# Reliability Analysis of 2-Component Standby Redundant System with Priority under Limited Information

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**Abstract.** Some theoretical evaluation methods of mean time to failure (MTTF) in 2-component standby redundant system with priority have been suggested under the prerequisite that the probability distributions of failure and repair times in each component are respectively specified by the explicit functions. However, this prerequisite might be considerably strict in practical scenes. If we cannot identify explicitly the probability distributions of failure and repair times in each component, the methods mentioned above are not applicable. In contrast, the mean and variance are the required minimum information on the probability distribution. In late years, under the limited information that only the mean and variance of failure and repair times in each component are specified, the evaluation method of MTTF in the 2-component standby redundant system with priority has been proposed. In this study, we expand the evaluation method of MTTF in the 2-component standby redundant system with priority under such a situation. We consider the evaluation of the variance of system failure time in the 2-component standby redundant system with priority in the case that only the mean and variance about failure and repair times are specified. Furthermore, based on the evaluated MTTF and variance of system failure time, the reliability analysis in the standby redundant system is addressed.

**Keywords:** mixed Erlang distribution, renewal theory, Markov process, system reliability

## 1. Introduction

With progress of technology, various systems have become more complex and complicated. Then, the society requires high reliability in various systems. Hence, the research and development in the reliability field is active. In the details, see textbooks in the reliability field such as Gertsbakh (2000) and Birolini (2010).

A 2-component redundant system is well-known as the most fundamental redundant model in the reliability theory. Then, the redundant system means the system consisting of plural components of the same function. However, every component in such a redundant system is not necessarily identical. Usually, such components constituting the redundant system have some different features such as capability and cost performance. Therefore, the order of priority among alternative components is considered in the situation of operating the entire system.

Zhang and Wang (2007) and Yuan and Meng (2011) have discussed a 2-component standby redundant system with priority in use. Also, Leung et al. (2011) have investigated a standby redundant system with priority in repair. As their results, some important reliability indices and optimal operating policy of components in 2-component redundant system have been suggested. Note that it has been assumed in their studies that all distributions about failure and repair times of each component are respectively specified as an exponential distribution.

On the other hand, Buzacott (1971) has considered the system consisting of priority and standby components, where the priority component is used whenever it is available. When the priority component falls in failure, it starts to be repaired immediately and the standby component starts to operate. If the standby component falls in failure during repairing the priority component, the entire system becomes down. Then, under the assumption that the respective failure and repair times of both components follow any specified probability distributions, Buzacott (1971) has derived a theoretical formula of mean time to failure (MTTF) of the entire system. In this case, the evaluation formulae of MTTF derived by Buzacott need any probability density functions (PDFs) and/or cumulative distribution functions (CDFs) about failure and repair times in the priority and standby components. Further, the calculations of Laplace transformation, convolution, and infinite series with respect to the distribution functions of failure and repair times are required. Hence, the numerical evaluation is not possible in all cases.

Further, Osaki (2002) has derived a theoretical formula of MTTF in the same system as Buzacott under the assumption that the failure and repair times in the priority component follow any probability distributions respectively but the failure time in the standby component follows an exponential distribution. In such a situation that the failure time in the standby component follows an exponential distribution, the standby component is considered to be equivalent to a new one whenever the standby component starts to operate, because of the memoryless property of exponential distribution. Therefore, MTTF evaluated by using the theoretical formula derived by Osaki brings frequently the overestimation.

Then, the evaluation formulae of MTTF derived by Buzacott and Osaki require any probability density functions (PDFs) and/or cumulative distribution functions (CDFs) about failure and repair times in the respective components. That is, the theoretical evaluation methods of MTTF of the 2-component standby redundant system with priority suggested by Buzacott and Osaki require the prerequisite that each probability distribution about failure and repair times is respectively specified by an explicit function. That is, the theoretical evaluation methods of MTTF of the 2-component standby redundant system with priority suggested by Buzacott and Osaki require the prerequisite that the probability distributions of failure and repair times in each component are respectively specified by the explicit functions. However, this prerequisite might be considerably strict in practical scenes. If we cannot identify explicitly the probability distributions about failure and repair times in each component, the methods mentioned above are not applicable. In contrast, the mean and variance are the required minimum information on characterizing the probability distribution.

In recent years, Takemoto and Arizono (2016) have proposed the evaluation method of MTTF in the 2-component standby redundant system with priority under the limited information that only mean and variance about failure and repair times are specified. Then, Takemoto and Arizono have considered the situation that the probability distributions about failure and repair times can be approximated as a mixed Erlang distribution. Under this situation, Takemoto and Arizono develop a new approximation procedure for computing the MTTF of the system by combining some results of the Markov analysis based on Erlang distributions.

In this study, we expand the evaluation method of MTTF in the 2-component standby redundant system with priority under such a situation. At first, we consider the evaluation of the variance of failure time in the 2-component standby redundant system with priority in the case that only mean and variance about failure and repair times of each component are specified. Furthermore, based on the evaluated MTTF and variance of system failure time, the system reliability analysis is addressed.

## 2. Details of 2-component Standby Redundant System with Priority

We consider the 2-component standby redundant system composed of Component 1 having priority and Component 2 which is a standby component. In this case, the component with priority means a component which is always used whenever it is available. In contrast, the standby component means a component which stands by whenever the component with priority is available. The outline of behavior in this system is illustrated in Figure 1. The solid line represents the operating status of each component. The dashed line represents the repair status of Component 1, the double line represents the standby status of Component 2. In addition, the dotted line connecting Component 1 and Component 2 represents the switching of components. Initially, at time $t = 0$, Component 1 is working and Component 2 is in the standby state. At this time, the standby time of the Component 2 is not included in its lifetime. When Component 1 fails, it starts to be repaired immediately and then Component 2 starts to work. If the repair of Component 1 has been completed and Component 2 is still working, then Component 1 starts to work immediately and Component 2 is in a standby state. However, if Component 2 fails during the repair of Component 1, the entire system goes down. The switching of components is reliably executed, and it is assumed that the switchover time is instantaneous.

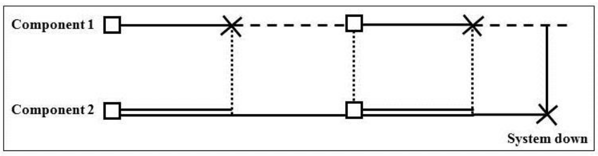


Figure 1: 2-component standby redundant system with priority.

## 3. Outcomes of Previous Literature

For the 2-component standby redundant system of Figure 1, Buzacott (1971) has derived MTTF under the situation that the failure and repair time distributions of each component are explicitly prescribed as respective arbitrary distributions. However, the procedure of Buzacott has required the calculations of Laplace transformation, convolution, and infinite series with respect to the distribution functions of failure and repair times. Hence, the numerical evaluation by using the procedure of Buzacott is not possible in all cases.

Further, under the situation that Component 2 has an exponential failure time distribution, Osaki (2002) has derived MTTF of the 2-component standby redundant system of Figure 1. In this case, the standby component is considered to be equivalent to a new standby component whenever it starts to be used because of the memoryless property of exponential distribution. Hence, the theoretical formula of MTTF by Osaki provides frequently the overestimation. All the distribution functions of the failure and repair times in each component should be explicitly specified for evaluating MTTF by using the theoretical formulae by Osaki and Buzacott. However, in some practical situations, it is difficult to know exactly the probability distributions of failure and repair times. On the other hand, the mean and variance are the general required minimum information on the probability distribution.

In recent years, Takemoto and Arizono (2016) have considered the evaluation of MTTF in the 2-component standby redundant system of Figure 1 under the following situation described by the limited information:

(i) The failure time distribution of Component 1 has the cumulative distribution function (CDF) $F_1(t)$ with mean $1/\lambda_1$. On the other hand, the mean and variance of the repair time of Component 1 are provided as $E_1$ and $V_1$, and the repair time distribution of Component 1 can be approximated as a mixed Erlang distribution with the CDF $G_1(t)$ of mean $E_1$ and variance $V_1$.

(ii) The mean and variance of the failure time of Component 2 are provided as $E_2$ and $V_2$, and the failure time distribution of Component 2 can be approximated as a mixed Erlang distribution with the CDF $F_2(t)$ of mean $E_2$ and variance $V_2$.

By applying the approximation technique by Keizer et al. (2001) to the repair time distribution of Components 1, Takemoto and Arizono (2016) have represented the following PDF $g_1(t)$ of the mixed Erlang distribution with the CDF $G_1(t)$ of mean $E_1$ and variance $V_1$:

$$g_1(t) = p_1\mu_1^{n_1}\frac{t^{n_1-1}}{(n_1-1)!}e^{-\mu_1 t} + (1-p_1)\mu_1^{n_1+1}\frac{t^{n_1}}{n_1!}e^{-\mu_1 t}, \tag{1}$$

where $n_1$, $p_1$ and $\mu_1$ denote parameters for approximation. Then, the approximation parameters $(n_1,\, p_1,\, \mu_1)$ are provided as follows:

$$\frac{1}{n_1+1} < \varphi_1^2 \le \frac{1}{n_1}, \tag{2}$$

$$p_1 = \frac{(n_1+1)\varphi_1^2 - \sqrt{(n_1+1)(1-n_1\varphi_1^2)}}{\varphi_1^2+1}, \tag{3}$$

$$\mu_1 = \frac{n_1+1-p_1}{E_1}, \tag{4}$$

where $\varphi_1$ means the coefficient of variation defined as $\varphi_1 = \sqrt{V_1}/E_1$. On deriving the approximation parameters $(n_1,\, p_1,\, \mu_1)$, let known mean $E_1$ and variance $V_1$ correspond to those of the mixed Erlang distribution, at first. Then, the approximation parameters $(n_1,\, p_1,\, \mu_1)$ are obtained from two values of mean $E_1$ and variance $V_1$ under the constraint that the value of mixed ratio $p_1$ is given between 0 and 1 and then the values of phases are integer and consecutive. Therefore, the relation of $0 < \varphi_1 \le 1$ is needed.

Similarly, the PDF $f_2(t)$ of the failure time distribution of Components 2 can be expressed as follows:

$$f_2(t) = p_2\lambda_2^{n_2}\frac{t^{n_2-1}}{(n_2-1)!}e^{-\lambda_2 t} + (1-p_2)\lambda_2^{n_2+1}\frac{t^{n_2}}{n_2!}e^{-\lambda_2 t}. \tag{5}$$

Then, $n_2$, $p_2$ and $\lambda_2$ are provided as follows:

$$\frac{1}{n_2+1} < \varphi_2^2 \le \frac{1}{n_2}, \tag{6}$$

$$p_2 = \frac{(n_2+1)\varphi_2^2 - \sqrt{(n_2+1)(1-n_2\varphi_2^2)}}{\varphi_2^2+1}, \tag{7}$$

$$\lambda_2 = \frac{n_2+1-p_2}{E_2}, \tag{8}$$

where $\varphi_2 = \sqrt{V_2}/E_2$, $0 < \varphi_2 \le 1$.

The approximate distributions with the PDFs $g_1(t)$ and $f_2(t)$ are provided as the mixed Erlang distributions, respectively. In this case, the repair time in Component 1 obeys the Erlang distribution with parameters $(n_1,\, \mu_1)$ in probability $p_1$. Similarly, the failure time in Component 2 obeys the Erlang distribution with parameters $(n_2,\, \lambda_2)$ in probability $p_2$. The Erlang distribution with $(n_1,\, \mu_1)$ is defined as the distribution of the sum of $n_1$ exponential variables with mean $1/\mu_1$. Similarly, the Erlang distribution with $(n_2,\, \lambda_2)$ is defined as the distribution of the sum of $n_2$ exponential variables with mean $1/\lambda_2$. If each time of failure and repair is considered to be divided into some exponential variables, the state transition can be explained by on the Markov process. Then, $MTTF_{(n_1,n_2)}$ is denoted as MTTF in the system with phases $n_1$ and $n_2$.

In this system, we can consider all states to be a renewal point in the Markov process. The definitions of states $S_{w,j}$ and $S_{i,j}$ are described as follows:

- $S_{w,j}$ : Component 1 is working and Component 2 is in standby, where Component 2 has the $j$ th degree of wear,
- $S_{i,j}$ : Component 1 is under repair and Component 2 is working, where Component 1 has the $i$ th degree of repair and Component 2 has the $j$ th degree of wear,
- $S_{i,n_2}$ : system down,

where $i=0,1,\ldots,\ n_1-1$, and $j=0,1,\ldots,\ n_2-1$.

The transition from state $S_{w,j}$ to state $S_{0,j}$ is governed by the failure time distribution of Component 1. The progress in repair in Component 1 is governed by the Erlang distribution composed of $n_1$ number of exponential variables. Hence, the transition from $S_{i,j}$ to $S_{i+1,j}$ and $S_{w,j}$ is governed by the exponential distribution with mean $1/\mu_1$. Similarly, the progress in wear in Component 2 is governed by the Erlang distribution composed of $n_2$ number of exponential variables. Then, the transition from $S_{i,j}$ to $S_{i,j+1}$ is governed by the exponential distribution with mean $1/\lambda_2$. Also, define $R_1(t)=1-F_1(t)$ as the reliability function of the Component 1. In addition, describe $\tilde{R}_1(s)$ as the Laplace transformation of $R_1(t)$, and so on.

On evaluating $MTTF_{(n_1,n_2)}$, Takemoto and Arizono (2016) have derived the probability $R_{i,j}(t)$ that the system is in state $S_{i,j}$ at time 0, and then the system is functioning at time $t$. Then, the probability $R_{i,j}(t)$ is classified into the following five cases:

1. The system is in state $S_{w,j}$ at time 0 and then the system is functioning at time $t$:

$$R_{w,j}(t)=R_1(t)+\int_0^t f_1(x)R_{0,j}(t-x)dx. \qquad (9)$$

2. The system is in state $S_{i,j}$, $j=0,1,\ldots,\ n_2-2$, at time 0 and then the system is functioning at time $t$:

$$R_{i,j}(t)=e^{-\mu_1 t}e^{-\lambda_2 t}+\mu_1\int_0^t e^{-\mu_1 x}e^{-\lambda_2 x}R_{i+1,j}(t-x)dx$$
$$+\lambda_2\int_0^t e^{-\lambda_2 x}e^{-\mu_1 x}R_{i,j+1}(t-x)dx. \qquad (10)$$

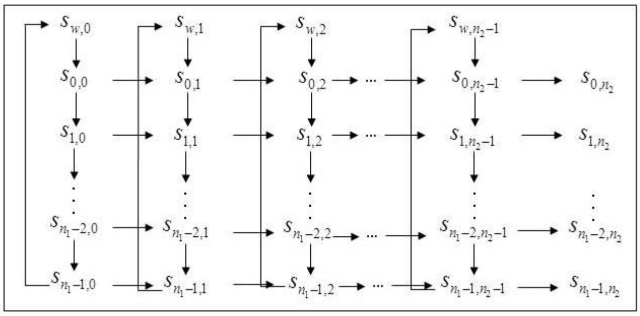


Figure 2: transition diagram in case of Erlang distributions with phases $n_1$ and $n_2$.

3. The system is in state $S_{n_1-1,j}$ at time 0 and then the system is functioning at time $t$:

$$R_{n_1-1,j}(t)=e^{-\mu_1 t}e^{-\lambda_2 t}+\mu_1\int_0^t e^{-\mu_1 x}e^{-\lambda_2 x}R_{w,j}(t-x)dx$$
$$+\lambda_2\int_0^t e^{-\lambda_2 x}e^{-\mu_1 x}R_{n_1-1,j+1}(t-x)dx. \qquad (11)$$

4. The system is in state $S_{i,n_2-1}$ at time 0 and then the system is functioning at time $t$:

$$R_{i,n_2-1}(t)=e^{-\mu_1 t}e^{-\lambda_2 t}+\mu_1\int_0^t e^{-\mu_1 t}e^{-\lambda_2 t}R_{i+1,n_2-1}(t-x)dx. \qquad (12)$$

5. The system is in state $S_{n_1-1,n_2-1}$ at time 0 and then the system is functioning at time $t$:

$$R_{n_1-1,n_2-1}(t)=e^{-\mu_1 t}e^{-\lambda_2 t}+\mu_1\int_0^t e^{-\mu_1 t}e^{-\lambda_2 t}R_{w,n_2-1}(t-x)dx. \qquad (13)$$

Then, by solving simultaneous equations based on the Laplace transform of equations (9)-(13), we have

$$\tilde{R}_{w,0}(s)=\tilde{u}(s)\sum_{i=0}^{n_2-1}\tilde{d}_i(s)+\tilde{a}(s)\tilde{v}(s)$$
$$\times\sum_{i=0}^{n_1-1}\sum_{j=0}^{n_2-1}\sum_{k=0}^{n_2-1-j}{}_{i+k}C_k\tilde{b}^i(s)\tilde{c}^k(s)\tilde{d}_j(s)\equiv\tilde{R}_{(n_1,n_2)}(s), \qquad (14)$$

where

$$\tilde{a}(s)=\frac{1}{s+\mu_1+\lambda_2}, \qquad (15)$$

$$\tilde{b}(s)=\frac{\mu_1}{s+\mu_1+\lambda_2}, \qquad (16)$$

$$\tilde{c}(s)=\frac{\lambda_2}{s+\mu_1+\lambda_2}, \qquad (17)$$

$$\tilde{d}_0(s)=1, \qquad (18)$$

$$\tilde{d}_j(s)=\tilde{w}(s)\sum_{k=0}^{j-1}{}_{n_1-1+j-k}C_{j-k}\tilde{c}^{j-k}(s)\tilde{d}_k(s). \qquad (19)$$

$$\tilde{u}(s)=\frac{\tilde{R}_1(s)}{1-\left\{1-s\tilde{R}_1(s)\right\}\tilde{b}^{n_1}(s)}, \qquad (20)$$

$$\tilde{v}(s)=\frac{1-s\tilde{R}_1(s)}{1-\left\{1-s\tilde{R}_1(s)\right\}\tilde{b}^{n_1}(s)}, \qquad (21)$$

and

$$\tilde{w}(s)=\frac{\left\{1-s\tilde{R}_1(s)\right\}\tilde{b}^{n_1}(s)}{1-\left\{1-s\tilde{R}_1(s)\right\}\tilde{b}^{n_1}(s)}, \qquad (22)$$

Based on the relationship of $MTTF_{(n_1,n_2)} = \tilde{R}_{(n_1,n_2)}(0)$, $MTTF_{(n_1,n_2)}$ can be obtained as follows:

$$
\begin{aligned}
MTTF_{(n_1,n_2)} &= \tilde{R}_{(n_1,n_2)}(0) \\
&= \frac{1/\lambda_1}{1-\tilde{b}^{n_1}(0)} \sum_{j=0}^{n_2-1} \tilde{d}_j(0) \\
&\quad + \frac{\tilde{a}(0)}{1-\tilde{b}^{n_1}(0)} \sum_{i=0}^{n_1-1} \sum_{j=0}^{n_2-1} \sum_{k=0}^{n_2-1-j} {}_{i+k}C_k \tilde{b}^i(0)\tilde{c}^k(0)\tilde{d}_j(0), \qquad (23)
\end{aligned}
$$

where

$$\tilde{a}(0) = \frac{1}{\mu_1+\lambda_2}, \qquad (24)$$

$$\tilde{b}(0) = \frac{\mu_1}{\mu_1+\lambda_2}, \qquad (25)$$

$$\tilde{c}(0) = \frac{\lambda_2}{\mu_1+\lambda_2}, \qquad (26)$$

and

$$\tilde{d}_0(0) = 1, \qquad (27)$$

$$\tilde{d}_j(0) = \frac{\tilde{b}^{n_1}(0)}{1-\tilde{b}^{n_1}(0)} \times \sum_{k=0}^{j-1} {}_{n_1-1+j-k}C_{j-k}\tilde{c}^{j-k}(0)\tilde{d}_k(0). \qquad (28)$$

Based on the similar way, $MTTF_{(n_1+1,n_2)}$, $MTTF_{(n_1,n_2+1)}$ and $MTTF_{(n_1+1,n_2+1)}$ can be derived under the combination of phases and probabilities in the respective Erlang distributions. Therefore, Takemoto and Arizono (2016) have defined MTTF of the entire system as follows:

$$
\begin{aligned}
MTTF = p_1p_2MTTF_{(n_1,n_2)} &+ (1-p_1)p_2MTTF_{(n_1+1,n_2)} \\
&+ p_1(1-p_2)MTTF_{(n_1,n_2+1)} \\
&+ (1-p_1)(1-p_2)MTTF_{(n_1+1,n_2+1)}. \qquad (29)
\end{aligned}
$$

## 4. Derivation of Variance in System Failure Time

In the derivation of MTTF by Takemoto and Arizono (2016) mentioned in the previous section, the variance of the failure time of Component 1 is not required. However, it is natural that the variance of the failure time of Component 1 is provided as the minimum information like the cases of other distributions. Accordingly, we revise the assumption (i) in Takemoto and Arizono (2016) as follows:

(i) The failure time distribution of Component 1 has the cumulative distribution function (CDF) $F_1(t)$ with mean $1/\lambda_1$ and "variance $V_{F_1}$". On the other hand, the mean and variance of the repair time of Component 1 are provided as $E_1$ and $V_1$, and the repair time distribution of Component 1 can be approximated as the mixed Erlang distribution with the CDF $G_1(t)$ of mean $E_1$ and variance $V_1$.

Then, we address the derivation of the variance of the failure time of the considered 2-component standby redundant system with priority.

First of all, denote the PDF of the failure time distribution of the entire system by $f(t)$. Then, the Laplace transformation for $f(t)$ is defined as

$$\tilde{f}(s) = \int_0^\infty e^{-st} f(t)dt. \qquad (30)$$

On the other hand, since the reliability function $R(t)$ of the entire system is expressed as

$$R(t) = 1 - \int_0^t f(\tau)d\tau, \qquad (31)$$

the relationship between the Laplace transformations $\tilde{f}(s)$ and $\tilde{R}(s)$ can be given as $\tilde{f}(s) = 1 - s\tilde{R}(s)$. Then, by applying the final-value theorem and the L'Hopital's rule to the differential calculus forms for this relationship, the following equations can be shown:

$$\frac{d}{ds}\tilde{f}(s)\Big|_{s=0} = -\tilde{R}(s)\Big|_{s=0} = -E[t] = -MTTF, \qquad (32)$$

$$\frac{d^2}{ds^2}\tilde{f}(s)\Big|_{s=0} = -2\frac{d}{ds}\tilde{R}(s)\Big|_{s=0} = E[t^2]. \qquad (33)$$

Accordingly, the variance $V[t]$ of the failure time distribution of the entire system can be obtained as

$$V[t] = \frac{d^2}{ds^2}\tilde{f}(s)\Big|_{s=0} - \left\{\frac{d}{ds}\tilde{f}(s)\Big|_{s=0}\right\}^2 \equiv \sigma^2. \qquad (34)$$

In the notations similar to the above, $MTTF_{(n_1,n_2)}$ in equation (23) is described as

$$MTTF_{(n_1,n_2)} = -\frac{d}{ds}\tilde{f}_{(n_1,n_2)}(s)\Big|_{s=0} = \tilde{R}_{(n_1,n_2)}(0). \qquad (35)$$

Furthermore, the MTTF of the entire system is given as

$$
\begin{aligned}
\mathrm{MTTF} = -\frac{d}{ds}\tilde{f}(s)\Big|_{s=0} &= -p_1p_2\frac{d}{ds}\tilde{f}_{(n_1,n_2)}(s)\Big|_{s=0} \\
&\quad - (1-p_1)p_2\frac{d}{ds}\tilde{f}_{(n_1+1,n_2)}(s)\Big|_{s=0} \\
&\quad - p_1(1-p_2)\frac{d}{ds}\tilde{f}_{(n_1,n_2+1)}(s)\Big|_{s=0} \\
&\quad - (1-p_1)(1-p_2)\frac{d}{ds}\tilde{f}_{(n_1+1,n_2+1)}(s)\Big|_{s=0}. \qquad (36)
\end{aligned}
$$

Moreover, we can derive the followings:

$$
\begin{aligned}
\frac{d^2}{ds^2}\tilde{f}(s)\Big|_{s=0} &= p_1p_2\frac{d^2}{ds^2}\tilde{f}_{(n_1,n_2)}(s)\Big|_{s=0} \\
&\quad + (1-p_1)p_2\frac{d^2}{ds^2}\tilde{f}_{(n_1+1,n_2)}(s)\Big|_{s=0}
\end{aligned}
$$

$$+ p_1(1-p_2)\frac{d^2}{ds^2}\tilde{f}_{(n_1,n_2+1)}(s)\big|_{s=0}$$

$$+(1-p_1)(1-p_2)\frac{d^2}{ds^2}\tilde{f}_{(n_1+1,n_2+1)}(s)\big|_{s=0}. \quad (37)$$

Then, we can evaluate the values of $d^2\tilde{f}_{n_1,n_2}(s)/ds^2\big|_{s=0}$, $d^2\tilde{f}_{(n_1+1,n_2)}/ds^2\big|_{s=0}$, $d^2\tilde{f}_{(n_1,n_2+1)}/ds^2\big|_{s=0}$ and $d^2\tilde{f}_{(n_1+1,n_2+1)}/ds^2\big|_{s=0}$. For reference, $d^2\tilde{f}_{n_1,n_2}(s)/ds^2\big|_{s=0}$ can be calculated by

$$\frac{d^2}{ds^2}\tilde{f}_{n_1,n_2}(s)\big|_{s=0}$$

$$=-2\frac{d}{ds}\tilde{u}(s)\big|_{s=0}\sum_{j=0}^{n_2-1}\tilde{d}_j(0)-2\tilde{u}(0)\sum_{j=0}^{n_2-1}\frac{d}{ds}\tilde{d}_j(s)\big|_{s=0}$$

$$+2\tilde{a}(0)\tilde{v}(0)\sum_{i=0}^{n_1-1}\sum_{j=0}^{n_2-1}\sum_{k=0}^{n_2-1-j}{}_{i+k}C_k\tilde{b}^i(0)\tilde{c}^k(0)$$

$$\times\left\{\tilde{d}_j(0)\tilde{a}(0)\left(1+i+k+\frac{h(0)}{\tilde{a}(0)}\right)-\frac{d}{ds}\tilde{d}_j(s)\big|_{s=0}\right\}, \quad (38)$$

where

$$\frac{d}{ds}\tilde{u}(s)\big|_{s=0}=-\frac{V_{F_1}+\left(\frac{1}{\lambda_1}\right)^2}{2\left\{1-\tilde{b}^{n_1}(0)\right\}}-\frac{\frac{1}{\lambda_1}\tilde{b}^{n_1}(0)\left\{n_1\tilde{a}(0)+\frac{1}{\lambda_1}\right\}}{\left\{1-\tilde{b}^{n_1}(0)\right\}^2}, \quad (39)$$

$$\frac{d}{ds}\tilde{d}_j(s)\big|_{s=0}=\frac{d}{ds}\tilde{w}(s)\big|_{s=0}\sum_{k=0}^{j-1}{}_{n_1-1+j-k}C_{j-k}\tilde{c}^{j-k}(0)\tilde{d}_k(0)$$

$$-\tilde{w}(0)\sum_{k=0}^{j-1}{}_{n_1-1+j-k}C_{j-k}(j-k)\tilde{a}(0)\tilde{c}^{j-k}(0)\tilde{d}_k(0)$$

$$+\tilde{w}(0)\sum_{k=0}^{j-1}{}_{n_1-1+j-k}C_{j-k}\tilde{c}^{j-k}(0)\frac{d}{ds}\tilde{d}_k(s)\big|_{s=0}. \quad (40)$$

$$\tilde{u}(0)=\frac{1/\lambda_1}{1-\tilde{b}^{n_1}(0)}, \quad (41)$$

$$\tilde{v}(0)=\frac{1}{1-\tilde{b}^{n_1}(0)}, \quad (42)$$

$$\tilde{w}(0)=\frac{\tilde{b}^{n_1}(0)}{1-\tilde{b}^{n_1}(0)}, \quad (43)$$

$$\frac{d}{ds}\tilde{w}(s)\big|_{s=0}=-\tilde{v}(0)h(0)\tilde{b}^{n_1}(0)-n_1\tilde{a}(0)\tilde{b}^{n_1}(0)\tilde{v}(0). \quad (44)$$

$$h(0)=\tilde{R}_1(0)+\frac{\tilde{b}^{n_1}(0)\tilde{R}_1(0)+n_1\tilde{a}(0)\tilde{b}^{n_1}(0)}{1-\tilde{b}^{n_1}(0)}. \quad (45)$$

Through similar ways, $d^2\tilde{f}_{n_1+1,n_2}(s)/ds^2\big|_{s=0}$, $d^2\tilde{f}_{n_1,n_2+1}(s)/ds^2\big|_{s=0}$ and $d^2\tilde{f}_{n_1+1,n_2+1}(s)/ds^2\big|_{s=0}$ can be obtained. Therefore, the variance of failure time of the 2-component standby redundant system with priority can be evaluated theoretically by using equation (34).

## 5. Numerical Analysis

In this section, we investigate the validity of our proposal on evaluating the variance in the 2-component standby redundant system with priority. Suppose that the mean and variance of failure time in Component 1 are given as $1/\lambda_1=100$ and $V_{F_1}=20^2$, respectively. Further, the mean and variance of repair time in Component 1 and failure time in Component 2 are given so as to be indicated in the respective tables. For an example, the result of numerical evaluation based on the proposed method established in section 4 is given in Table 1. Furthermore, the verification is given using computer simulations under the situation that the log-normal and Weibull distributions as probability distributions about failure and repair times are assumed. The log-normal distribution and Weibull distribution are well known in the reliability field because the characteristics of failure and repair times are represented. From Table 1, the validity of our proposed method has been confirmed in the respective situations. Indeed, it is found that the evaluations of the variance in the 2-component standby redundant system with priority based on the proposed method have the sufficient accuracy.

## 6. Evaluation of System Reliability

In this section, we address an idea for evaluating the reliability of the 2-component standby redundant system with priority. We have the approximation of the mean time to failure based on the evaluation proposed by Takemoto and Arizono (2016), and then the approximation of the variance of failure time based on the evaluation method proposed in this study. Therefore, under the parametric model, we can identify approximately the reliability function based on some kind of lifetime distributions which reproduces simultaneously the mean time MTTF and variance $\sigma^2$ of the 2-component standby redundant system with priority. In this study, we employ the Weibull distribution as one of lifetime distributions which reproduces simultaneously the mean time MTTF and variance $\sigma^2$.

The reliability functions indicated by the Weibull distribution against data of numerical simulations of Case I-VIII in the case of $(E_2,V_2)=\left(50.0,(0.5E_2)^2\right)$ in Table 1 are illustrated in Figure 3. From Figures 3, it has been confirmed that the shape of the reliability function by the Weibull distribution almost resembles that of each simulation result. However, it is thought that the result of the identification of the reliability function is influenced by each distribution applied as a model of the reliability function. On the other hand, we treat the nonparametric situations that the mean and variance are provided as the minimum information on the probability distributions of failure and repair times. Therefore, we further consider the novel evaluation for the system reliability based on the mean time MTTF and variance $\sigma^2$ of the 2-component standby redundant system with priority under the nonparametric model.

## 7. Remaining Life Time of System

In this section, we consider the expected remaining life time at time $T$ of the 2-component standby redundant system with priority based on the nonparametric technique. Then, the remaining life time at time $T$ is presented by

$$[x-T]^{+} = \max\{x-T, 0\}. \tag{46}$$

Table 1: Verification of the variance $\sigma^2$ based on the proposed method by simulations with 1 million iterations in the case of $(E_1, V_1) = \left(5.0, (0.3E_1)^2\right)$

| $E_2, V_2$ | Proposed method | Simulation results | | | |
|---|---|---|---|---|---|
| | | Case I | Case II | Case III | Case IV |
| | | $F_1(t)$: log-normal<br>$G_1(t)$: Weibull<br>$F_2(t)$: Weibull | $F_1(t)$: log-normal<br>$G_1(t)$: Weibull<br>$F_2(t)$: log-normal | $F_1(t)$: log-normal<br>$G_1(t)$: log-normal<br>$F_2(t)$: Weibull | $F_1(t)$: log-normal<br>$G_1(t)$: log-normal<br>$F_2(t)$: log-normal |
| $50.0, (0.5E_2)^2$ | 292465.291 | 289779.210 | 294759.032 | 290413.745 | 290966.264 |
| $50.0, (0.7E_2)^2$ | 558257.220 | 557139.465 | 569608.411 | 557023.833 | 561032.732 |
| $100.0, (0.5E_2)^2$ | 1140937.059 | 1129793.051 | 1137191.644 | 1129610.708 | 1151950.435 |
| $100.0, (0.7E_2)^2$ | 2204239.476 | 2196239.801 | 2230236.793 | 2202804.356 | 2267401.746 |

| $E_2, V_2$ | Proposed method | Simulation results | | | |
|---|---|---|---|---|---|
| | | Case V | Case VI | Case VII | Case VIII |
| | | $F_1(t)$: Weibull<br>$G_1(t)$: log-normal<br>$F_2(t)$: log-normal | $F_1(t)$: Weibull<br>$G_1(t)$: log-normal<br>$F_2(t)$: Weibull | $F_1(t)$: Weibull<br>$G_1(t)$: Weibull<br>$F_2(t)$: log-normal | $F_1(t)$: Weibull<br>$G_1(t)$: Weibull<br>$F_2(t)$: Weibull |
| $50.0, (0.5E_2)^2$ | 292465.291 | 291057.089 | 290063.087 | 293373.707 | 290184.675 |
| $50.0, (0.7E_2)^2$ | 558257.220 | 573281.521 | 556797.015 | 568342.824 | 556387.989 |
| $100.0, (0.5E_2)^2$ | 1140937.059 | 1136515.926 | 1129405.074 | 1143249.873 | 1131275.560 |
| $100.0, (0.7E_2)^2$ | 2204239.476 | 2275518.462 | 2193825.005 | 2260847.283 | 1131275.560 |

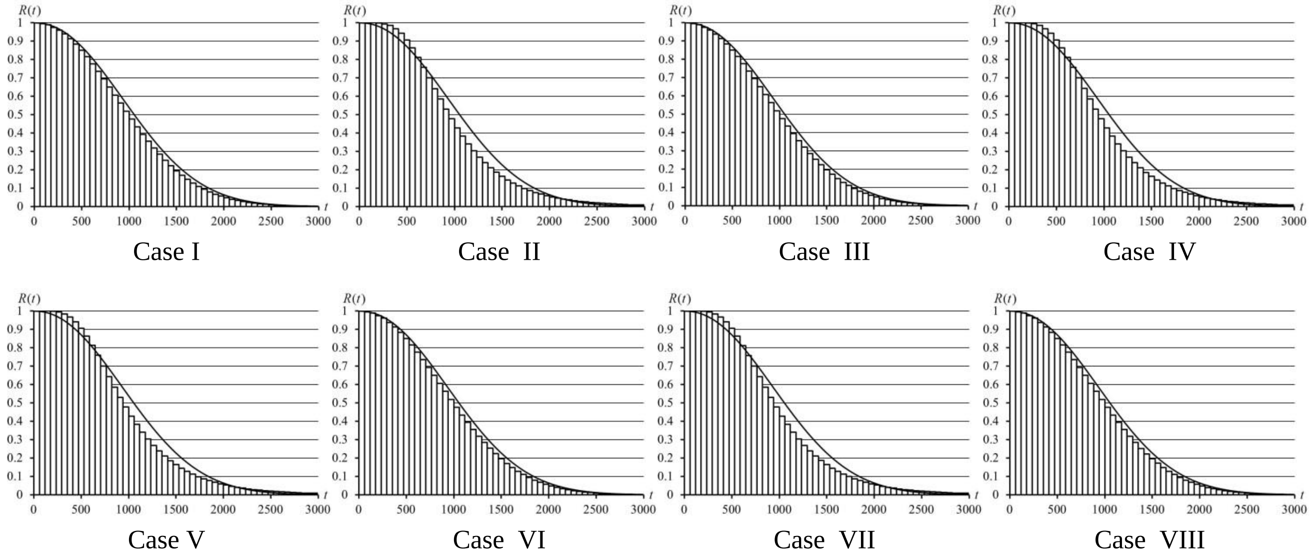


Figure 3: Reliability functions for Cases I-VIII

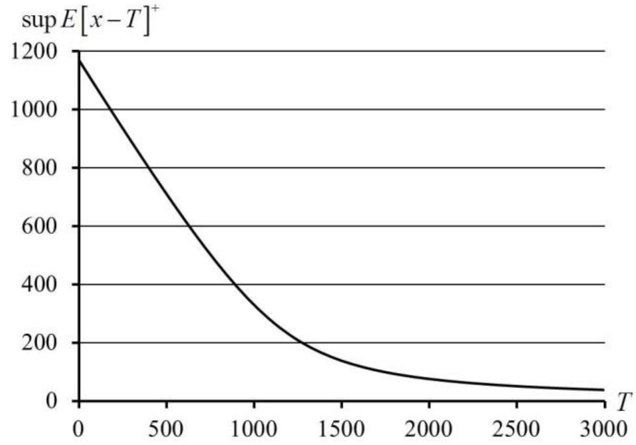


Figure 4: The expected remaining life of the system.

By adopting the inequality of Cauchy-Schwarz (see Steele, 2004) under the nonparametric technique, the expected remaining life time $E[x-T]^+$ at time $T$ can be expressed as follows:

$$E[x-T]^+ \le \frac{1}{2}\left\{\sqrt{\sigma^2+(MTTF-T)^2}+MTTF-T\right\} \equiv \sup E[x-T]^+ . \tag{47}$$

Then, the right hand $\sup E[x-T]^+$ of equation (47) means the supremum of the expected remaining life time at time $T$. Figure 4 shows the change of $\sup E[x-T]^+$ for $T$ under the condition of $(1/\lambda_1, V_{F_1}) = (100, 20^2)$, $(E_1, V_1) = \left(5.0, (0.3E_1)^2\right)$ and $(E_2, V_2) = \left(50.0, (0.5E_2)^2\right)$.

Also, the supremum of the expected remaining life time at time $T$ is interpreted as optimistic expected remaining life time at time $T$. Accordingly, if the optimistic expected remaining life time at time $T$ is less than the required remaining life time, the preventive maintenance of the system should be enforced. Therefore, we can employ the supremum of the expected remaining life time at time $T$ as the reliability index for the preventive maintenance of the system.

## 8. Conclusion

In this study, we have addressed the evaluation method of the variance of the failure time in the 2-component standby redundant system with priority. Then, by expanding the evaluation method proposed by Takemoto and Arizono (2016), the evaluation method of the variance of the failure time in the 2-component standby redundant system with priority has been successfully established. Further, the identification of the reliability function based on some kind of the parametric models has been investigated. Moreover, based on the concept of nonparametric techniques, supremum of the expected remaining life time of the system has been derived. Additionally, the capability in the supremum of the expected remaining life time as the reliability index for decision making in the preventive maintenance of the system has been indicated.

Then, it would like to be the future subjects to establish the way of the optimal preventive maintenance of the system by using the reliability indices such as the reliability function of the system and/or the supremum of the expected remaining life time in the system.

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