Impacts of capital constraints on retailer's integrated ordering and pricing policies towards seasonal products

Jinzhao Shi
School of Management
Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi 710049, China
Department of Systems Engineering & Engineering Management
City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China
Tel: (+86) 187-0676-0470, Email: shi901226@stu.xjtu.edu.cn

Richard Y.K. Fung †
Department of Systems Engineering & Engineering Management
City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China
Tel: (+852) 3442-8413, Email: richard.fung@cityu.edu.hk

Ju'e Guo
School of Management
Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi 710049, China
Tel: (+86) 029-8266-5049, Email: guojue@mail.xjtu.edu.cn

Abstract. This paper studies the impacts of capital constraints on retailer's integrated ordering and pricing policies, and explores how these impacts can be regulated by different demand uncertainty levels. The simplest supply chain comprising one manufacturer and one retailer is considered. The retailer is embedded with zero internal capital, but it can be financed by external banks. Studies show that when the market size is extremely small, retailers tend not to borrow capital to order. Otherwise, it will borrow to order the optimal order quantity, and sell at a uniquely determined price. By comparing the optimal policies between the capital-constrained retailers and the well-funded retailers, it is found that the capital-constrained retailers tend to opt for ordering a small quantity even at a higher price when the demand uncertainty level is relatively low. However, when the demand uncertainty is relatively high, capital-constrained retailers are likely to set a lower selling price than the well-funded ones, implying that the pricing policy of capital-constrained retailers can be influenced by different demand uncertainty levels.

Keywords: ordering and pricing; capital constraints; demand uncertainty; seasonal product

1. INTRODUCTION

With the rapid advance of science and technology, more and more products become fashionable or seasonal goods are subject to have shorter sales cycle, lower salvage value and higher demand uncertainty (Hu et al., 2014). For instance, cut flowers, clothing and even smartphones are all common products with fashionable or seasonal attributes, and inventory management of these products is complicated (Li et al., 2012; Chen et al., 2014). What is more, the vast of enterprises in the industry are small and medium-sized enterprises (SMEs), often with shortage of liquid funds (Sun, 2015). Under the influence of global financial crisis, enterprises all over the world are threatened by capital shortage even bankruptcy, in manufacturers, distributors and retailers, etc. Therefore, in the last decade, interfaces of operations and financial decisions receive substantial interest and the so called "capital-constrained newsvendor" problem receives widely increasing attentions (Kouvelis and Zhao, 2012).

With "capital-constrained newsvendor" problem, many scholars study the retailer's integrated decision on operations and financing in the presence of capital constraint, including optimal order quantity (Dada and Hu, 2008), purchase timing (Yan and Wang, 2014; Feng et al., 2014), financing equilibrium (Jing et al., 2012; Jing and

Seidmann, 2014; Chen, 2015) and so on. These researches are all based on classical newsvendor models where market prices and demand distributions are exogenous. However, in most practical cases, market demands are price-dependent, thus characterizing effects of price into the newsvendor problem is necessary (Petruzzi and Dada, 1999; Li et al., 2012). For instance, demand uncertainty and demand-price elasticity of smartphones are both relatively high, so it is crucial for smartphone companies (e.g., Samsung, Huawei, Xiaomi) to make an integrated decision on capacity and pricing before releasing a new smartphone to the market.

Considering random price-dependent market demands, this paper combines pricing decision into the "capital-constrained newsvendor" problem and investigates retailer's integrated decisions on operations, marketing and financing, which is not observed in existing literature. Furthermore, the impacts of capital constraint on retailer's optimal ordering and pricing policies are investigated by comparing the optimal policies between the capital-constrained and the well-funded retailers, and how these impacts can be regulated by different demand uncertainty levels is also studied.

Table 1: Summary of notations

Notation	Definition
a	demand intercept which represents market size
b	price coefficient which represents slope of the linear demand curve
ε	random term in demand function
$-A$, A	lower and upper bound of ε respectively whose absolute values measure the level of demand uncertainty
$f(\varepsilon)$, $F(\varepsilon)$	PDF and CDF of ε respectively
r	loan interest rate
D	market demand
w	unit wholesale price announced by the manufacturer
p	unit selling price set by the retailer
Q	order quantity of the retailer
Q^*, p^*	optimal order quantity and selling price of the well-funded retailer
Q^{**}, p^{**}	optimal order quantity and selling price of the capital-constrained retailer
z	stocking factor whose definition will be explained in the ensuing chapters
z^*, z^{**}	optimal stocking factor of well-funded retailer and capital-constrained retailer respectively
π_{wr}, π_{cr}	profit of well-funded retailer and capital-constrained retailer respectively

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

This paper considers a supply chain comprising one manufacturer and one retailer. The retailer purchases single seasonal products from the manufacturer before the selling season, and then sells them to the customers. The market demand is randomly price-dependent and is formulated linearly as $D(p,\varepsilon)=a-bp+\varepsilon$, where $\varepsilon\in[-A,A]$ is a random term with a uniform distribution, whose PDF and CDF are $f(\varepsilon)$ and $F(\varepsilon)$ respectively, and a, b, A are all positive (Petruzzi and Dada, 1999; Li et al., 2012; Wang and Chen, 2015) [①]. The retailer may face two different kinds of financial statuses, i.e., well-funded and capital-constrained. For convenience, a capital-constrained retailer's internal capital level is normalized to zero without loss of generality (Jing and Seidmann, 2014; Chen, 2015), and it can borrow from external banks at interest rate r ($0\le r\le 1$). With risk neutral assumption, a well-funded retailer's objective is to determine the optimal order quantity and selling price to maximize the expected profit from a given product, while a capital-constrained retailer should make a financing decision before making the ordering and pricing decisions. For seasonal products, salvage value of leftovers and penalty cost of shortages are all assumed to be zero (Kouvelis and Zhao, 2012; Jing and Seidmann, 2014; Chen, 2015). The main model notations are summarized in Table 1. Two crucial assumptions are made as follows.

Assumption 1. $a-wb-A>0$, which means that the market demand is always positive supposing the product is sold at a wholesale price w. A similar assumption is made by Li et al. (2012) to ensure the market size a is not too small.

Assumption 2. $A\le wb$, which means that the random term ε alone cannot bring a positive demand even though the product is sold at a wholesale price w. It must be pointed out that this assumption is made to ensure the

① Normally, $\varepsilon\in[A,B]$ is adopted in related literatures. Here, we adopt $\varepsilon\in[-A,A]$ to reduce the amount of parameters as well as measure the demand uncertainty level by absolute value of A which describes the range of the random fluctuation.

demand uncertainty level is not too high, and it is critical for yielding regular conclusions in this paper. When $A > wb$, results will be more intricate and cannot be derived by strict mathematical proofs, hence it is further studied using numerical experiments in Section 5.2.

3. MODEL FORMULATIONS AND SOLUTIONS

3.1 Well-Funded Retailer Case

The well-funded retailer case is studied first as a benchmark where the retailer is assumed to be embedded with sufficient internal capital to support its purchase decision. The retailer's objective is to determine the optimal order quantity Q^* and optimal selling price p^* to maximize the expected profit. The problem is formulated as follows:

$$\max_{Q,p} \pi_{wr} = pE\min\left(Q \wedge D\right) - wQ \tag{1}$$

Based on $E\min\left(Q \wedge D\right) = Q - E\left(Q - D\right)^+$ where $E\left(Q-D\right)^+ = \max\left(Q - D, 0\right)$, Eq. (1) can be converted as:

$$\max_{Q,p} \pi_{wr} = (p - w)Q - pE(Q - D)^+ \tag{2}$$

Defining a substitution variable z named *stocking factor*: $z = Q - (a - bp)$ where $z \in [-A, A]$ (Petruzzi and Dada, 1999; Li et al., 2012). If the realized value of random term ε is larger than z, then market demand exceeds order quantity and shortages occur; otherwise, market demand is smaller than order quantity with leftovers. Then, the problem of determining optimal quantity Q^* and price p^* can be converted into the problem of determining optimal stocking factor z^* and price p^*, and Eq. (2) can be further transformed into:

$$\max_{z,p} \pi_{wr} = (p - w)(z + a - bp) - p\Lambda(z) \tag{3}$$

Where $\Lambda(z) = \int_{-A}^{z} (z - \varepsilon) f(\varepsilon) d\varepsilon$. The second-order derivative of π_{wr} with respect to p can be given as:

$$\frac{\partial^2 \pi_{wr}}{\partial p^2} = -2b \tag{4}$$

Obviously, Eq. (4) is strictly negative, thus π_{wr} is always concave in p^* for any given z. Therefore, a two-step optimization method can be used to solve the problem. Firstly, supposing z is given, the unique optimal $p^*(z)$ can be obtained from the first-order derivative of π_{wr} concerning p. Substituting $p^*(z)$ into Eq. (3), then the objective function will contain only one decision variable z. Once the optimal stocking factor z^* is solved, the optimal price can be obtained as $p^*(z^*)$, and the optimal order quantity can be determined by $Q^* = a - bp^*(z^*) + z^*$.

Theorem 1. *For the well-funded retailer, the optimal stocking factor* z^* *is uniquely determined by*

$$F(z^*) = \frac{H(z^*) - wb}{H(z^*) + wb} \tag{5}$$

the optimal price p^* *is determined by*

$$p^*(z^*) = \frac{H(z^*) + wb}{2b} \tag{6}$$

and the optimal order quantity Q^* *is thus obtained as*

$$Q^* = z^* + a - bp^*(z^*) \tag{7}$$

where $H(z^*) = a + z^* - \Lambda(z^*)$ *and* $\Lambda(z^*) = \int_{-A}^{z^*} (z^* - \varepsilon) f(\varepsilon) d\varepsilon$.

Proof. See Appendix A.

3.2 Capital-constrained Retailer Case

Following the convention in existing "capital-constrained newsvendor" literature, the capital-constrained retailer's internal capital endowment is normalized to zero without loss of generality (Jing and Seidmann, 2014; Chen, 2015). The retailer has access to financing from external banks at loan interest rate r, which is exogenously given, and $r \in [0,1]$. The retailer should make a decision on whether to opt for financing or not, then determine the optimal order quantity Q^{**} and the optimal selling price p^{**}. At the end of the selling season, the retailer repays the loan with the sales revenue. The optimization problem can be described as follows:

$$\max_{Q,p} \pi_{cr} = pE\min\left(Q \wedge D\right) - wQ(1 + r) \tag{8}$$

Eq. (8) becomes

$$\max_{Q,p} \pi_{cr} = (p - w(1+r))Q - pE(Q-D)^{+} \tag{9}$$

Then, by applying the same *stocking factor method* presented in Section 3.1, Eq. (9) can be further transformed into

$$\max_{z,p} \pi_{cr} = (p - w(1+r))(z + a - bp) - p\Lambda(z) \tag{10}$$

Where $\Lambda(z)=\int_{-A}^{z}(z-\varepsilon)f(\varepsilon)d\varepsilon$. For any given z, π_{cr} is still concave in p. Hence the same two-step optimization method presented in Section 3.1 can be applied to solve Eq. (10). Theorem 2 can be presented as follows:

Theorem 2. *For capital-constrained retailers,*

(a) If the market size a *satisfies* $a > A + wb(1+r)$, *the optimal stocking factor* z^{**} *can be uniquely determined by*

$$F(z^{**}) = \frac{H(z^{**}) - wb(1+r)}{H(z^{**}) + wb(1+r)} \tag{11}$$

the optimal price p^{**} *is determined by*

$$p^{**}(z^{**}) = \frac{H(z^{**}) + wb(1+r)}{2b} \tag{12}$$

and the optimal order quantity Q^{**} *can be given as*

$$Q^{**} = z^{**} + a - bp^{**}(z^{**}) \tag{13}$$

where $H(z^{**}) = a + z^{**} - \Lambda(z^{**})$ *and* $\Lambda(z^{**})=\int_{-A}^{z^{**}}(z^{**}-\varepsilon)f(\varepsilon)d\varepsilon$.

(b) If market size a *satisfies* $A + wb < a \le A + wb(1+r)$, *retailers will unlikely borrow capital to order any quantity.*

Proof. See Appendix B.

4. MODEL COMPARATIVE ANALYSES

When the market size is extremely small, i.e., $A + wb < a \le A + wb(1+r)$, the optimal order quantity of capital-constrained retailers becomes zero, thus pricing is meaningless. So, comparative analyses of the optimal policies between the well-funded retailer and the capital-constrained retailer are conducted only under the case of $a > A + wb(1+r)$, in most cases.

From Theorem 2, it can be observed that the optimal policies of the capital-constrained retailers are also affected by the loan interest rate r. The optimal stocking factor z^{**} and loan interest rate r can be described as follows.

Proposition 1. *For the capital-constrained retailer, the optimal stocking factor* z^{**} *is strictly decreasing in the loan interest rate* r, *i.e.,* $\partial z^{**} / \partial r < 0$.

Proof. See Appendix C.

Based on Proposition 1, the optimal policies between the capital-constrained retailers and the well-funded retailers can be compared, and the results are further explained in Theorem 3.

Theorem 3. *If market size satisfies* $a > A + wb(1+r)$,

(a) The optimal order quantity of capital-constrained retailers is no more than that of the well-funded ones, i.e. $Q^{**} \le Q^{*}$.

(b) The optimal price of capital-constrained retailers is no less than that of the well-funded ones, i.e. $p^{**} \ge p^{*}$.

Proof. See Appendix D.

Theorem 3 reveals that when demand uncertainty level is relatively low, i.e. $A \le wb$, the capital-constrained retailers tend to conduct a "higher price but smaller quantity" policy comparing with the well-funded ones. The rationale can be inferred as follows: the loan interest rate causes the increase in unit purchase cost, which leads to the increase in selling price. Then, a higher price leads to the decrease in market demand, so a smaller quantity is necessary to control the expected leftovers. This is the best combined ordering and pricing strategy of the capital-constrained retailers to deal with a relatively low demand uncertainty.

However, when the demand uncertainty level is relatively high, i.e. $A > wb$, the conclusions in Theorem 3 will not always yielded. Theoretical results in this case cannot be obtained due to obstacles in mathematical proofs. Therefore, the latter case is further studied using numerical simulations in Section 5.2. Results show that capital-constrained retailers are likely to set a lower price than that of the well-funded ones in some special cases, in contrary to the conclusion in Theorem 3(b).

5. NUMERICAL ANALYSES

The optimal solutions in Theorems 1 and 2 are all embodied in implicit functions which are not intuitional.

Numerical analyses in this section will contribute to a better understanding of the conclusions with two main objectives, (1) validate the conclusions in Section 4, and (2) further explore the case of high demand uncertainties.

5.1 Verification of Theoretical Results

To validate the results of Theorem 3, the following parameters are used: $A=5$, $w=3$, $b=2$, in order to satisfy Assumption 2 (i.e., $A \le wb$). The market size a should be chosen at a certain value that satisfies $a > A + wb(1+r)$ for any possible value of $r \in [0,1]$, thus $a = 50$ is chosen as an example to meet this condition.

Figure 1 shows the change of stocking factor z^{**} with regard to loan interest rate r. Obviously, the optimal stocking factor is decreasing with increasing loan interest rate, which verifies Proposition 1.

Figures 2 and 3 describe the relations between the optimal policies of the capital-constrained retailers and the well-funded retailers. Simulation results show that the optimal order quantity of the capital-constrained retailers is equal or less than that of the well-funded ones, while the optimal price of the capital-constrained retailers is equal or more than that of the well-funded ones, consistent with the conclusions in Theorem 3. Moreover, it can be observed that the equivalent cases occur only when the loan interest rate equals zero, and as the loan interest rate rises, the difference between the optimal solutions will widen. It should also be noted that the optimal solutions in Theorem 2 vary non-linearly with the loan interest rate, even though the results of numerical simulations look linear (see the star-solid lines in Figures 2 and 3).

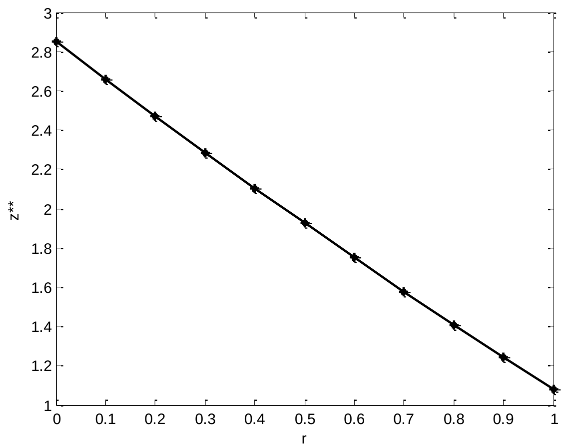

Figure 1: Stocking factor with loan interest rate varying

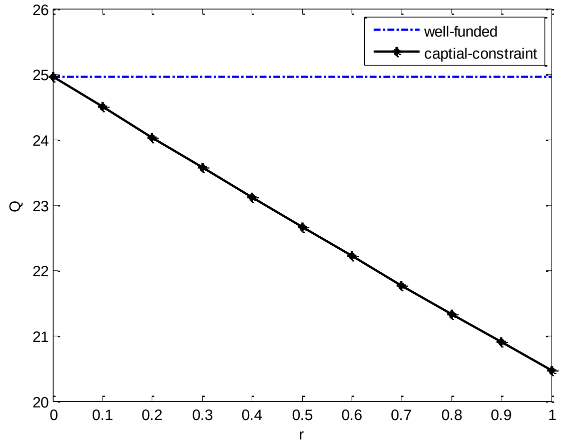

Figure 2: Comparison of the optimal order quantities

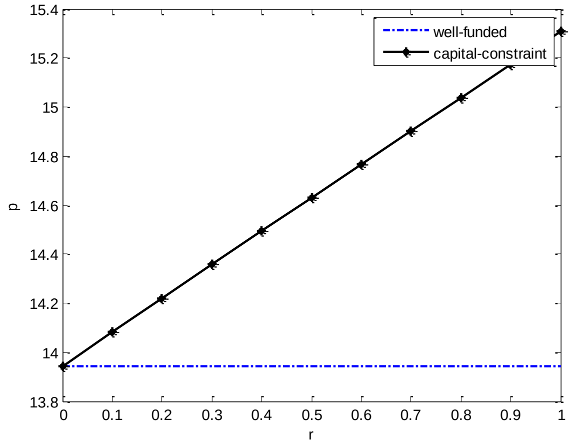

Figure 3: Comparison of the optimal prices

5.2 Extension for the case $A > wb$

Consistent with Section 5.1, $w=3$, $b=2$ are used. With a high demand uncertainty, i.e. $A > wb$, Table 2 shows the applicability of conclusions in Theorem 3. Results show that the conclusion in Theorem 3(a) is always applicable in a high demand certainty scenario. It can be concluded that capital-constrained retailers always choose a lower order quantity than that of the well-funded ones, independent of the demand uncertainty level. However, the conclusion in Theorem 3(b) is inapplicable when demand uncertainty level falls within a specific interval (i.e. approximate 8.00~32.00).

The inapplicable case is then further studied. For Theorem 3(b), when the demand uncertainty level is higher than 8.00 but lower than 32.00, a small interval of market size a (see Table 3) make the result $p^{**} < p^{*}$ when assigned with specific values of loan interest rate r, contrary to the result in Theorem 3(b) when $A \le wb$. An example is shown in Figure 4 with $A = 20$ and a=33. It can be observed that the case $p^{**} < p^{*}$ will occur with some higher values of r, ranging 0.83~1.00.

Table 3: The interval of market size incurs $p^{**} < p^{*}$ under $A > wb$

A	6.00-8.00	8.10	10.00	20.00	30.00	31.90	≥32.00
a	—	(20.10, 20.15)	(22.00, 22.83)	(32.00, 33.95)	(42.00, 42.47)	(43.90, 43.92)	—

Table 2: Applicability of Theorem 3 when $A > wb$

A	Theorem 3(a)	Theorem 3(b)
6.00~8.00	√	√
8.00~32.00	√	×
≥32.00	√	√

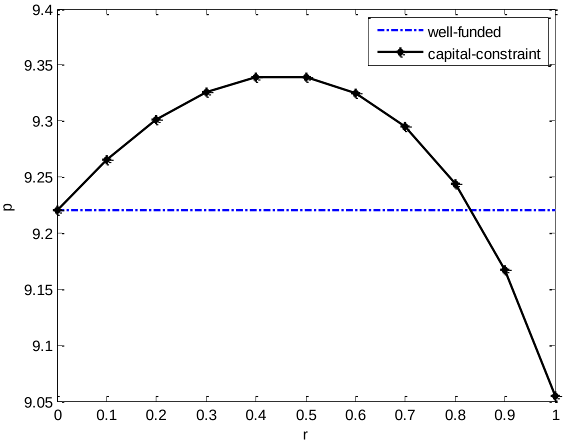

Figure 4: Comparison of the optimal prices (A=20, a=33)

In summary, capital-constrained retailers are likely to set a lower price than that of the well-funded ones when demand uncertainty level falls within a subinterval of $A > wb$, different from the result obtained in the case when $A \leq wb$ where the capital-constrained retailers always set a higher price than that of the well-funded ones. Therefore, it can be further concluded that the impact of capital constraints on a retailer's optimal pricing decision can be regulated at different demand uncertainty levels.

6. Conclusions

This paper combines pricing decision making into the "capital-constrained newsvendor" problem and investigates retailer's integrated ordering and pricing policies in the presence of capital constraints. Results show that when the market size is extremely small, the retailers will not borrow from the external bank for ordering. Otherwise, they will borrow to purchase an optimal order quantity and the selling price can be uniquely determined.

The impacts of capital constraints on retailer's ordering and pricing policies are also investigated. Results show that when the demand uncertainty level is relatively low, capital-constrained retailers will always adopt a "higher price but smaller quantity" policy. However, when demand uncertainty level is relatively high, capital-constrained retailers are more likely to set a lower price than that of the well-funded ones, meaning that the impact of capital constraints on retailer's pricing decisions can be regulated by different demand uncertainty levels.

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APPENDICES

Appendix A. Proof of Theorem 1

Based on Eq. (4), for any given z, π_{wr} is concave in p, so the unique optimal $p^*(z)$ can be obtained by solving $\partial\pi_{wr}/\partial p=0$, i.e.

$$p^*(z)=\frac{a+z-\square(z)+wb}{2b} \tag{14}$$

Then, substituting $p^*(z)$ into Eq. (3) and taking the first-order derivative of π_{wr} with respect to z, based on the chain rule, yields

$$\begin{aligned}&\frac{d\pi_{wr}}{dz}=\frac{\partial\pi_{wr}}{\partial z}+\frac{\partial\pi_{wr}}{\partial p^*(z)}\frac{dp^*(z)}{dz}=\\&\frac{1}{2b}\left((a+z-\square(z)+wb)(1-F(z))-2wb\right)=\frac{1}{2b}G(z)\end{aligned} \tag{15}$$

The second-order derivative of $G(z)$ is

$$\frac{\partial^2 G(z)}{\partial z^2}=-3f(z)(1-F(z))\le 0 \tag{16}$$

Thus $G(z)$ is concave and unimodal in z. Further, since $G(A)=-2wb<0$ and $G(-A)=a-wb-A>0$ (Assumption 1), there always exists a unique $z^*\in[-A,A]$ that satisfies $G(z^*)=0$. Obviously, when $z<z^*$, $d\pi_{wr}/dz>0$; and when $z>z^*$, $d\pi_{wr}/dz<0$. By setting Eq. (15) to zero, a unique z^* that maximizes π_{wr} can be obtained as shown in Eq. (5) where $H(z^*)=a+z^*-\square(z^*)$. Then, the optimal price $p^*(z^*)$ is determined by substituting z^* into Eq. (14), and the optimal order quantity is $Q^*=a-bp^*(z^*)+z^*$ based on the definition of stocking factor z.

Appendix B. Proof of Theorem 2

For any given z, $\partial^2\pi_{cr}/\partial p^2=-2b<0$, so π_{cr} is also concave in p. The unique optimal $p^{**}(z)$ can be obtained by solving $\partial\pi_{cr}/\partial p=0$. Then, by substituting $p^{**}(z)$ into Eq. (10) and taking the first-order derivative of π_{cr} with respect to z, based on the chain rule,

$$\frac{d\pi_{cr}}{dz}=\frac{1}{2b}\Big((a+z-\square(z)+wb(1+r))(1-F(z))-2wb(1+r)\Big)$$
$$=\frac{1}{2b}G(z) \tag{17}$$

The second-order derivative of $G(z)$ is the same as shown in Eq. (16), so $G(z)$ is concave and unimodal in z. Furthermore, it can be easily calculated that $G(A)=-2wb(1+r)<0$ and $G(-A)=a-A-wb(1+r)$.

Case (a): When $a>A+wb(1+r)$, yields $G(-A)>0$, the optimal stocking factor can be uniquely determined, and the proof is similar to that of Theorem 1.

Case (b): When $A+wb<a\le A+wb(1+r)$, yields $G(-A)\le 0$. Since $G'(-A)=1-f(-A)(a-A+wb(1+r)) <1-wb/A-wbr/2A<0$ (Assumptions 1 and 2), it can be concluded that $G(z)\le 0$ in the interval $[-A,A]$, thus π_{cr} is decreasing in z over $[-A,A]$ and the optimal stocking factor is $z^{**}=-A$. Then, the optimal selling price and order quantity can be determined as $p^{**}(-A)=(a-A+wb(1+r))/2b$ and $Q^{**}=(a-A-wb(1+r))/2$ respectively. Since $a\le A+wb(1+r)$, yields $Q^{**}\le 0$. Because order quantity must be non-negative, i.e. $Q^{**}\ge 0$, and based on $Q^{**}=(a+z^{**}+\square(z^{**})-wb)/2$, a larger Q^{**} requires a larger z^{**} causing a decrease in π_{cr}. Thus, $Q^{**}=0$ is the optimal decision for case (b).

Appendix C. Proof of Proposition 1

From Eq. (11), it can be derived that

$$\frac{\partial z^{**}}{\partial r}=\frac{wb(1+F(z^{**}))}{(1-F(z^{**}))^2-f(z^{**})H(z^{**})-wb(1+r)f(z^{**})} \tag{18}$$

As the numerator is positive, set the denominator be $T(z^{**})$, taking the first-order derivative of $T(z^{**})$ with respect to z^{**}, yields $\partial T(z^{**})/\partial z^{**}=-3f(z^{**})(1-F(z^{**}))$ which is non-positive. Since $a>A+wb(1+r)$, the maximum value of $T(z^{**})$ is

$$T(-A)=\frac{3A-a-wb(1+r)}{2A}<\frac{A-wb(1+r)}{A}\le 0 \tag{19}$$

Hence, $T(z^{**})$ is always negative over $[-A,A]$. Thus, Eq. (18) is negative, and the proof is completed.

Appendix D. Proof of Theorem 3

Proof for part (a). From Eq. (5) and (11), when $r=0$, $Q^{**}=Q^{*}$. Since $r\in[0,1]$ and Q^{*} is independent of r, as long as $\partial Q^{**}/\partial r<0$ is proved, $Q^{**}\le Q^{*}$ yields. From Eq. (12) and (13),

$$\frac{\partial Q^{**}}{\partial r}=\frac{1}{2}\left((1+F(z^{**}))\frac{\partial z^{**}}{\partial r}-wb\right) \tag{20}$$

Based on Proposition 1, Eq. (20) is negative.

Proof for part (b). Similar to the proof for part (a), when $r=0$, $p^{**}=p^{*}$ holds. Hence, as long as $\partial p^{**}/\partial r\ge 0$ is proved, $p^{**}\ge p^{*}$ will be automatically proved. From Eq. (12), taking the first-order derivative of p^{**} with respect to r, yields

$$\frac{\partial p^{**}}{\partial r}=\frac{1}{2b}((1-F(z^{**}))\frac{\partial z^{**}}{\partial r}+wb) \tag{21}$$

Substituting Eq. (18) into Eq. (21), yields

$$\frac{\partial p^{**}}{\partial r}=\frac{1}{2b}\left(\frac{wb(1-F^2(z^{**}))}{(1-F(z^{**}))^2-f(z^{**})H(z^{**})-wb(1+r)f(z^{**})}+wb\right)$$
$$=\frac{1}{2b}R(z^{**}) \tag{22}$$

Proving $\partial p^{**}/\partial r\ge 0$ is equivalent to proving $R(z^{**})\ge 0$ as follows

$$(1-F^2(z^{**}))\le f(z^{**})H(z^{**})+wb(1+r)f(z^{**})-(1-F(z^{**}))^2 \tag{23}$$

Further, Eq. (23) can be reduced to

$$U(z^{**})=z^{**^2}-10Az^{**}+9A^2-4A(a+wb(1+r))\le 0 \tag{24}$$

Next, Eq. (24) will be proved to be always true for $z^{**} \in [-A, A]$. It is clear that when $z^{**}=A$, $U(A)=-4A(a+wb(1+r))<0$. From the proof of case (a) of Theorem 2, it can be inferred that $z^{**}=-A$ occurs only when the market size infinitely approaches to its lower bound, i.e. $a \to A+wb(1+r)$. Further, $a > A+2wb$ must hold in order to ensure that $a > A+wb(1+r)$ for any possible $r \in [0,1]$. Thus, $a \to A+wb(1+r)$ occurs only when $a \to A+2wb$ in conjunction with $r \to 1$. Therefore,

$$U(-A)=20A^2-4A(A+2wb+wb(1+1))=16A(A-wb) \quad (25)$$

Based on Assumption 2, Eq. (25) is non-positive. Since $U(z^{**})$ is a quadratic function with convexity, it can be concluded that $U(z^{**}) \le 0$ over $[-A, A]$. Eq. (24) has been proved to be always true, ensuring that $\partial p^{**} / \partial r \ge 0$ holds in Eq. (22). Theorem 3 is thus proved.