# Reconsideration of a negotiation procedure for a buyback contract in a supply chain

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**Abstract.** There are some studies about a negotiation procedure between a supplier and a retailer in a supply chain model with a buyback contract. The negotiation procedure in the above studies consists of the following three successive steps: (i) show such a requirement for contract parameters that the supplier and retailer simultaneously have incentives to conclude the contract, (ii) present a condition of contract parameters guaranteeing both optimality of the supplier and retailer, and (iii) determine a unique combination of contract parameters using Nash bargaining theory. Through the successive steps (i)-(iii), some candidates of contract parameters have been screened step by step, and eventually, optimal contract parameters as the result of negotiation have been uniquely decided in step (iii). However, the preceding studies of the negotiation procedure stated above have not given a full explanation for the role of which steps (i) and (ii) fill in the bargaining solution of step (iii) in the mathematical sense. Therefore, in this study, we reconsider the negotiation procedure in the preceding studies based on a supply chain model with a buyback contract. Through the reconsideration, we confirm the usefulness of the negotiation procedure consisting of three successive steps in the preceding studies.

**Keywords:** Buyback contract, Cooperative game theory, Coordination approach, Incentive compatible condition, Nash bargaining solution

## 1. Introduction

In a usual business deal, a stock risk due to unsold goods at a retail store is imposed on a retailer. When a retailer avoids the stock risk due to unsold goods, the retailer might reduce order quantity. As the result, market requests will not be satisfied enough and the market might shrink. One of solutions to mitigate the stock risk of a retailer is a buyback contract (Cachon, 2003). In a buyback contract, a supplier buys back unsold goods in a retailer at a portion of the wholesale price. Through this contract, the supplier assures a certain portion of the stock risk for unsold goods and divides the stock risk with the retailer. Hence, the retailer increase the order quantity and market requests will be satisfied more. As the result, the buyback contract can be expected to increase whole profit of a supply chain composed of the supplier and retailer. Hence, the buyback contract has been studied by many researchers until now such as Wu (2013), Hafezalkotob and Makui (2014), and Liu et al. (2015).

As a contract technique of determining contract parameters, the coordination approach has been considered (Cachon, 2003). The coordination approach has derived such contract parameters that maximize a total expected profit of the supply chain composed of the supplier and retailer. In case of not realizing the maximization of the total profit in supply chain, the contract parameters are considered to be inappropriate for coordination. On one hand, the traditional coordination approach has not considered how to divide the profit between the supplier and retailer. In addition, the traditional coordination approach has not shown incentive (or some necessary conditions) to agree on the contract from the viewpoint of the individual supply chain member, i.e., supplier and retailer respectively.

In recent years, Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have suggested a negotiation procedure between a supplier and a retailer in a buyback contract under a model for a publishing supply chain in Japan. In a usual buyback contract model, order quantity, wholesale and buyback prices as contract parameters are determined. Their studies have determined three contract parameters on the negotiation procedure constituted of the following three successive steps:

- (i) show such a requirement for the contract parameters that the supplier and retailer simultaneously have incentives to conclude the contract,
- (ii) present a condition of the contract parameters guaranteeing both optimality of the supplier and retailer,
- (iii) determine a unique combination of the contract parameters using Nash bargaining theory (Nash, 1950).

As a detail of step (i), Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have shown some requirements for the contract parameters such that expected profits of the supplier and retailer under the buyback contract become greater than those under a usual deal, respectively. They have called the requirements an incentive compatible condition. In step (ii), they have obtained combinations of the contract parameters which maximize the supplier's and retailer's expected profits respectively and simultaneously. Then, it has been confirmed that the combinations of the contract parameters in step (ii) are equivalent to those which maximize a total expected profit of the supply chain composed of the supplier and retailer. This procedure is called collaborative coordination approach against the traditional coordination approach (Cachon, 2003). Through the successive steps (i)-(iii), the candidates of the contract parameters have been screened step by step, and eventually, optimal contract parameters as the result of negotiation have been uniquely decided in step (iii).

While Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have suggested to apply the above three successive steps to the determination of the contract parameters, they have not given a full explanation for the role of which steps (i) and (ii) fill in the bargaining solution of step (iii) in the mathematical sense. Hence, we reconsider the negotiation procedure in the preceding studies under a supply chain model with a buyback contract in this study. Through the reconsideration, we confirm the usefulness of the negotiation procedure consisting of three successive steps in the preceding studies.

## 2. Model setting

In this study, we consider a supply chain model with a buyback contract. First, we describe a mathematical model of the supply chain in order to investigate the negotiation procedure in the preceding studies (Arizono and Takemoto, 2012; Takemoto and Arizono, 2013). The notations for describing the model in this study are defined as follows:

- $p$ : retail price,
- $k_w p$ : wholesale price (decision variable),
- $k_b p$ : buyback price (decision variable),
- $k_c p$ : original cost,
- $k_d p$ : disposal cost,
- $\pi_R$ : expected profit function of retailer,
- $\pi_S$ : expected profit function of supplier,
- $\pi_A$ : total expected profit function of supply chain,
- $x$ : quantity of demand,
- $f(x)$ : probability density function of $x$,
- $q$ : order quantity (decision variable).

Note that wholesale price, buyback price, original cost and disposal cost are given on basis of the retail price $p$ for convenience. In this case, it is reasonable that the following relations are satisfied: $0 < k_c < k_w < 1$, and $0 < k_b < k_w$. Then, since all the prices and costs are given as ratios to the retail price $p$, we can treat the model in case of $p = 1$ without loss of generality.

Next, we formulate the expected profit functions of the retailer and supplier using the notations mentioned above. The expected profit function of the retailer $\pi_R(k_w, k_b, q)$ is defined as

$$\pi_R(k_w, k_b, q) = (1-k_w)q - (1-k_b)S(q), \tag{1}$$

where $S(q)$ means the quantity of unsold goods and is given as follows:

$$S(q) = \int_0^q (q-x) f(x) dx. \tag{2}$$

Similarly, the expected profit function of the supplier $\pi_S(k_w, k_b, q)$ is defined as

$$\pi_S(k_w, k_b, q) = (k_w - k_c)q - (k_b + k_d)S(q). \tag{3}$$

By using Eqs.(1) and (3), the total expected profit function of the supply chain $\pi_A(k_w, k_b, q)$ is defined as

$$\begin{aligned}\pi_A(k_w, k_b, q) &= \pi_R(k_w, k_b, q) + \pi_S(k_w, k_b, q) \\ &= (1-k_c)q - (1+k_d)S(q). \end{aligned} \tag{4}$$

## 3. Negotiation procedure in the preceding researches

As stated in section 1, Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have addressed the negotiation procedure to determine collaboratively the contract parameters in the buyback contract between the supplier and retailer. In this section, we show a process of determining the contract parameters on the negotiation procedure in these studies.

At first, Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have assumed the postulate that the retailer decides the order quantity because of facing consumer's demand directly. Therefore, the order quantity $q$ which maximizes the expected profit function of the retailer, $\pi_R(k_w, k_b, q)$, is obtained from the following equation:

$$\frac{\partial \pi_R(k_w, k_b, q)}{\partial q} = 0. \tag{5}$$

Note that $q$ depends on $k_w$ and $k_b$, that is, $q$ is a function of $k_w$ and $k_b$. For convenience, the relation of $q$, $k_w$ and $k_b$ is defined as $k_w \equiv k_w(q, k_b)$. As the result, $\pi_R(k_w, k_b, q)$ and $\pi_S(k_w, k_b, q)$ are redefined as $\pi_R(q, k_b)$ and $\pi_S(q, k_b)$.

Provided that $q$ is determined by the retailer, step (i) is considered under the condition of Eq.(5). Since step (i) show some requirements for the contract parameters such that the expected profits of the supplier and retailer under the buyback contract become greater than those under a usual deal, respectively, the following relationships need to be satisfied:

$$\begin{cases} \pi_R(q, k_b) - \pi_R^0 \geq 0, \\ \pi_S(q, k_b) - \pi_S^0 \geq 0, \end{cases} \tag{6}$$

where $\pi_S^0$ and $\pi_R^0$ express the expected profits of the supplier and retailer under a usual deal, respectively. They have called the relationships of Eq.(6) the incentive compatible condition.

In step (ii), the combinations of the contract parameters which maximize the supplier's and retailer's expected profit functions simultaneously are obtained. Therefore, the following relationships need to be considered:

$$\begin{cases} \dfrac{\partial \pi_R(q, k_b)}{\partial q} = 0, \\ \dfrac{\partial \pi_S(q, k_b)}{\partial q} = 0. \end{cases} \tag{7}$$

In this case, the following relationship is also satisfied:

$$\frac{\partial \pi_R(q, k_b)}{\partial q} + \frac{\partial \pi_S(q, k_b)}{\partial q} = 0. \tag{8}$$

That is, any combinations of the contract parameters which satisfy the conditions in Eq.(7) are also those of the contract parameters which maximize the total expected profit of the supply chain composed of the supplier and retailer.

Finally, in step (iii), Nash bargaining solution is applied to the buyback contract model. In the Nash bargaining solution, optimal contract parameters are given as a combination of the contract parameters which maximize the following equation:

$$T(q, k_b) = \left\{\pi_R(q, k_b) - \pi_R^0\right\}\left\{\pi_S(q, k_b) - \pi_S^0\right\}. \tag{9}$$

Eq.(9) is called Nash products in a common sense. Also, the combination of $(\pi_R^0, \pi_S^0)$ is particularly called the disagreement point of bargaining because $(\pi_R^0, \pi_S^0)$ means the profits in the case that the conclusion of contract was not achieved.

Throughout successive steps (i), (ii) and (iii), the supplier and retailer can have the unique combination of the contract parameters to conclude the buyback contract. On one hand, the respective concepts of steps (i), (ii) and (iii) are independent mutually. In particular, the concept of the incentive compatible condition in step (i) has been considered originally by Arizono and Takemoto (2012) and Takemoto and Arizono (2013). Further, the collaborative coordination approach in step (ii) is seen as the concept to modify the traditional coordination approach by Arizono and Takemoto (2012).

## 4. Reconsideration of negotiation procedure from the viewpoint of Nash bargaining theory

Arizono and Takemoto (2012) and Takemoto and Arizono (2013) have indicated the negotiation process consisting of the successive three steps in order to agree to the contract between the retailer and supplier. However, these studies haven't sufficiently discussed what kind of role steps (i) and (ii) fill in the Nash bargaining solution in step (iii). In the negotiation procedure consisting of steps (i)-(iii), the candidates of contract parameters have been screened step by step in steps (i) and (ii). Hence, it is unclear that the combination of contract parameters eventually obtained by the negotiation procedure is consistent with the combination of contract parameters obtained by using just Nash bargaining theory. That is, the combination of contract parameters eventually obtained by the negotiation procedure may not be global optimal but be local optimal. Therefore, we need to reconfirm the global optimality of the solution obtained by the negotiation procedure.

In this section, we show a process of determining the combination of the contract parameters uniquely by Nash bargaining theory only. That is, we begin from step (iii) in this section. From this reconsideration, we confirm the role of which step (i) and (ii) fill in the bargaining solution of step (iii) in the mathematical sense. Based on Eqs.(1) and (3), Nash product is defined as follow:

$$T(k_w, k_b, q) = \left\{\pi_R(k_w, k_b, q) - \pi_R^0\right\}\left\{\pi_S(k_w, k_b, q) - \pi_S^0\right\}. \tag{10}$$

Then, we consider the maximization problem of $T(k_w, k_b, q)$ in Eq.(10) in order to obtain the unique combination of the contract parameters $(k_w, k_b, q)$.

At first, we obtain first-order derivatives about $T(k_w, k_b, q)$ in $k_w, k_b,$ and $q$, respectively. The following equations are obtained by differentiating Eq.(10) partially with respect to $k_w, k_b$, and $q$, respectively:

$$\begin{aligned}\frac{\partial T(k_w,k_b,q)}{\partial k_w}&=\frac{\partial \pi_R(k_w,k_b,q)}{\partial k_w}\times\left\{\pi_S(k_w,k_b,q)-\pi_S^0\right\}\\&\quad+\left\{\pi_R(k_w,k_b,q)-\pi_R^0\right\}\times\frac{\partial \pi_S(k_w,k_b,q)}{\partial k_w}\\&=q\left\{\pi_R(k_w,k_b,q)-\pi_R^0\right\}-\left\{\pi_S(k_w,k_b,q)-\pi_S^0\right\}q,\end{aligned}\tag{11}$$

$$\begin{aligned}\frac{\partial T(k_w,k_b,q)}{\partial k_b}&=\frac{\partial \pi_R(k_w,k_b,q)}{\partial k_b}\times\left\{\pi_S(k_w,k_b,q)-\pi_S^0\right\}\\&\quad+\left\{\pi_R(k_w,k_b,q)-\pi_R^0\right\}\times\frac{\partial \pi_S(k_w,k_b,q)}{\partial k_b}\\&=\left\{\pi_S(k_w,k_b,q)-\pi_S^0\right\}S(q)\\&\quad-\left\{\pi_R(k_w,k_b,q)-\pi_R^0\right\}S(q),\end{aligned}\tag{12}$$

$$\begin{aligned}\frac{\partial T(k_w,k_b,q)}{\partial q}&=\frac{\partial \pi_R(k_w,k_b,q)}{\partial q}\times\left\{\pi_S(k_w,k_b,q)-\pi_S^0\right\}\\&\quad+\left\{\pi_R(k_w,k_b,q)-\pi_R^0\right\}\times\frac{\partial \pi_S(k_w,k_b,q)}{\partial q}.\end{aligned}\tag{13}$$

Then, first-order conditions in the contract parameters which maximize $T(k_w,k_b,q)$ are given as follows:

$$\begin{cases}\dfrac{\partial T(k_w,k_b,q)}{\partial k_w}=0,\\[2mm]\dfrac{\partial T(k_w,k_b,q)}{\partial k_b}=0,\\[2mm]\dfrac{\partial T(k_w,k_b,q)}{\partial q}=0.\end{cases}\tag{14}$$

Then, a combination of the contract parameter $(k_w,k_b,q)$ which satisfies the relationships of Eq.(14) at the same time are a candidate of contract parameters giving the maximum in $T(k_w,k_b,q)$. We denote $(k_w,k_b,q)$ satisfying the relationships of Eq.(14) as $(k_w^*,k_b^*,q^*)$.

Then, from $\partial T(k_w,k_b,q)/\partial k_w=0$ in Eq.(14), the following relation can be derived:

$$\pi_R(k_w^*,k_b^*,q^*)-\pi_R^0=\pi_S(k_w^*,k_b^*,q^*)-\pi_S^0.\tag{15}$$

Similarly, the same relationship as Eq.(15) is derived from $\partial T(k_w,k_b,q)/\partial k_b=0$ in Eq.(14). It is noted in Eq. (15) that the incremental profit by the contract is equally divided between the retailer and supplier. Further, from the relationship of Eqs.(13), (15), and $\partial T(k_w,k_b,q)/\partial q=0$ in Eq.(14), $(k_w^*,k_b^*,q^*)$ needs to be satisfied either of the following relations :

$$\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q}+\frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q}=\frac{\partial \pi_A(k_w^*,k_b^*,q^*)}{\partial q}=0.\tag{16}$$

or

$$\pi_R(k_w^*,k_b^*,q^*)-\pi_R^0=\pi_S(k_w^*,k_b^*,q^*)-\pi_S^0=0.\tag{17}$$

Therefore, $(k_w^*,k_b^*,q^*)$ has the relations as Eqs.(15) and (16) or Eqs.(15) and (17).

First, we consider a case where the relationships of Eqs.(15) and (16) are satisfied. Then, from Eq.(4) regarding the relation of Eq.(16) , we have

$$\int_0^q f(x)dx=\frac{1-k_c}{1+k_d}.\tag{18}$$

Because $k_c$ and $k_d$ are a constant, $q^*$ can be uniquely obtained from the relationship in Eq.(18) regardless of $k_b$ and $k_w$.

Further, we consider second-order derivatives of $T(k_w,k_b,q)$. The Hessian matrix for $T(k_w,k_b,q)$ with respect to $(k_w^*,k_b^*,q^*)$ is defined as follows:

$$H(k_w^*,k_b^*,q^*)=\begin{bmatrix}\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w^2}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w\partial k_b}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w\partial q}\\[2mm]\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b\partial k_w}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b^2}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b\partial q}\\[2mm]\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q\partial k_w}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q\partial k_b}&\dfrac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q^2}\end{bmatrix},\tag{19}$$

where the following notation is adopted:

$$\left.\frac{\partial T(k_w,k_b,q)}{\partial k_w}\right|_{(k_w,k_b,q)=(k_w^*,k_b^*,q^*)}\equiv\frac{\partial T(k_w^*,k_b^*,q^*)}{\partial k_w}$$

Further, each element of Eq.(19) is obtained as follows:

$$\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w^2} = -2q^{*2} < 0, \tag{20}$$

$$\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w \partial k_b} = \frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b \partial k_w} = 2q^* S(q^*) > 0, \tag{21}$$

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w \partial q} &= \frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q \partial k_w} \\ &= q^*\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} - \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q}\right) \\ &\quad + \left[\left\{\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0\right\} - \left\{\pi_S(k_w^*,k_b^*,q^*) - \pi_S^0\right\}\right],\end{aligned} \tag{22}$$

$$\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b^2} = -2\left\{S(q^*)\right\}^2 < 0, \tag{23}$$

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b \partial q} &= \frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q \partial k_b} \\ &= -\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} - \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q}\right) S(q^*) \\ &\quad - \left[\left\{\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0\right\} - \left\{\pi_S(k_w^*,k_b^*,q^*) - \pi_S^0\right\}\right] \\ &\quad \times \int_0^{q^*} f(x)dx,\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q^2} &= \frac{\partial^2 \pi_R(k_w^*,k_b^*,q^*)}{\partial q^2}\left\{\pi_S(k_w^*,k_b^*,q^*) - \pi_S^0\right\} \\ &\quad + 2\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q}\frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q} \\ &\quad + \left\{\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0\right\}\frac{\partial^2 \pi_S(k_w^*,k_b^*,q^*)}{\partial q^2}.\end{aligned} \tag{25}$$

When $H(k_w^*,k_b^*,q^*)$ is a negative definite matrix, the combination of $(k_w^*,k_b^*,q^*)$ gives the maximum value in Eq.(10). A negative definite matrix is defined from the relationship of $(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t < 0$ for any combinations of $(k_w,k_b,q)$, where $t$ means transposition operation. By proving the relation for $H(k_w^*,k_b^*,q^*)$, we show that the combination of $(k_w^*,k_b^*,q^*)$ gives the maximum value in Eq.(10).

In each element of Eq.(19), the signs of Eqs.(20), (21) and (23) are already known. Further, by considering the relationship of Eq.(15), Eqs.(22) and (24) can be transformed into the following relations:

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_w \partial q} &= \frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q \partial k_w} \\ &= q^*\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} - \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q}\right),\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial k_b \partial q} &= \frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q \partial k_b} \\ &= -\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} - \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q}\right) S(q^*)\end{aligned} \tag{27}$$

However, we cannot judge the sign of $(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t$ because the sign of Eqs.(25), (26) and (27) are not known yet at this time. Then, we consider a situation where the following relationship is satisfied:

$$\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} - \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q} = 0. \tag{28}$$

By considering the relationship of Eq.(28), both values of Eqs.(26) and (27) are equivalent to 0. In addition, from the relationship of Eq.(16) which is derived by $\partial T(k_w,k_b,q)/\partial q = 0$ in Eq.(14), the following relation can be derived:

$$\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q} = \frac{\partial \pi_S(k_w^*,k_b^*,q^*)}{\partial q} = 0. \tag{29}$$

The relationship of Eq.(29) correspond to the relationships of Eqs.(5) and (7) in section 3. Hence, we confirm that the order quantity $q^*$ which is derived from the maximization of Nash product is consistent with the order quantity obtained by the negotiation procedure in section 3. Then, from Eq.(25) by considering the relationship of Eqs.(15) and (29), we obtain the following relation:

$$\begin{aligned}\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q^2} &= \left\{\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0\right\} \\ &\quad \times \left(\frac{\partial^2 \pi_R(k_w^*,k_b^*,q^*)}{\partial q^2} + \frac{\partial^2 \pi_S(k_w^*,k_b^*,q^*)}{\partial q^2}\right).\end{aligned} \tag{30}$$

Using each element derived above, we have the following equation:

$$\begin{aligned}&(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t \\ &= -2\left[k_w q^* - k_b S(q^*)\right]^2 + \left\{\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0\right\} \\ &\quad \times \left(\frac{\partial^2 \pi_R(k_w^*,k_b^*,q^*)}{\partial q^2} + \frac{\partial^2 \pi_S(k_w^*,k_b^*,q^*)}{\partial q^2}\right) q^2.\end{aligned} \tag{31}$$

Because the expected profit functions of the retailer and supplier $\pi_R(k_w,k_b,q)$ and $\pi_S(k_w,k_b,q)$ are concave functions against $q$, the second-order derivatives of $\pi_R(k_w,k_b,q)$ and $\pi_S(k_w,k_b,q)$ are given as negative values. Further, the relationships of $k_b < k_w$ and $S(q^*) < q^*$ are always satisfied. When the relationship of $\pi_R(k_w^*,k_b^*,q^*) - \pi_R^0 \geq 0$ is satisfied, the relation of $(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t < 0$ is also satisfied. When the following incentive compatible condition is satisfied:

$$\begin{cases} \pi_R(k_w^*,k_b^*,q^*) - \pi_R^0 \geq 0, \\ \pi_S(k_w^*,k_b^*,q^*) - \pi_S^0 \geq 0. \end{cases} \tag{32}$$

It is found that the combination of $(k_w^*,k_b^*,q^*)$ gives the maximum value in Eq.(10).

On the other hand, we consider a case where the relationships of Eqs.(15) and (17) are satisfied. From the relationship of Eq.(17) under the relationship of Eq.(28), Eq.(25) is transformed into the following relation:

$$\frac{\partial^2 T(k_w^*,k_b^*,q^*)}{\partial q^2} = 2\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q}\right)^2. \tag{33}$$

$(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t$ is calculated as follow:

$$\begin{aligned} &\left(k_w,k_b,q\right)H(k_w^*,k_b^*,q^*)\left(k_w,k_b,q\right)^t \\ &\quad = -2\left\{k_w q^* - k_b S(q^*)\right\}^2 + 2\left(\frac{\partial \pi_R(k_w^*,k_b^*,q^*)}{\partial q}\right)^2 q^2. \end{aligned} \tag{34}$$

When the relation of $\partial \pi_R(k_w^*,k_b^*,q^*)/\partial q = 0$ is satisfied, the relation of $(k_w,k_b,q)H(k_w^*,k_b^*,q^*)(k_w,k_b,q)^t < 0$ is satisfied. In this case, the relation of Eq.(29) can be also obtained from the relationship of Eq.(28).

As the results mentioned above, we have confirmed that Nash bargaining solution gives the maximum value in Eq.(10) when Eqs.(29) and (32) are satisfied. That is, Eq.(6) in step (i) and Eqs.(7) and (8) in step (ii) are prerequisite conditions for obtaining Nash bargaining solution. Hence, the negotiation procedure as steps (i), (ii), and (iii) is a useful approach.

## 5. Concluding remarks

In this study, we have reconsidered the negotiation procedure in Arizono and Takemoto (2012) and Takemoto and Arizono (2013) using the supply chain model with the buyback contract. In the preceding studies, their negotiation procedure consists of three successive steps. In the first and second steps, some requirements of the contract parameters to conclude the contract has been shown, and then the unique combination of the contract parameters has been obtained using Nash bargaining solution in the third step. However, they haven't given full explanation about the role of which the first and second steps fill in the bargaining solution at the third step in the mathematical sense. In this study, we have confirmed that the requirements in the first and second steps are prerequisite conditions for obtaining Nash bargaining solution in the third step.

The traditional coordination approach has derived such contract parameters that maximize the total expected profit of the supply chain composed of the supplier and retailer. However, the traditional coordination approach has not considered how to divide the profit between the supplier and retailer. In this point, the Nash bargaining theory provides a solution of allocation in the profit between the supplier and retailer. But, the Nash bargaining solution does not considered incentive (or some necessary conditions) to agree on the contract from the viewpoint of the individual supply chain member, i.e., supplier and retailer respectively. The negotiation procedure in Arizono and Takemoto (2012) and Takemoto and Arizono (2013) has considered the incentives in the supplier and retailer to agree on the contract. Then, the negotiation procedure has shown some necessary conditions in the contract parameters to agree on the contract. Further, the rationality and optimality in the negotiation procedure has been proved in this study. As the result, we have reconfirmed the usefulness of the negotiation procedure in Arizono and Takemoto (2012) and Takemoto and Arizono (2013).


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## References


Arizono, I. and Takemoto, Y., A Solution of Contract Problem in Book Trade Supply Chain Systems Based on Distribution Free Approach, in *Pioneering Supply Chain Design – A Comprehensive Insight into Emerging Trend, Technologies and Applications (eds. Blecker, T., Kersten, W., and Ringle, C. M.)*, 75-87, Josef Eul Verlag, 2012.

Cachon, G. P. (2003) Supply Chain Coordination with Contract, in *Handbooks in Operations Research and Management Science, A.G. de Kok, S. Graves, (eds.)*, Elsevier.

Hafezalkotob A., and Makui A. (2014) Network design of a decentralized distribution supply chain:Analysis of non-cooperative equilibrium vs. coordination with discount or buyback mechanism, *Scientia Iranica. Transaction E, Industrial Engineering*, **21**(3), 988-1006.

Liu, X., Xu. Q., and Xu, L. X. (2015) A Literature Review on Supply Chain Contracts Selection and Coordination under

Competing Multi Manufacturers, *International Journal of Business and Management*, **10** (7), 196-207.
Nash, J. F. (1950) The Bargaining Problem, *Econometrica*, 28, 155-162.
Takemoto, Y. and Arizono, I. (2013) Collaborative Sale Contract in Publishing Supply Chain, *Innovation and Supply Chain Management*, **7** (2), 52-57.
Wu D. (2013) Coordination of competing supply chains with news-vendor and buyback contract, *International Journal of Production Economics*, **144** (1), 1-13.