Profitability and Risk Analysis for Investment Alternatives on *C*-*R* Domain

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Abstract. This paper investigates a method for evaluating profitability and risk for multiple investment alternatives, for both cases of consistent return over a planned period and fluctuating return year by year. The paper first examines a method for evaluating a single alternative from the viewpoint of profitability and safety. Then it proceeds to the evaluation of multiple mutually exclusive alternatives, out of which the best one is selected. The paper proposes *C*-*R* domain which comprises initial investment and annual return on each of horizontal and vertical axis. On this domain, expected values of net present profit and annual mean profit are represented. Then the procedure for analyzing and evaluating profitability and risk is discussed, and the validity of the proposed method is examined by using numerical examples.

Keywords: Profitability, Risk, Safety, Multiple investment alternatives, *C*-*R* domain

1. INTRODUCTION

Current uncertainties in factors related to investment alternatives, such as initial investment and annual return, require manufacturing companies to pay careful attention to methods for evaluating profitability and rigorousness against expected risks.

Methods for evaluating economic performance for a set of multiple investment alternatives, for both cases of stable return over the planning horizon, and of fluctuating return year by year, are the main area of focus of this paper. This problem has been investigated in the field of engineering economy, and basic procedures have been clarified and modified/extended in previous research (Senju *et al*., 1982, 1986, 1994; Nakamura, 1985, 2002). Further, general economic evaluation procedures with consideration of risk have been discussed in previous research (Kono, 2003, 2009, 2010, 2015).

The paper presents the basic model for analysis in the next section, and then proceeds to the case of stable return in Section 3, to be followed by the case of fluctuating return in Section 4. Then simple numerical examples in Sections 5 examine the effectiveness of the methods proposed in Section 3 and 4.

2. MODEL FORMULATION

The paper assumes the following investment alternative:

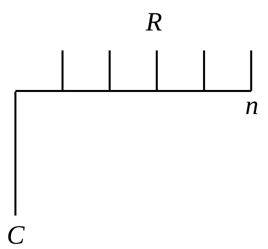

Figure 2.1: Investment alternative with stable return

Where each notation refers to

- C: amount of initial investment
- R: annual return (increase of cash inflow and/or decrease in cash outflow)
- n: period of investment
- i: interest rate to be used as hurdle rate in profit calculation

The above figure represents consistent return type. **Figure 2-2** represents another case of fluctuating return year by year, where R_j means return for the *j-th* year.

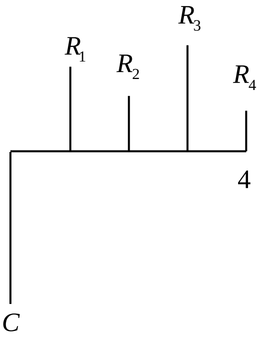

Figure 2.2: Investment alternative with fluctuating return

3. THE CASE OF CONSISTENT RETURN

3.1 Representation of Profit on the *C-R* Domain

In this case, the net present value *P* and annual mean profit M can be obtained by the following equations.

$$P = R \times M \rightarrow P_{n}^{i} - C, \text{ and} \tag{3.1}$$

$$M = R - C \times P \rightarrow M_{n}^{i}. \tag{3.2}$$

where $M \rightarrow P_{n}^{i}$ is called uniform series present worth factor obtained by $\frac{1+i^{\,n}-1}{i\ 1+i^{\,n}}$ and, $P \rightarrow M_{n}^{i}$ is called capital recovery factor defined by $\frac{i\ 1+i^{\,n}}{1+i^{\,n}-1}$.

For the purpose of representing such profit values as *P* and M, this paper proposes a domain whose horizontal axis corresponds to the amount of initial investment *C*, and vertical axis refers to the annual return *R*. This domain is hereafter referred to as *C-R* domain. Then, an investment alternative with initial investment *C* and annual return *R* is represented as a point as shown in **Figure 3.1**. Depicting a line from (0,0) whose slope corresponds to $P \rightarrow M_{n}^{i}$, annual profit M of the investment alternative, which is obtained by statement (3,2), can be represented as in **Figure3.1**.

Here, it is clear that the values of $P \rightarrow M_{n}^{i}$ and $M \rightarrow P_{n}^{i}$ are mutually inverse and holds the next statement

$$P \rightarrow M_{n}^{i} = \frac{1}{M \rightarrow P_{n}^{i}}. \tag{3.3}$$

Therefore, on the *C-R* domain, the line with the value of $M \rightarrow P_{n}^{i}$ can be represented as shown in **Figure 3.2**.

This implies that the net present value *P*, defined by statement (3,1), can be represented as the horizontal length on the *C-R* domain as in **Figure 3.3**.

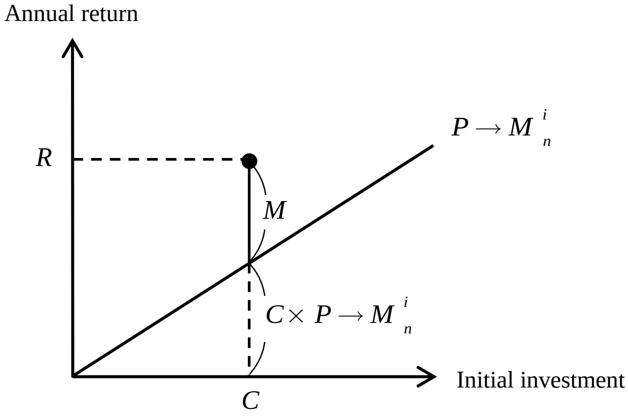

Figure 3.1: Representation of an investment alternative on the *C-R* domain

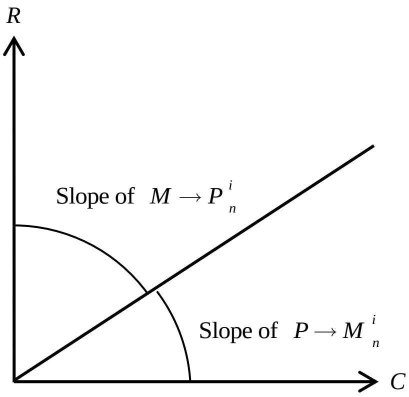

Figure 3.2: The values of $P \rightarrow M_{n}^{i}$ and $M \rightarrow P_{n}^{i}$ on the *C-R* domain

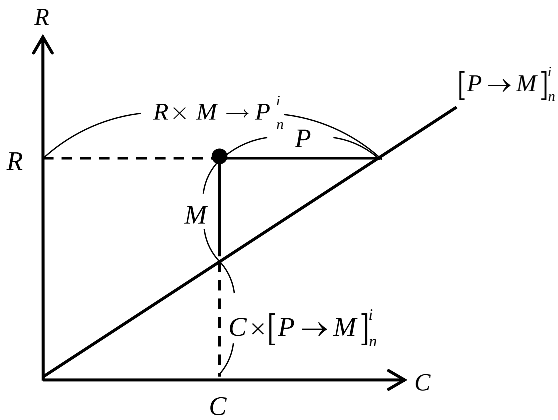

Figure 3.3: Representation of *P* on the *C-R* domain

3.2 Representation of IRR and Payback Period on the *C-R* Domain

Along with the increase in interest rate, the slope of the line of $P \rightarrow M{}_{n}^{i}$ becomes steeper. Since IRR (Internal Rate of Return) is defined to be the interest rate which makes the value of net profit M zero, the following statement is satisfied.

$$r \,|\, M\ \ r\ = 0. \tag{3.4}$$

It follows,

$$M\ \ r\ = R - C \times\ P \rightarrow M{}_{n}^{r} = 0, \tag{3.5}$$

Therefore,

$$P \rightarrow M{}_{n}^{r} = \frac{R}{C}. \tag{3.6}$$

Then, the value of IRR is depicted on the *C-R* domain as shown is **Figure 3.4**.

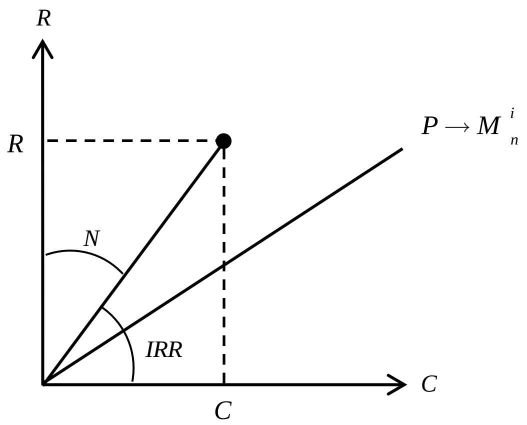

Figure 3.4: IRR and payback period N on the *C-R* domain

On the other hand, as the period n becomes smaller, the value of $P \rightarrow M{}_{n}^{i}$ becomes larger. The value of payback period N is given by the period where the net profit is zero, and thus described by the next statement.

$$P\ \ N\ = R \times\ M \rightarrow P{}_{N}^{i} - C = 0, \tag{3.7}$$

Therefore, it follows,

$$P \rightarrow M{}_{N}^{i} = \frac{C}{R}. \tag{3.8}$$

Then the value of N can be described on the *C-R* domain as in **Figure 3.4**.

Made in Figure 3.4 implies that, among multiple alternatives, one with higher IRR always achieves shorter payback period. Therefore if we evaluate alternatives based on IRR, naturally ones with shorter payback period are selected. It should be noted, however, that selection by IRR is identical with selection by payback period, which leads to selection of alternatives of low risk, not necessarily guaranteeing high economic profit.

3.3 Evaluation of Risk

The paper then analyzes the risk of investment alternatives. In this paper, risks encompass those relating to increase in interest rate, increase in initial investment, and decrease in annual return.

Along with the increase in interest rate, the value of $P \rightarrow M{}_{n}^{i}$ increases. Therefore, the slope of the line of $P \rightarrow M{}_{n}^{i}$ on the *C-R* domain becomes steeper. In the same context, if the value of C is increased to $\alpha C\ \ \alpha > 1$, the annual profit is decreased to

$$M = R - \alpha C \times\ P \rightarrow M{}_{n}^{r}. \tag{3.9}$$

This implies that the decrease in net annual profit can be evaluated in the same context as the case of increase in interest rate.

On the other hand, the decrease in annual profit from original value R to $\beta R\ \ \beta < 1$ can be evaluated by the next statement.

$$\begin{aligned} M &= \beta R - C \times\ P \rightarrow M{}_{n}^{i} \\ &= \beta\left(R - \frac{C}{\beta} \times\ P \rightarrow M{}_{n}^{i}\right). \end{aligned} \tag{3.10}$$

For all the profit on the line connecting (0,0) and plot (*C*, *R*), the decrease in annual return at the ratio β decreases the profit equally. Therefore, it is clear that the risk against decrease in annual return can be evaluated in the same logic as the former two cases.

From the above discussion, risk against expected changes can be evaluated simultaneously on the *C-R* domain, by the line connecting the point (0,0) and each plot (*C*, *R*). The alternative with larger value of *R*/*C* (namely, steeper slope) is more rigorous in terms of risk. But it should be noted that the alternative with higher risk aversion level does not guarantee economic profitability.

3.4 Evaluation of Multiple Alternative

The paper then examines the case of selecting the best one out of mutually exclusive alternatives, which can be delineated in **Figure 3.5**.

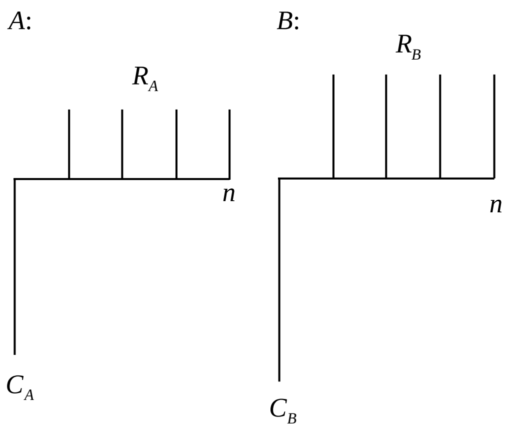

Figure 3.5: Mutually exclusive alternatives

The two alternatives can be represented on the *C*-*R* domain as shown in **Figure 3.6**.

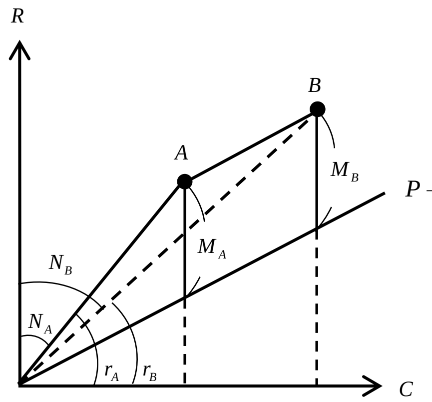

Figure 3.6: Multiple alternatives on the *C*-*R* domain

When line segments connecting (0,0), (C_A,R_A) and (C_B,R_B) create convex, it should be noted that alternative *A* with smaller initial investment always guarantees larger IRR and shorter payback period, although the value of net annual profit for *B* may be larger than *A*. From the viewpoint of risk of increase in interest rate, increase by the same ratio in initial investment, and/or same ratio of decrease in annual return, alternative *A* is more rigorous than alternative *B*.

What requires attention is that, when comparing more than three alternatives, there might be cases where line segments connecting alternatives form concave as in **Figure 3.7**. In such a case, even if the interest rate fluctuates and the slope of $P \to M_n^i$ is changed, the profit for an alternative *B* is always smaller than alternatives *A* or *C*. Thus, the alternative *B* becomes disqualified in terms of profitability. It follows that the set of qualified alternatives on the *C*-*R* domain creates convex line segments as in **Figure 3.8**.

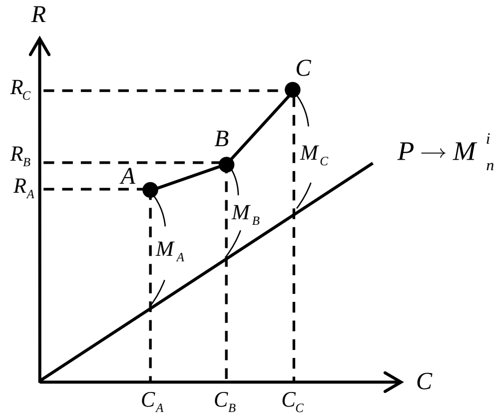

Figure 3.7: A case of concave line segments

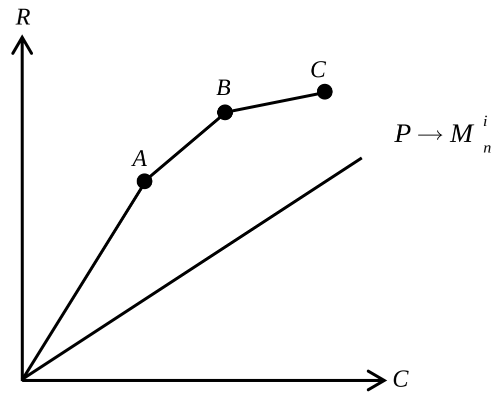

Figure 3.8: Convex line segments on the *C*-*R* domain

4. THE CASE OF INCONSISTENT RETURN

4.1 Representation of Profit on the *C*-*R* Domain

First, the cash flow pattern under investigation is described as in **Figure 4.1**.

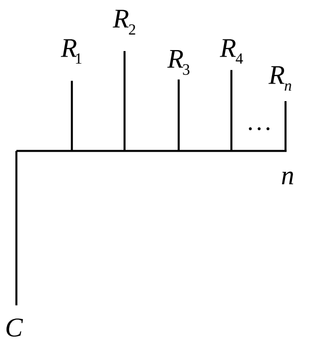

Figure 4.1: Cash flow pattern for the case of inconsistent return

In this case, the net present profit P can be calculated by the next statement, whereas net annual profit is dependent on the return pattern and cannot be obtained directly.

$$P = \sum_{j=1}^{n} \frac{R_j}{(1+i)^{j}} - C \,. \tag{4.1}$$

On the C-R domain, where the vertical axis is converted to the sum of annual return $\sum_{j=1}^{n} R_j$, depicting the line with slope 1 from (0,0), the value of P can be represented as shown in **Figure 4.2**.

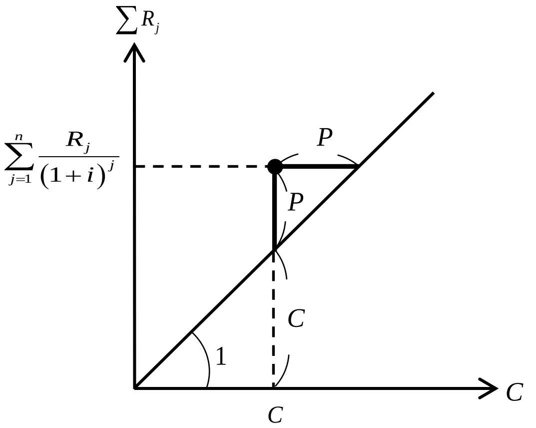

Figure 4.2: Representation of net present profit on the C-R domain

It should be noted that the values of IRR and payback period are dependent on the pattern of annual return, and therefore cannot be represented on the C-R domain for the case of inconsistent return over the horizon.

4.2 Evaluation of Multiple Alternatives

This section analyzes the comparison of multiple alternatives, as illustrated in **Figure 4.3**.

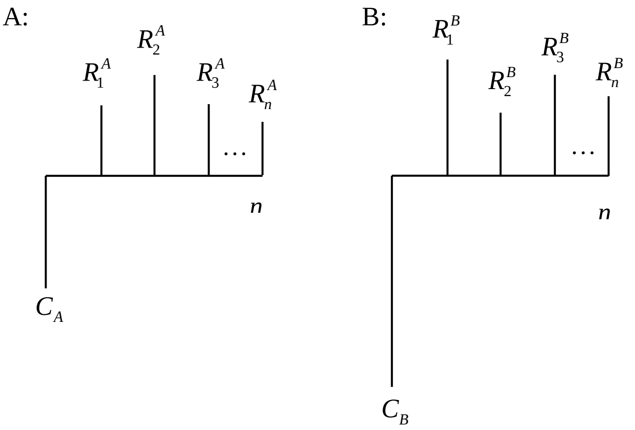

Figure 4.3: Multiple alternatives

The net present profit for alternatives A and B can be calculated by the following statements, where subscripts and superscripts refer to the name of respective alternatives.

$$P_A = \sum_{j=1}^{n} \frac{R_j^A}{(1+i)^{j}} - C_A \,. \tag{4.2}$$

$$P_B = \sum_{j=1}^{n} \frac{R_j^B}{(1+i)^{j}} - C_B \,. \tag{4.3}$$

Then, these values can be represented on the C-R domain as in **Figure 4.3**. Profitability can be evaluated by the length of P_A and P_B as in **Figure 4.4**.

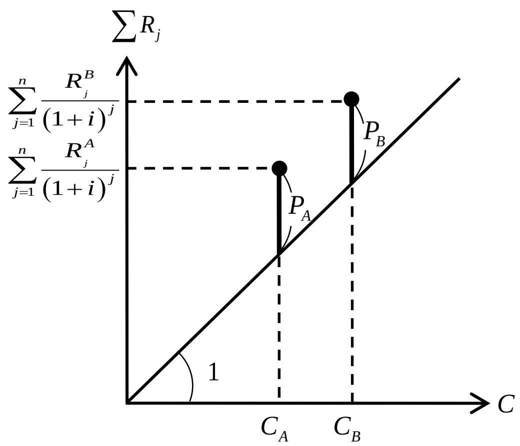

Figure 4.4: Multiple alternatives on the C-R domain

For the purpose of evaluating rigidity under uncertainties, this section considers the case of increase in the same ratio in initial investment $C \to \alpha C, \alpha > 1$ and decrease in the same ratio in annual return $R_j \to \beta R_j,\ \beta < 1,\ j = 1, 2, ..., n$.

In case where the value of initial investment increases from C to α C, as shown in **Figure 4.5**, when the plot A reaches plot A' , the net profit becomes zero. Thus, the BEP for α , denoted by α^* , can be given by the next statement.

$$\alpha^* = \frac{C+P}{C} = \frac{\sum_{j=1}^{n} \frac{R_j}{(1+i)^{j}}}{C} \,. \tag{4,4}$$

On the other hand, if the value of $\frac{\sum_{j=1}^{n} R_j}{(1+i)^{j}}$ is decreased, the plot A on the C-R domain moves downward. And if it reaches A' (refer to **Figure 4.6**), then the net present profit becomes zero. Therefore, BEP β^* for annual return decrease can be given by the next statement.

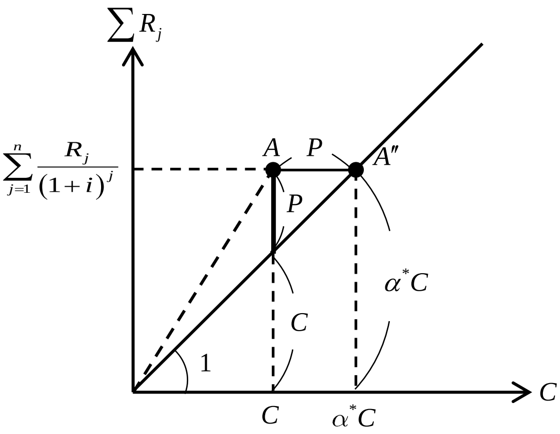

Figure 4.5: BEP α^* on the *C-R* domain

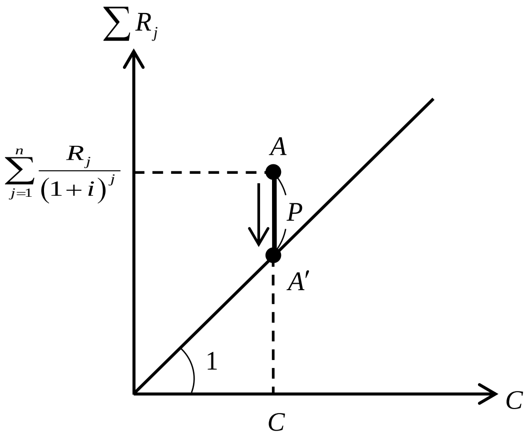

Figure 4.6: BEP β^* on the *C-R* domain

$$\beta^* = \frac{C}{\sum_{j=1}^{n} \frac{R_j}{\left(1+i\right)^{j}}}. \tag{4.5}$$

Statements (4.4) and (4.5) show that α^* and β^* are mutually inverse, satisfying the next equation.

$$\alpha^* \beta^* = 1 \tag{4.6}$$

4.3 Elimination of Disqualified Alternatives

This section examines the case of multiple alternatives represented on the *C-R* domain, where line segments connecting adjacent plots may be concave.

As regards the increase or decrease in value of *C* for the same ratio among candidate alternatives, its impact can be a slope change of the line starting from (0,0), upward with investment increase and downward with investment decrease. For both cases, alternative *B* cannot attain maximum value of *P*, being less than either *A* or *C*.

In the same context, decrease or increase in annual return for the same ratio over each year among candidate alternatives, can be evaluated by the line connecting (0,0) and each plot. Any plot on the same line gets the same impact from the change in annual return.

Therefore, the decrease (or increase) in annual return can be evaluated by the shift of the line with slope 1 starting (0,0) to upward (return decrease) or downward (return increase). In any case, alternative *B* in **Figure 4.7** cannot achieve the maximum profit, and can be therefore disqualified in selecting most profitable alternative.

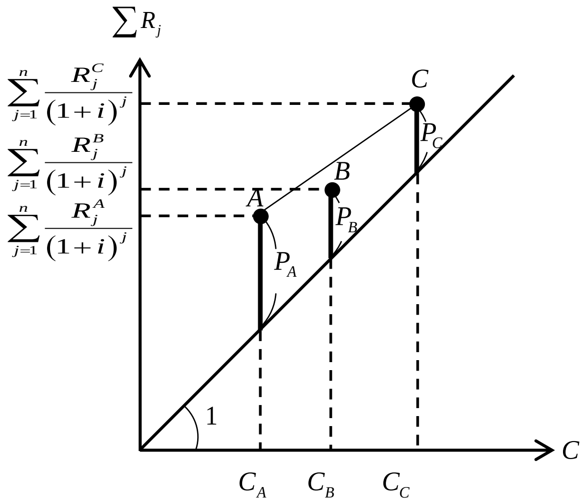

Figure 4.7: Disqualified alternatives on the *C-R* domain

Above discussion leads to a conclusion that a set of qualified alternatives, not from the viewpoint of profit but from that of risk aversion under uncertainty in initial investment and annual return, can create a set of convex line segments as shown in **Figure 4.8**.

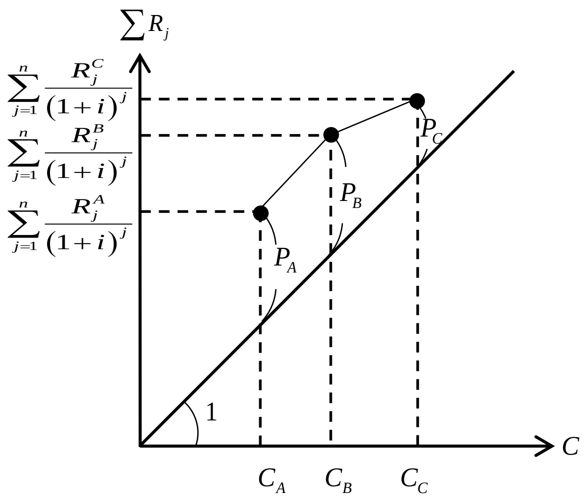

Figure 4.8: Convex set or qualified alternatives on the *C-R* domain

5. NUMERICAL EXAMPLES

5.1 The Case of Consistent Return

This section considers the following three alternatives, with the interest rate i=10%. Annual mean profit M for each alternative can be obtained as follows:

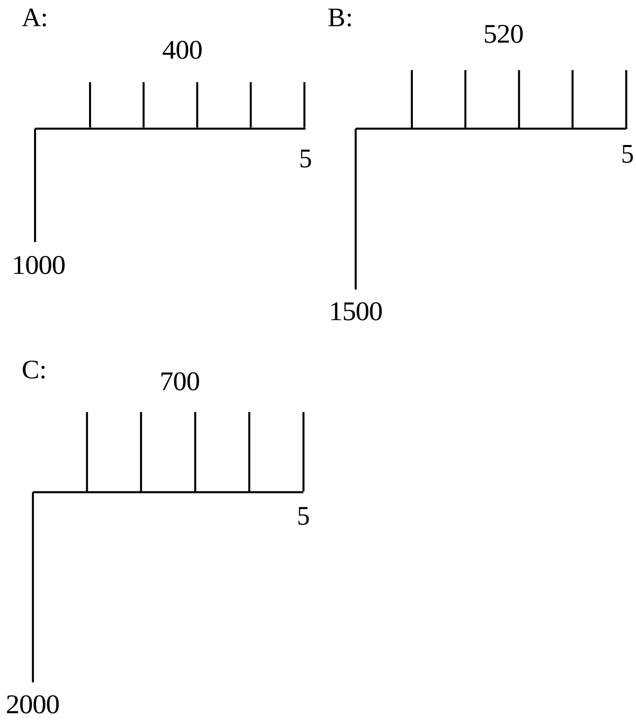

Figure 5.1: Numerical example for the case of consistent return

$$M_A = 400 - 1000 \times P \to M\,{}_{5}^{10} = 136.2 \quad (5.1)$$

$$M_B = 520 - 1500 \times P \to M\,{}_{5}^{10} = 124.3 \quad (5.2)$$

$$M_C = 700 - 2000 \times P \to M\,{}_{5}^{10} = 172.4 \quad (5.3)$$

Therefore, alternative *C* is most profitable. But this calculation cannot analyze robustness against uncertainties. Then each alternative should be plotted on the *C*-*R* domain, which is shown in **Figure 5.2**.

It is clear from this figure that line segments connecting plots *A*, *B*, and *C* are concave, and therefore, alternative *B* is disqualified. The figure also shows that the slope connecting plots *A* and *C* is 0.3. It follows that if the interstate rate is increased to satisfy $P \to M\,{}_{5}^{i} > 0.3$, that is, i>16%, alternative *A* becomes more profitable than alternative *C*.

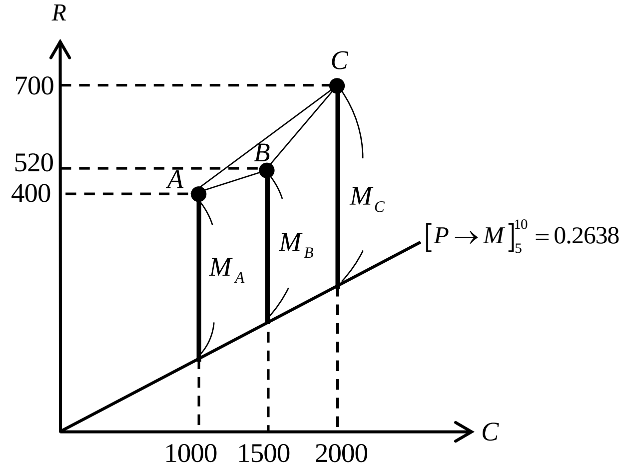

Figure 5.2: Plots on the *C*-*R* domain

It can also be confirmed that, when initial investment is increased in the ratio α for all alternatives, *A* and *C* become equally profitable when α^* satisfies.

$$400 - \alpha^* \times 1000 \times P \to M\,{}_{5}^{10} = 700 - \alpha^* \times 2000 \times P \to M\,{}_{5}^{10}. \quad (5.4)$$

Then, α^* obtained is 1.137. In the same context, if annual return is decreased in the same ratio β for all alternatives, β^* in which alternatives *A* and *C* are equally profitable is given by $\beta^* = \frac{1}{\alpha^*} = 0.879$.

5.2 The Case of Inconsistent Return

This section assumes the following numerical example (i=10%).

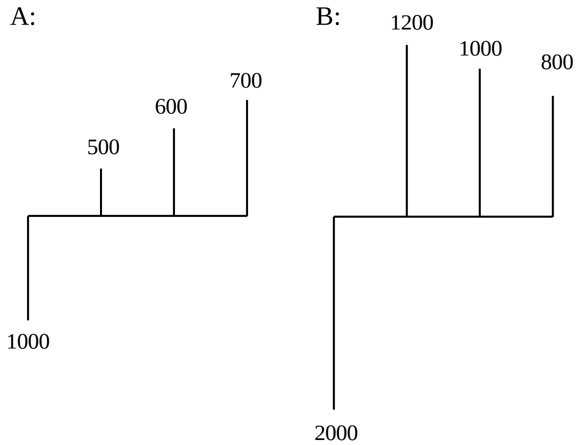

Figure 5.3: Numerical Example for fluctuating annual return

The net present value of profit is given by:

$$P_A = \frac{500}{1.1} + \frac{600}{1.1^2} + \frac{700}{1.1^3} - 1000 = 580.5$$

$$P_B = \frac{1200}{1.1} + \frac{1000}{1.1^2} + \frac{800}{1.1^3} - 2000 = 518.4$$

Two alternatives can be represented on the *C-R* domain as in **Figure 5.4**. It can be confirmed that P_A is larger than P_B in this figure.

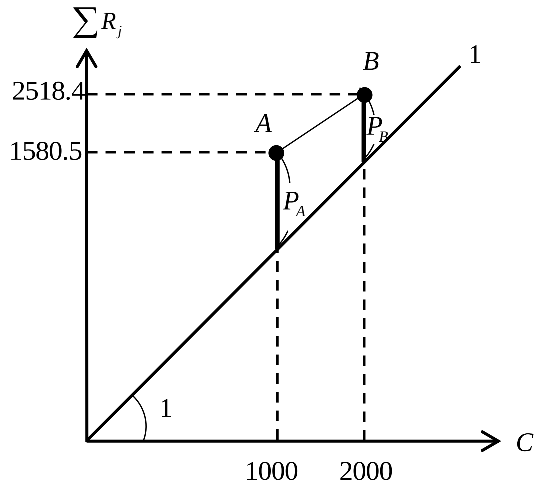

Figure 5.4: Representation of profit on the *C-R* domain

The slope of the line segment connecting plots *A* and *B* is 0.938. It follows that if initial investment is decreased for both alternatives to 93.8% from the current estimation, profit for both alternatives becomes equal at the value 642.6. In the same context, if the annual return for each of both alternatives increases up to 1/0.938=1.066 from the current estimation, both alternatives become break-even. If the expected change risk is lower, then we can select alternative *A* after consideration of risk under consideration. Thus, the proposed analysis procedure on the *C-R* domain helps economic evaluation and selection of alternatives under uncertainties.

6. CONCLUDING REMARKS

This paper investigated a problem of evaluating profitability and risk of investment alternatives, for both cases of consistent return and fluctuating return over the planning horizon. Major outcome of this paper is the procedure of visually evaluating profitability and risk on the *C-R* domain. Especially, risk evaluation on the *C-R* domain helps practical decision making under uncertain situations. In this context, this paper has practical purpose in addition to theoretical validity of analysis.

The paper simply denoted annual return by *R* (or R_j for the *j-th* year). However, it actually comprises increase of income, such as sales increase, or decrease of production cost, including material cost and processing cost. Therefore, return *R* can be divided into such factors as sales volume, production volume, unit sales price, and unit variable cost. Then, previous research outcomes in the field of engineering economy applying total-cost unit-cost domain, and/or capacity surplus and shortage distinction, can be combined to the analysis on the *C-R* domain. Further research to combine these results of analysis to help practical decision making is left as a topic for future research.

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