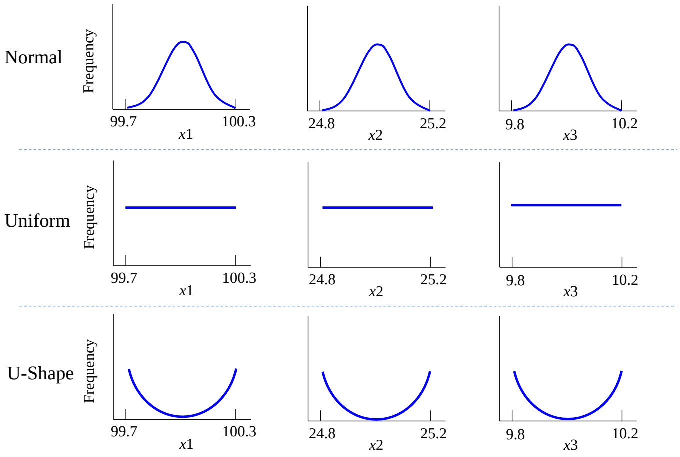

Figure 3. Mass production simulation assuming normal, uniform or U-shape distribution for each size.

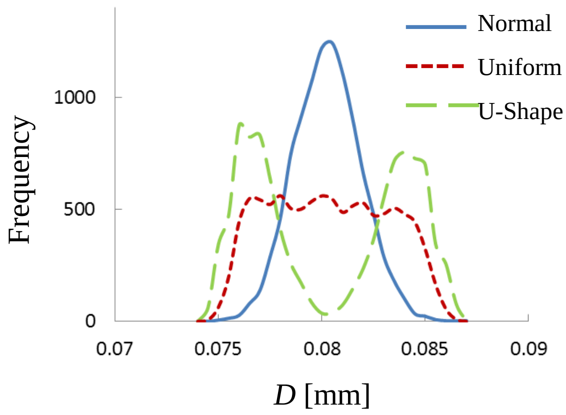

Figure 4. Simulation result of performance distribution ion for each distribution assumptions.

designer can calculate the limit of displacement. If all of the dimensions are nominal value, then D is 0.008. The maximum value of D is 0.0865, when $x_1 = 100.3$, $x_2 = 24.8$ and $x_3 = 9.8$. The minimum value of D is 0.0741, when $x_1 = 99.7$, $x_2 = 25.2$ and $x_3 = 10.2$.

In other word, if tolerances are set as shown in Fig. 2, the displacement value of the cantilever falls in the range between 0.0737 and 0.0859. However, the distribution of the displacement cannot be controlled. Although normal distribution is assumed, its parameters mean and standard deviation are not controllable. Usual methods additionally assume that the mean is at center of tolerance and the standard deviation is one sixth of the tolerance range. However, at manufacturing stage, who controls or verifies the distribution? This assumption is based on a nominal manufacturing process. If the process ignore the distribution control, quality loss of both customer and manufacturer will be increased. Therefore, designer should use STI to control the distribution.

In the following subsections, two simulations are conducted to verify an effectiveness of STI specification. One is the case in which the only tolerance is specified, and another is the case in which the tolerance and STI are specified.

4.2 Simulation with only Tolerance

First, mass production simulation is conducted in the case when only tolerance is specified. The three type of distributions, normal, uniform and U-shape, are assumed and those means dimension sizes are also assumed to be centered. Furthermore, standard deviations of distributions of dimension sizes are set to $T/6$ for normal distribution. This assumption is often used in statistical tolerancing. Lot size is set to 10,000, and a number of lot in simulation l is set to 100. Under these conditions, the frequency distribution is obtained as shown in Fig. 4. The result could be useful if the sizes of dimensions follow normal distribution with the desired parameters. However, those parameters usually change in mass production. If designers do not specify other condition, the distribution of the product performance could be generated as shown in Fig. 4. Although the result of the normal distribution seems to be enough good, who controls the distribution to be nominal in actual manufacturing process? That is not designers but manufacturers. Therefore, limitation of the distribution, that is STI, should be specified to control product performance at design stage. STI also cannot directly control shape of size distribution, but indirectly control its mean and standard deviation. If the STI are specified on design drawing, mean and standard deviation of product performance is limited.

5. CONCLUSIONS

In this paper, an efficiency of the STI specification are explained through case study using cantilever model. The cantilever performance is defined as the maximum tips displacement, which are depended on size of outlines. Changing shapes of size distributions, it is confirmed that tolerance cannot control performance distribution. As a result, product performance should be managed by rather the STI than tolerances..

ACKNOWLEDGMENTS

This study was supported by JSPS KAKENHI Grant Number 26870774.

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