Optimal Operation with Flexible Ordering Policy for Green Supply Chain Considering the Uncertainties in Product Demand and Collection Quantity of Used Products

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Abstract. This paper discusses a green supply chain(GSC) from collection of used products through remanufacturing of them to sales of products produced of the remanufactured parts, with a buyer, a manufacturer, and a recycler. To manage the GSC's operation, it is necessary for GSC members to consider (i) the uncertainty in product demand, (ii) the uncertainty in collection quantity of used products, and (iii) a variety of quality of the parts extracted from used products. Here, the optimal operation for a GSC with a flexible ordering policy that the retailer's order quantity is between the minimum order quantity and the maximum order quantity with the manufacturer. The optimal decisions for the minimum order quantity, the maximum order quantity, recycling incentive, and lower limit of quality level, are made in the decentralized GSC(DGSC) and the integrated GSC(IGSC). DGSC optimizes the expected profit of each member, while IGSC optimizes that of the whole system. Numerical analysis clarifies the effectivity of the flexible ordering policy by comparing the optimal operation in IGSC with the flexible ordering policy with that with a traditional ordering policy. The benefit of supply chain coordination adopting Nash bargaining solution for shifting from DGSC to IGSC is shown.

Keywords: green supply chain, uncertainties in product demand and collection quantity of used products, quality of recyclable parts, flexible ordering policy, game theory

1. INTRODUCTION

From social concerns about 3R(reduce, reuse, recycle) activity worldwide, it is urgently-needed to construct a green supply chain (GSC) from collection of used products through recycling of them to sales of products using the recycled parts (Fleischmann et al., 1997; Guide et al., 2003; Watanabe and Kusukawa, 2014). It is necessary to consider a variety of quality of used products after collecting them from markets/customers. The following previous papers: Ferguson et al. (2009), Aras et al. (2004), Guide et al. (2003) and Watanabe and Kusukawa 2014, verified how the uncertainty in quality of used products affected the optimal production planning on recycled parts/products and new parts/products in a GSC. Watanabe and Kusukawa (2014) discussed a GSC with a retailer paying an incentive for collection of used products from customers, and a manufacturer recycling parts from the used products and producing the products. They verified how the uncertainty in product demand and quality of used products affected the optimal operation and the expected profit in a GSC. However, this previous paper didn't discuss the uncertainty in collection quantity of used products for collection incentive. Also, they didn't discuss the collection of used products and remanufacturing conducted by a professional recycler.

In order to restrain the uncertainty in production yield in a forward supply chain, Hu et al. (2013) presented the optimal flexible ordering policy with the optimal minimum order quantity and the optimal maximum order quantity.

The motivation of this paper is to incorporate (i) the uncertainties in product demand and collection quantity of used products and (ii) flexible ordering policy with the optimal minimum order quantity and the optimal maximum order quantity into the modelling and the theoretical analysis in a GSC, and determine the optimal operation. Concretely, this paper discusses a GSC from collection of used products through remanufacturing of them to sales of products produced of the remanufactured parts, with a buyer, a manufacturer, and a recycler. To manage the GSC's operation, it is necessary for GSC members to consider (i) the uncertainty in product demand, (ii) the uncertainty in collection quantity of used products, and (iii) a variety of quality of the parts extracted from used products. Here, the optimal operation for a GSC with a flexible ordering policy (FOP) that the retailer's order quantity is between the minimum order quantity and the maximum order quantity with the manufacturer. The optimal decisions for the minimum order quantity, the maximum order quantity, recycling incentive, and lower limit of quality level, are made under the decentralized GSC (DGSC) and the integrated GSC (IGSC). DGSC optimizes the expected profit of each member, while IGSC optimizes that of the whole system. Numerical analysis clarifies the effectivity of FOP by comparing the optimal operation in IGSC with FOP with that with a traditional ordering policy (TOP). The benefit of supply chain coordination adopting Nash bargaining solution (Nagarajan and Sonic, 2008) for shifting from DGSC to IGSC is shown.

The contribution of this paper provides the following managerial insights: (i) the decision procedure for the optimal operation in a GSC considering the uncertainties in product demand and collection quantity of used products can be derived, (ii) FOP can reduce the influence of the uncertainties in product demand and collection quantity of used products, (iii) supply chain coordination adopting Nash bargaining solution enables the shift from DGSC to IGSC.

2. MODEL DESCRIPTIONS

2.1 Operational Flow of a GSC with FOP

A GSC with a retailer, a manufacturer, and a recycler is considered. A single type of products such as consumer electronics (mobile phone, personal computer), semiconductor, and electronic component is sold in a market.

The operational flow of the GSC with FOP is as follows.

(1) A recycler collects used products whose quantity is x_c at the unit cost c_c.
(2) The recycler disassembles the used products to a single type of parts and inspects the parts at the unit cost c_a. After that, the parts are classified into quality level $\ell(0 \le \ell \le 1)$. The recycler remanufactures the parts with quality level which is greater than or equal to the lower limit of quality level $u(0 \le u \le 1)$, $u \le \ell \le 1$, at the unit cost $c_r(\ell)$.
(3) The parts with quality level which is less than u, $0 \le \ell < u$, are disposed at the unit cost c_d.
(4) The recycler sells all of the remanufactured parts whose quantity is x_r to a manufacturer at the unit price w_r.
(5) The manufacturer pays recycling incentive t per unit of remanufactured parts to the recycler.
(6) If x_r is less than a minimum product order quantity q, the manufacturer procures new parts whose quantity is $q - x_r$ from an external supplier at the unit cost c_n.
(7) The manufacturer produces the products whose quantity is d at the unit cost c_m and sells them to a buyer at the unit wholesale price w_m. If x_r is less than q, d is equal to q. If x_r is greater than or equal to q and less than or equal to a maximum product order quantity Q, d is equal to x_r. If x_r is greater than Q, d is equal to Q.
(8) If x_r is greater than Q, the excess remanufactured parts whose quantity is $x_r - Q$ are sold at the unit salvage value p_s in a disposal market.
(9) The buyer sells the products in a market at the unit sales price p_m during a single period. Also, the buyer incurs the inventory holding cost h_m per unit of unsold products for product demand x and the shortage penalty cost s_m per unsatisfied product demand.

2.2 Model Assumptions

(1) The expected collection quantity of used products, $A(t)$, varies as to the recycling incentive t. In general, the higher t is, the more used products the recycler can collect from a market. Therefore, $A(t)$ has the following characteristic: $\partial A(t)/\partial t > 0$. Here, from the aspect of the manufacturer's profit, the feasible range of t is $0 \le t \le w_m - c_m - w_r$.
(2) Collection quantity of used products, x_c, is modeled as
$$x_c = A(t) + \varepsilon, \tag{1}$$
where ε is a random variable. ε follows a probabilistic distribution with the probability density function (PDF) $h(\varepsilon)$ and the cumulative distribution function (CDF) $H(\varepsilon)$. Here, the expectation $\mathrm{E}[\varepsilon] = 0$.
(3) The variability of quality level ℓ of the parts extracted from used products is modeled as a probabilistic distribution with PDF $g(\ell)$ and CDF $G(\ell)$.
(4) The remanufacturing cost per unit of the parts, $c_r(\ell)$, varies as to the quality level ℓ of the parts. The lower quality level is, the higher $c_r(\ell)$ is. Here, $\ell = 0$ indicates the worst quality level, and $\ell = 1$ indicates the best quality level. Therefore, $c_r(\ell)$ has the following characteristic: $\partial c_r(\ell)/\partial \ell < 0$.
(5) The quality of remanufactured parts is the same as that of new parts.
(6) The unit salvage value p_s is lower than the unit procurement cost of new parts, c_n, and the unit procurement cost of remanufactured parts, w_r.
(7) The variability of product demand x is modeled as a probabilistic distribution with PDF $f(x)$ and CDF $F(x)$.

3. FORMULATION OF EXPECTED PROFITS

In this section, the expected profits of a buyer, a manufacturer, and a recycler, and the whole system are formulated based on Section 2.

First, from Subsection 2.1 (2) and Subsection 2.2 (2), the quantity of remanufactured parts, x_r, is calculated as

$$x_r = \int_u^1 x_c g(\ell)\,d\ell = \int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell. \tag{2}$$

Next, from Subsection 2.1 (7), the wholesale quantity of products from the manufacturer to the buyer, d, is given by

$$d = \begin{cases} q & (x_r < q), \\ x_r & (q \le x_r \le Q), \\ Q & (Q < x_r). \end{cases} \tag{3}$$

Note that if $Q = q$, then $d = q$, or the GSC with FOP is corresponding to the GSC with TOP.

3.1 Expected Profits in a GSC with FOP

The buyer(B)'s profit consists of the product sales, the inventory holding cost of unsold products, shortage penalty cost of unsatisfied product demand, and the procurement cost of products. The buyer's profit for the minimum product order quantity q, the maximum product order quantity Q, the recycling incentive t, and the lower limit of quality level u, $\pi_B(q,Q,t,u)$, is formulated as

$$\begin{aligned} \pi_B(q,Q,t,u) &= p_m \min(d,x) \\ &-h_m \max(d-x,0) - s_m \max(x-d,0) - w_m d. \end{aligned} \tag{4}$$

By taking expectation with respect to the product demand x and the random variable ε, the expected profit of the buyer for q, Q, t, and u, $\mathrm{E}[\pi_B(q,Q,t,u)]$, is derived as

$$\begin{aligned} &\mathrm{E}\left[\pi_B\{q,Q,t,(u \ne 1)\}\right] = p_m \int_{-A(t)}^{L} \mathrm{E}_x\left[\min(q,x)\right] h(\varepsilon)\,d\varepsilon \\ &+p_m \int_L^U \mathrm{E}_x\left[\min\left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell, x\right]\right] h(\varepsilon)\,d\varepsilon \\ &+p_m \int_U^\infty \mathrm{E}_x\left[\min(Q,x)\right] h(\varepsilon)\,d\varepsilon \\ &-h_m \int_{-A(t)}^{L} \mathrm{E}_x\left[\max(q-x,0)\right] h(\varepsilon)\,d\varepsilon \\ &-h_m \int_L^U \mathrm{E}_x\left[\max\left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell - x, 0\right]\right] h(\varepsilon)\,d\varepsilon \\ &-h_m \int_U^\infty \mathrm{E}_x\left[\max(Q-x,0)\right] h(\varepsilon)\,d\varepsilon \\ &-s_m \int_{-A(t)}^{L} \mathrm{E}_x\left[\max(x-q,0)\right] h(\varepsilon)\,d\varepsilon \\ &-s_m \int_L^U \mathrm{E}_x\left[\max\left[x - \int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell, 0\right]\right] h(\varepsilon)\,d\varepsilon \\ &-s_m \int_U^\infty \mathrm{E}_x\left[\max(x-Q,0)\right] h(\varepsilon)\,d\varepsilon \\ &-w_m \int_{-A(t)}^{L} q h(\varepsilon)\,d\varepsilon - w_m \int_L^U \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon \\ &-w_m \int_U^\infty Q h(\varepsilon)\,d\varepsilon, \end{aligned} \tag{5}$$

$$\begin{aligned} &\mathrm{E}\left[\pi_B\{q,Q,t,(u=1)\}\right] = p_m \int_{-A(t)}^{\infty} \mathrm{E}_x\left[\min(q,x)\right] h(\varepsilon)\,d\varepsilon \\ &-h_m \int_{-A(t)}^{\infty} \mathrm{E}_x\left[\max(q-x,0)\right] h(\varepsilon)\,d\varepsilon \\ &-s_m \int_{-A(t)}^{\infty} \mathrm{E}_x\left[\max(x-q,0)\right] h(\varepsilon)\,d\varepsilon \\ &-w_m \int_{-A(t)}^{\infty} q h(\varepsilon)\,d\varepsilon. \end{aligned} \tag{6}$$

Here, L and U denote

$$L = \left\{ q \Big/ \int_u^1 g(\ell)\,d\ell \right\} - A(t), \tag{7}$$

$$U = \left\{ Q \Big/ \int_u^1 g(\ell)\,d\ell \right\} - A(t), \tag{8}$$

respectively, but L and U for $u=1$ are undefined. If $u=1$, $x_r = 0$, or $d = q$ regardless of ε from Eq. (3). Therefore, $\mathrm{E}[\pi_B(q,Q,t,u)]$ for $u=1$ is Eq. (6) and is unaffected by Q. Also, $\mathrm{E}_x[\cdot]$ denotes an expectation with respect to x.

The manufacturer(M)'s profit consists of the product wholesales, the manufacturing cost of products, the procurement cost of remanufactured parts, the recycling incentive to the recycler, the procurement cost of new parts, and the salvage sales of remanufactured parts. The manufacturer's profit for q, Q, t, and u, $\pi_M(q,Q,t,u)$ is formulated as

$$\begin{aligned} \pi_M(q,Q,t,u) &= w_m d - c_m d - w_r x_r - t x_r \\ &-c_n \max(q - x_r, 0) + p_s \max(x_r - Q, 0). \end{aligned} \tag{9}$$

By taking expectation with respect to ε, the expected profit of the manufacturer for q, Q, t, and u, $\mathrm{E}[\pi_M(q,Q,t,u)]$, is derived as

$$\begin{aligned} &\mathrm{E}\left[\pi_M\{q,Q,t,(u \ne 1)\}\right] = w_m \int_{-A(t)}^{L} q h(\varepsilon)\,d\varepsilon \\ &+w_m \int_L^U \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon \\ &+w_m \int_U^\infty Q h(\varepsilon)\,d\varepsilon - c_m \int_{-A(t)}^{L} q h(\varepsilon)\,d\varepsilon \\ &-c_m \int_L^U \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon - c_m \int_U^\infty Q h(\varepsilon)\,d\varepsilon \\ &-w_r \int_{-A(t)}^{\infty} \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon \\ &-t \int_{-A(t)}^{\infty} \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon \\ &-c_n \int_{-A(t)}^{L} \left[q - \int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell\right] h(\varepsilon)\,d\varepsilon \\ &+p_s \int_U^\infty \left[\int_u^1 \{A(t)+\varepsilon\} g(\ell)\,d\ell - Q\right] h(\varepsilon)\,d\varepsilon, \end{aligned} \tag{10}$$

$$\mathrm{E}\left[\pi_M\left\{q,Q,t,(u=1)\right\}\right]=\left(w_m-c_m-c_n\right)\int_{-A(t)}^{\infty}qh(\varepsilon)d\varepsilon. \quad (11)$$

As with $\mathrm{E}\left[\pi_B(q,Q,t,u)\right]$, $\mathrm{E}\left[\pi_M(q,Q,t,u)\right]$ for $u=1$ is Eq. (11) and is unaffected by Q.

The recycler(R)'s profit consists of the sales of remanufactured parts, the recycling incentive from the manufacturer, the remanufacturing cost of the parts, the disposal cost of the parts, the disassembly and inspection cost of used products, and the collection cost of used products. The recycler's profit for t and u, $\pi_R(t,u)$, is formulated as

$$\pi_R(t,u)=w_r x_r+tx_r-\int_u^1 c_r(\ell)x_c g(\ell)d\ell$$
$$-c_d\int_0^u x_c g(\ell)d\ell-c_a x_c-c_c x_c. \quad (12)$$

By taking expectation with respect to ε, the expected profit of the recycler for t and u, $\mathrm{E}\left[\pi_R(t,u)\right]$, is derived as

$$\mathrm{E}\left[\pi_R(t,u)\right]$$
$$=w_r\int_{-A(t)}^{\infty}\left[\int_u^1\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$+t\int_{-A(t)}^{\infty}\left[\int_u^1\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$-\int_{-A(t)}^{\infty}\left[\int_u^1 c_r(\ell)\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$-c_d\int_{-A(t)}^{\infty}\left[\int_0^u\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$-c_a\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon-c_c\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon. \quad (13)$$

The expected profit of the whole system for q, Q, t, and u, $\mathrm{E}\left[\pi_S(q,Q,t,u)\right]$ is calculated from the sum of the buyer's expected profit, the manufacturer's expected profit, and the recycler's expected profit, as

$$\mathrm{E}\left[\pi_s\left\{q,Q,t,(u\neq1)\right\}\right]=\mathrm{E}\left[\pi_B\left\{q,Q,t,(u\neq1)\right\}\right]$$
$$+\mathrm{E}\left[\pi_M\left\{q,Q,t,(u\neq1)\right\}\right]+\mathrm{E}\left[\pi_R(t,u)\right], \quad (14)$$
$$\mathrm{E}\left[\pi_s\left\{q,Q,t,(u=1)\right\}\right]=\mathrm{E}\left[\pi_B\left\{q,Q,t,(u=1)\right\}\right]$$
$$+\mathrm{E}\left[\pi_M\left\{q,Q,t,(u=1)\right\}\right]+\mathrm{E}\left[\pi_R(t,u)\right]. \quad (15)$$

3.2 Expected Profit in a GSC with TOP

As described in Section 3, if $Q=q$, then $d=q$, or the GSC with FOP is corresponding to the GSC with TOP.

Therefore, replacing Q with q in Eqs. (14) and (15), the expected profit of the whole system in the GSC with TOP, $\mathrm{E}\left[\pi_S^T(q,t,u)\right]$, can be derived as

$$\mathrm{E}\left[\pi_S^T\left\{q,t,(u\neq1)\right\}\right]=p_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\min(q,x)\right]h(\varepsilon)d\varepsilon$$
$$-h_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\max(q-x,0)\right]h(\varepsilon)d\varepsilon$$
$$-s_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\max(x-q,0)\right]h(\varepsilon)d\varepsilon-c_m\int_{-A(t)}^{\infty}qh(\varepsilon)d\varepsilon$$
$$-c_n\int_{-A(t)}^{L}\left[q-\int_u^1\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$+p_s\int_L^{\infty}\left[\int_u^1\left\{A(t)+\varepsilon\right\}g(\ell)d\ell-q\right]h(\varepsilon)d\varepsilon$$
$$-\int_{-A(t)}^{\infty}\left[\int_u^1 c_r(\ell)\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$-c_d\int_{-A(t)}^{\infty}\left[\int_0^u\left\{A(t)+\varepsilon\right\}g(\ell)d\ell\right]h(\varepsilon)d\varepsilon$$
$$-c_c\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon-c_a\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon, \quad (16)$$

$$\mathrm{E}\left[\pi_S^T\left\{q,t,(u=1)\right\}\right]=p_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\min(q,x)\right]h(\varepsilon)d\varepsilon$$
$$-h_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\max(q-x,0)\right]h(\varepsilon)d\varepsilon$$
$$-s_m\int_{-A(t)}^{\infty}\mathrm{E}\left[\max(x-q,0)\right]h(\varepsilon)d\varepsilon$$
$$-c_m\int_{-A(t)}^{\infty}qh(\varepsilon)d\varepsilon-c_n\int_{-A(t)}^{\infty}qh(\varepsilon)d\varepsilon$$
$$-c_d\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon$$
$$-c_c\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon-c_a\int_{-A(t)}^{\infty}\left\{A(t)+\varepsilon\right\}h(\varepsilon)d\varepsilon. \quad (17)$$

4. OPTIMAL OPERATIONS IN DGSC

In DGSC, the optimal decision approach for a Stackelberg game (Nagarajan and Sonic. 2008) is adopted.

This paper regards a buyer, a manufacturer, and a recycler, as the first leader, the second leader, and the follower, of the decision-making in DGSC, respectively. Then, the buyer determines the optimal minimum product order quantity q_D and the optimal maximum product order quantity Q_D so as to maximize the expected profit of the buyer, $\mathrm{E}\left[\pi_B(q,Q,t,u)\right]$. The manufacturer determines the optimal recycling incentive t_D so as to maximize that of the manufacturer under q_D and Q_D, $\mathrm{E}\left[\pi_M(q_D,Q_D,t,u)\right]$. The recycler determines the optimal lower limit of quality level u_D so as to maximize that of the recycler under t_D, $\mathrm{E}\left[\pi_R(t_D,u)\right]$.

4.1 Optimal Minimum Product Order Quantity and Optimal Maximum Product Order Quantity

The first partial derivatives of $\mathrm{E}\left[\pi_B(q,Q,t,u)\right]$ with respect to q and Q are respectively derived as

$$\partial\mathrm{E}\left[\pi_B\left\{q,Q,t,(u\neq1)\right\}\right]/\partial q$$
$$=\left\{\left(p_m+s_m-w_m\right)-\left(p_m+h_m+s_m\right)F(q)\right\}H(L), \quad (18)$$

$$\partial \mathrm{E}\left[\pi_B\{q,Q,,t,(u=1)\}\right]/\partial q$$
$$=(p_m+s_m-w_m)-(p_m+h_m+s_m)F(q), \quad (19)$$

$$\partial \mathrm{E}\left[\pi_B\{q,Q,t,(u\neq 1)\}\right]/\partial Q$$
$$=\{(p_m+s_m-w_m)-(p_m+h_m+s_m)F(Q)\}\{1-H(U)\}. \quad (20)$$

Here, note that Q doesn't affect $\mathrm{E}\left[\pi_B\{q,Q,t,(u=1)\}\right]$ from Eq. (6). Eqs. (5) and (6) are monomodal with respect to q due to the following three characteristics: Eqs. (18) and (19) are positive if $0<F(q)<(p_m-w_m+s_m)/(p_m+h_m+s_m)$, Eqs. (18) and (19) are negative if $(p_m-w_m+s_m)/(p_m+h_m+s_m)<F(q)\leq 1$, and $F(q)$ is a monotonically increasing function with respect to q. Therefore, the optimal minimum product order quantity q_D is determined regardless of u as the following unique solution to maximize $\mathrm{E}\left[\pi_B(q,Q,t,u)\right]$:

$$q_D=F^{-1}\{(p_m+s_m-w_m)/(p_m+h_m+s_m)\}. \quad (21)$$

As with q_D, the optimal maximum product order quantity Q_D is determined as the following unique solution to maximize $\mathrm{E}\left[\pi_B(q,Q,t,u)\right]$:

$$Q_D=F^{-1}\{(p_m+s_m-w_m)/(p_m+h_m+s_m)\}. \quad (22)$$

From the above analyses, $q_D=Q_D$ can be derived. Therefore, even if FOP is introduced into DGSC, it results in TOP. This means that FOP doesn't function in DGSC. The reason is that the buyer isn't rewarded by FOP. Here, q_D and Q_D are unaffected by t and u.

4.2 Optimal Recycling Incentive and Optimal Lower Limit of Quality Level

Under the optimal minimum product order quantity q_D and the optimal maximum product order quantity Q_D determined in Subsection 4.1, the recycling incentive t and the lower limit of quality level u are optimized.

The first partial derivative of $\mathrm{E}\left[\pi_R(t,u)\right]$ with respect to u is derived as

$$\partial \mathrm{E}\left[\pi_R(t,u)\right]/\partial u$$
$$=\{c_r(u)-(w_r+t+c_d)\}g(u)\{A(t)+\mathrm{E}[\varepsilon]\}. \quad (23)$$

Eq. (23) is zero if the following equation:

$$c_r(u)-(w_r+t+c_d)=0 \quad (24)$$

is satisfied. Here, as described in Subsection 2.2 (4), $c_r(u)$ is a monotonically decreasing function with respect to u. Therefore, the provisional lower limit of quality level for t, $u_D(t)$, is determined as follows:

i. If $c_r(0)-(w_r+t+c_d)<0$, $u_D(t)=0$,
ii. If $c_r(1)-(w_r+t+c_d)>0$, $u_D(t)=1$,
iii. Otherwise, $u_D(t)=c_r^{-1}(w_r+t+c_d)$.

Also, it is clarified that $u_D(t)$ is unaffected by q_D and Q_D.

It is difficult to derive the optimal solution for t, t_D, by mathematical analysis. In this paper, t_D is determined by the numerical search. The decision procedure for the optimal recycling incentive t_D and the optimal lower limit of quality level u_D is shown as follows:

[Step 1] Substitute t and $u_D(t)$ into the expected profit of the manufacturer, $\mathrm{E}\left[\pi_M(q_D,Q_D,t,u)\right]$, with varying t within $0\leq t\leq w_m-c_m-w_r$.
[Step 2] Determine the optimal combination of t and $u_D(t)$, which maximizes $\mathrm{E}\left[\pi_M\{q_D,Q_D,t,u_D(t)\}\right]$, as (t_D,u_D).

5. OPTIMAL OPERATIONS IN IGSC

In IGSC, the optimal minimum product order quantity q_I, the optimal maximum product order quantity Q_I, the optimal recycling incentive t_I, the optimal lower limit of quality level u_I are determined so as to maximize the expected profit of the whole system, $\mathrm{E}\left[\pi_S(q,Q,t,u)\right]$.

5.1 Optimal Minimum Product Order Quantity and Optimal Maximum Product Order Quantity

The first partial derivatives of $\mathrm{E}\left[\pi_S(q,Q,t,u)\right]$ with respect to q and Q are respectively derived as

$$\partial \mathrm{E}\left[\pi_S\{q,Q,t,(u\neq 1)\}\right]/\partial q$$
$$=\{(p_m+s_m-c_m-c_n)-(p_m+h_m+s_m)F(q)\}H(L), \quad (25)$$

$$\partial \mathrm{E}\left[\pi_S\{q,Q,t,(u=1)\}\right]/\partial q$$
$$=(p_m+s_m-c_m-c_n)-(p_m+h_m+s_m)F(q), \quad (26)$$

$$\partial \mathrm{E}\left[\pi_S\{q,Q,t,(u\neq 1)\}\right]/\partial Q$$
$$=\{(p_m+s_m-c_m-p_s)-(p_m+h_m+s_m)F(Q)\}$$
$$\times\{1-H(U)\}. \quad (27)$$

Here, note that Q doesn't affect $\mathrm{E}\left[\pi_S\{q,Q,t,(u=1)\}\right]$ from Eqs. (6), (11), and (15). Eqs. (14) and (15) are monomodal with respect to q due to the following three characteristics: Eqs. (25) and (26) are positive if $0<F(q)<(p_m+s_m-c_m-c_n)/(p_m+h_m+s_m)$, Eqs. (25) and (26) are negative if $(p_m+s_m-c_m-c_n)/(p_m+h_m+s_m)<F(q)\leq 1$, and $F(q)$ is a monotonically increasing function with respect to q. Therefore, the optimal minimum product order quantity q_I is determined regardless of u as the following unique solution to maximize $\mathrm{E}\left[\pi_S(q,Q,t,u)\right]$:

$$q_I=F^{-1}\{(p_m+s_m-c_m-c_n)/(p_m+h_m+s_m)\}. \quad (28)$$

As with q_I, the optimal maximum product order quantity Q_I is determined as the following unique solution to maximize $\mathrm{E}\left[\pi_S(q,Q,t,u)\right]$:

$$Q_I=F^{-1}\{(p_m+s_m-c_m-p_s)/(p_m+h_m+s_m)\}. \quad (29)$$

Also, due to $\partial F(x)/\partial x\geq 0$ and $p_s<c_n$, $q_I<Q_I$ is derived. Therefore, FOP functions in IGSC unlike DGSC.

5.2 Optimal Recycling Incentive and Optimal Lower Limit of Quality Level

Under the optimal minimum product order quantity q_I and the optimal maximum product order quantity Q_I determined in Subsection 5.1, the recycling incentive t and the lower limit of quality level u are optimized. Unlike DGSC, it is difficult to derive the optimal solutions for t and u, t_I and u_I, by mathematical analysis. In this paper, t_I and u_I are determined by the numerical search. The decision procedure for the optimal recycling incentive t_I and the lower limit of quality level u_I is shown as follows:

[Step 1] Substitute t and u into the expected profit of the whole system, $\mathrm{E}\left[\pi_S(q_I,Q_I,t,u)\right]$ with varying t within $0\le t\le w_m-c_m-w_r$ and u within $0\le u\le 1$.

[Step 2] Determine the optimal combination of t and u, which maximizes $\mathrm{E}\left[\pi_S(q_I,Q_I,t,u)\right]$, as (t_I,u_I).

5.3 Optimal Operation in IGSC with TOP

In this section, the optimal operation in IGSC with TOP is discussed. $\mathrm{E}\left[\pi_S^T(q,t,u)\right]$ is the concave function with respect to q. The provisional product order quantity for t and $u\neq 1$, $q_I^T\{t,(u\neq 1)\}$, is determined as q which satisfies the first order condition of $\mathrm{E}\left[\pi_S^T(q,t,u)\right]$ with respect to q:

$$\begin{aligned}&(p_m+s_m-c_m-p_s)-(p_m+h_m+s_m)F(q)\\&-(c_n-p_s)H\left\{q\Big/\int_u^1 g(\ell)d\ell-A(t)\right\}=0.\end{aligned}\tag{34}$$

Also, $q_I^T\{t,(u=1)\}$ is determined as

$$q_I^T\{t,(u=1)\}=F^{-1}\{(p_m+s_m-c_m-c_n)/(p_m+h_m+s_m)\}.\tag{35}$$

The optimal solutions for t and u, t_I^T and u_I^T, are determined by the numerical search. The decision procedure for the optimal product order quantity q_I^T, the optimal recycling incentive t_I^T, and the optimal lower limit of quality level u_I^T, is shown as follows:

[Step 1] Substitute $q_I^T(t,u)$, t, and u, into the expected profit of the whole system, $\mathrm{E}\left[\pi_S^T(q,t,u)\right]$ with varying the recycling incentive within $0\le t\le w_m-c_m-w_r$ and the lower limit of quality level within $0\le u\le 1$.

[Step 2] Determine the optimal combination of $q_I^T(t,u)$, t, and u, which maximizes $\mathrm{E}\left[\pi_S^T(q,t,u)\right]$, as (q_I^T,t_I^T,u_I^T).

6. SUPPLY CHAIN COORDINATION

In general, by shifting from the optimal operation in DGSC to that in IGSC, the expected profit of the whole system improves. However, it is not guaranteed that the expected profit of each member improves. Nevertheless, it is desirable to shift from the optimal operation in DGSC to that in IGSC from the aspect of the total optimization of the expected profit.

This section discusses supply chain coordination (SCC) among the buyer, the manufacturer, and the recycler, to guarantee the improvement in the expected profit of each member. This paper coordinates the unit procurement cost of remanufactured parts, w_r, and the unit wholesale price of products, w_m, as the Nash bargaining solution (w_r^{Nash},w_m^{Nash}). The coordinated parameters (w_r^{Nash},w_m^{Nash}) are determined so as to maximize $T(w_r^{Nash},w_m^{Nash})$ in Eq. (36) with satisfying the constrained conditions in Eqs. (37), (38), and (39), as follows:

$$\begin{aligned}&T\left(w_r^{Nash},w_m^{Nash}\right)\\&=\left\{\mathrm{E}\left[\pi_B\left(w_m^{Nash}\mid q_I,Q_I,t_I,u_I\right)\right]-\mathrm{E}\left[\pi_B\left(w_m\mid q_D,Q_D,t_D,u_D\right)\right]\right\}\\&\times\left\{\mathrm{E}\left[\pi_M\left(w_r^{Nash},w_m^{Nash}\mid q_I,Q_I,t_I,u_I\right)\right]\right.\\&\left.-\mathrm{E}\left[\pi_M\left(w_r,w_m\mid q_D,Q_D,t_D,u_D\right)\right]\right\}\\&\times\left\{\mathrm{E}\left[\pi_R\left(w_r^{Nash}\mid t_I,u_I\right)\right]-\mathrm{E}\left[\pi_R\left(w_r\mid t_D,u_D\right)\right]\right\},\end{aligned}\tag{36}$$

$$\begin{aligned}&\mathrm{E}\left[\pi_B\left(w_m^{Nash}\mid q_I,Q_I,t_I,u_I\right)\right]\\&-\mathrm{E}\left[\pi_B\left(w_m\mid q_D,Q_D,t_D,u_D\right)\right]>0,\end{aligned}\tag{37}$$

$$\begin{aligned}&\mathrm{E}\left[\pi_M\left(w_r^{Nash},w_m^{Nash}\mid q_I,Q_I,t_I,u_I\right)\right]\\&-\mathrm{E}\left[\pi_M\left(w_r,w_m\mid q_D,Q_D,t_D,u_D\right)\right]>0,\end{aligned}\tag{38}$$

$$\mathrm{E}\left[\pi_R\left(w_r^{Nash}\mid t_I,u_I\right)\right]-\mathrm{E}\left[\pi_R\left(w_r\mid t_D,u_D\right)\right]>0.\tag{39}$$

Eqs. (37), (38), and (39), are constraint conditions to guarantee that the expected profit of each member in IGSC with SCC is higher than that in DGSC.

7. NUMERICAL ANALYSES

This section investigates how (i) the uncertainty in product demand, (ii) the uncertainty in collection quantity of used products, and (iii) a variety of quality of the parts extracted from used products, affect the optimal operation and the expected profits in the GSC with FOP by providing numerical examples. Also, in order to clarify the effectivity of FOP, the expected profit of the whole system under the optimal operation in IGSC with FOP is compared with that with TOP. Moreover, the benefit of supply chain coordination (SCC) adopting Nash bargaining solution is shown.

The following numerical examples are used as system parameters in a GSC: $p_m=150$, $h_m=15$, $s_m=175$, $w_m=70$, $c_m=10$, $c_n=40$, $p_s=10$, $w_r=20$, $c_d=5$, $c_a=3$, $c_c=1$. $A(t)$ and $c_r(\ell)$ are respectively set as $A(t)=500+50t$ and $c_r(\ell)=40(1-0.9\ell)$, satisfying the characteristics in Section 2. The product demand x follows the normal distribution with mean $\mu_x=1000$ and variance $\sigma_x^2=300^2$. The random variable ε follows the normal distribution with mean $\mu_\varepsilon=0$ and variance $\sigma_\varepsilon^2=100^2$. The quality level of the parts extracted from used products, ℓ, follows the beta distribution with parameters (a,b).

The PDF $g(\ell\mid a,b)$ of the beta distribution is provided as

$$g(\ell\mid a,b)=\left\{\Gamma(a+b)/\left(\Gamma(a)\Gamma(b)\right)\right\}\ell^{a-1}(1-\ell)^{b-1},\tag{40}$$

Table 1: The optimal operation in each of DGSC and IGSC with FOP in the four cases of quality distribution of the parts

QD	GSC	Optimal operations with FOP				$\mathrm{E}[x_r]$
		q	Q	t	u	
Case 1	DGSC	1202	1202	8	0.16	758
	IGSC	1262	1380	28	0.31	1311
Case 2	DGSC	1202	1202	7	0.18	777
	IGSC	1262	1380	23	0.29	1314
Case 3	DGSC	1202	1202	6	0.20	777
	IGSC	1262	1380	22	0.38	1349
Case 4	DGSC	1202	1202	8	0.16	792
	IGSC	1262	1380	20	0.18	1274

where $\Gamma(\cdot)$ denotes the gamma function. This paper provides the following four cases of quality distribution (QD) $B(\ell \mid a,b)$ of the parts:

Case 1 $B(\ell \mid 1,1)$: a case where quality of the parts are distributed uniformly,

Case 2 $B(\ell \mid 2,2)$: a case where there are many parts with middle quality level,

Case 3 $B(\ell \mid 3,2)$: a case where there are many parts with the relatively high quality level,

Case 4 $B(\ell \mid 2,3)$: a case where there are many parts with the relatively low quality level.

Table 1 shows the optimal operation in each of DGSC and IGSC with FOP in the four cases of quality distribution of the parts. In Table 1, $\mathrm{E}[x_r]$ denotes the expectation of quantity of remanufactured parts. From Table 1, it can be seen that the optimal recycling incentive t_I and the optimal lower limit of quality level u_I in IGSC are higher than t_D and u_D in DGSC in any case. From Eq. (2), the higher t is, the larger quantity of remanufactured parts, x_r, is, and the higher u is, the smaller x_r is. As a result, $\mathrm{E}[x_r]$ in IGSC is larger than that in DGSC. Therefore, the recycling activity can be promoted in IGSC.

Table 2 and Table 3 respectively show the influences of the uncertainty in product demand and that in collection quantity of used products on the expected profit of the whole system under the optimal operation in IGSC with each of FOP and TOP in Case 2 of QD of the parts. Here, π_S^{T*} denotes the expected profit of the whole system under the optimal operation in IGSC with TOP, and π_S^{*} denotes that under the optimal operation in IGSC with FOP. Also, the improvement rate of the expected profit of the whole system, P (%), appearing in Table 2 is calculated as

$$P = \left(\pi_S^{*} - \pi_S^{T*}\right) \big/ \pi_S^{T*} \times 100. \tag{41}$$

From Table 2, it can be seen that as the standard deviation of product demand x, σ_x, increases, π_S^{T*} and π_S^{*} decrease, and P improves. As with product demand, from Table 3, it can be seen that as the standard deviation of the random variable ε of collection quantity of used products, σ_ε, increases, π_S^{T*} and π_S^{*} decrease, and P improves. Therefore, it is verified that FOP can reduce the decrement in the expected profit of the whole system caused by increment in the uncertainties in product demand and collection quantity of used products. In addition, as the uncertainties increase, the collection incentive t is higher, or the IGSC tends to collect more used products.

Table 4 shows each member's expected profit and the result of SCC in IGSC with FOP in Case 2 of QD of the parts. From Table 4, the expected profit of the buyer and that of the manufacturer under the optimal operation in IGSC without SCC are lower than those in DGSC, respectively, but the expected profit of each member in IGSC with SCC is higher than that in DGSC. Therefore, SCC can make the GSC shift from DGSC to IGSC. Here, these results in Case 2 are the same as those in the other cases of QD.

8. CONCLUSIONS

This paper discussed a green supply chain (GSC) from collection of used products through remanufacturing of them to sales of products produced of the remanufactured parts, with a buyer, a manufacturer, and a recycler. The optimal operations in DGSC and IGSC with TOP and FOP were proposed. Also, it was clarified that FOP didn't function in DGSC, but it improved the expected profit of the whole system in IGSC. In the decision-making process, not only a variety of quality of the parts extracted from used products and the uncertainties in product demand and collection quantity of used products were considered. In numerical analyses, the influences of these uncertainties on the optimal operation and the expected profits in the GSC with FOP were investigated, and the effectivity of FOP was clarified by comparing the expected profit of the whole system under the optimal operation in IGSC with FOP with that with TOP. Results of the mathematical analyses and the numerical analyses in this paper verified the following managerial insights: (i) the decision procedure for the optimal operation in a GSC considering not only the uncertainties in product demand and collection quantity of used products could be derived, (ii) FOP could reduce the influence of the uncertainties in product demand and collection quantity of used products, (iii) supply chain coordination adopting Nash bargaining solution enabled the shift from DGSC to IGSC.

As future researches, it will be necessary to discuss introduction of the following topics:

- A contract which causes FOP to function in DGSC
- Decision-making for a target value of collection quantity of used products by a recycler and new frameworks to encourage the recycler to increase the collection quantity of used products
- Supply chain coordination adopting another method

Table 2: Influences of the uncertainty in product demand on the expected profit of the whole system under the optimal operation in IGSC with each of TOP and FOP in Case 2 of QD of the parts

σ_x	Ordering policy	Optimal operations in IGSC				Expected profit of the whole system	Improvement rate P (%)
		q	Q	t	u		
200	TOP	1211		20	0.28	$\pi_S^{T*} =$ 97882	0.27
	FOP	1175	1253	20	0.28	$\pi_S^{*} =$ 98149	
300	TOP	1315		23	0.29	$\pi_S^{T*} =$ 90030	0.37
	FOP	1262	1380	23	0.29	$\pi_S^{*} =$ 90365	
400	TOP	1418		25	0.28	$\pi_S^{T*} =$ 82424	0.52
	FOP	1349	1506	25	0.28	$\pi_S^{*} =$ 82848	

Table 3: Influences of the uncertainty in collection quantity of used products on the expected profit of the whole system under the optimal operation in IGSC with each of TOP and FOP in Case 2 of QD of the parts

σ_ε	Ordering policy	Optimal operations in IGSC				Expected profit of the whole system	Improvement rate P (%)
		q	Q	t	u		
50	TOP	1314		22	0.27	$\pi_S^{T*} =$ 90512	0.30
	FOP	1262	1380	22	0.27	$\pi_S^{*} =$ 90781	
100	TOP	1315		23	0.29	$\pi_S^{T*} =$ 90030	0.37
	FOP	1262	1380	23	0.29	$\pi_S^{*} =$ 90365	
150	TOP	1315		23	0.29	$\pi_S^{T*} =$ 89553	0.41
	FOP	1262	1380	23	0.29	$\pi_S^{*} =$ 89920	

Table 4: Each member's expected profit and the result of SCC in IGSC with FOP in Case 2 of QD of the parts

Member	DGSC	IGSC without SCC	IGSC with SCC	w_r^{Nash}	w_m^{Nash}
Buyer's expected profit	47598	45139(-2459)	50408(+2810)	5	66
Manufacturer's expected profit	34145	22130(-12015)	36573(+2428)		
Recycler's expected profit	1123	23096(+21973)	3384(+2261)		
Whole system's expected profit	82866	90365(+7499)	90365(+7499)		

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