Multi-service Facility Location with Applications to the Recycling Industry

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Abstract. The raising environmental awareness has caused recycling a major part of our daily lives. As waste recycling has become an important business around the world, the recycling companies actually set the factories costly. In order to solve this problem, we propose a mathematical model, called Multi-service Facility Location. In this model, each facility has the ability to provide at most p types of distinct services, and each client has different requirements from the p types of services. The objective is to select a subset of facilities and identify its corresponding service assignment to clients such that the requirements of each client can be satisfied, and the total cost, including the facility setup cost, service cost and connection cost is minimized. Based on previous studies, we design a local search heuristic algorithm with theoretical analysis, and prove that our algorithm has a theoretical locality gap of three for this problem. Moreover, the implementation of the algorithm for the recycling industry in Taiwan demonstrates its efficiency and effectiveness, which can assist recycling companies in Taipei area in making suitable decisions to setup their factories and services in a better way.

Keywords: Facility location, multi-service, approximation algorithm

1. INTRODUCTION

According to the data from Environmental Protection Administration, Taiwan is now one of the world's top recyclers; precisely, the national recycling rate becomes 55 percent while the rate was only 5.87 percent 20 years ago. The critical reason to the extraordinary success in recycling is due to a large number of recycling companies (Yang, 2009) which are private enterprises. However, when taking a closer look at these recycling firms, it is not hard to discover that their locations are highly overlapped. If the firms could effectively rearrange the resources that used to set up the facilities in a centralized manner, the setup cost could be reduced significantly. Hence, this study aims to investigate this problem by transforming it to the well-known facility location problem.

During the past decades, there has been a considerable amount of research on the facility location problem and its variations in the operations research and computer science communities. The uncapacitated facility location problem (UFLP) is the most basic facility location problem. However, considering the aspect of logistic and distribution centers in which large-scale and multiple services may be provided so that the conventional model of the uncapacitated facility location problem is not suitable for such applications in the real world. For example, we consider the relationship between facilities and retailers. A retailer can provide many kinds of goods, but due to the limitation of space, the demand of markets, and so on, make it too difficult to offer all kinds of goods. One of the most efficient ways is to provide the specific types of goods to satisfy a given set of requirements of clients while minimizing the total cost. Yu (2012) then proposed a model called the multi-service center problem in which the total distance between each client and its corresponding facility that offers a service to the client is incorporated. In Yu's(2012) model, each facility provides only one kind of service. By contrast, in recycling industry, each recycling company may provide several types of recycle services.

Therefore, for the purpose of fitting the recycling business into a new generalization of the facility location problem, we proposed a new model called the multi-service facility location problem. In a distribution network, each facility has the ability to provide at most p kinds of distinct services for clients, where every facility may or may not have a given capacity limit. Each client is associated with different requirements for the p services, and a client may connect to many facilities to get services. The goal is to select a subset of facilities and to identify its corresponding service assignment to clients such that the requirements of each client can be satisfied, and the total cost, including the facility setup cost, service cost and connection cost which is usually measured by the metric distance between facilities and clients, is minimized.

2. RELATED PREVIOUS STUDIES

This section is divided into three parts for further explanation. The first part is about the benefits gained by the coalition of recycling companies. The second part presents the different variations of facility location problems. The last part looks closely into the method of how to solve the problem by using local search heuristics.

2.1 DISCUSSION OF FRANCHISE RECYCLING COMPANIES

In Yang's (2009) thesis, he focused on twelve recycling companies in the middle area of Taiwan. By considering transportation cost, labor cost, facility cost, taxes and so on, he provided a profit table that showed significant differences between franchise recycling company and traditional recycling company. Though the franchise recycling company started to earn profit not until the third coalition company joined, however, the profit later on upsurge in a rapid speed whenever a new coalition company joined. When the 12th coalition company joined, the profit is 3 times more than the total profit of traditional 12 individual recycling companies.

The reason of the huge contrast among them was due to the reduction of costs. The resources, including money, trucks and facilities, could be managed in an efficient way when the companies collaborate. Hence, the coalition pattern could not only make more profit, but also save the environment by lowering the pollutions in recycling industry.

2.2 DISCUSSION OF FACILITY LOCATION PROBLEMS

There has been a considerable amount of research on the facility location problem and its variations in the field of operational research and approximation algorithms. The uncapacitated facility location problem (UFLP) is the most basic facility location problem, in which the objective is to locate identical facilities in potential sites and assign each client to a facility such that the total cost, that is, the cost of opening facilities and connecting the clients is minimized.

Shmoys et al. (1997) gave the first constant factor approximation algorithm to the uncapacitated facility location problem. After that, a great deal of approximation algorithms had been proposed (Arya et al., 2004; Byrka and Aardal, 2010; Charikar and Guha, 1999; Chudak and Shmoys, 2003; Jain et al., 2003; Jain et al., 2001; Korupolu et al., 2000; Li, 2013; Mahdian et al., 2006; Sviridenko, 2002).

There were many di□erent approaches to the uncapacitated facility location problem. Mahdian et al. (2006) gave an algorithm with 1.52-approximation factor for UFLP which could be implemented in quasi-linear time. Their algorithm combined the greedy algorithm of Jain et al. (2002) and Jain et al. (2003) with the idea of cost scaling. Chudak and Shmoys (2003) used linear programming rounding to get a 1.736-approximation factor for UFLP. Sviridenko (2002) also used linear programming rounding to get a 1.58-approximation factor for UFLP.

Based on the work of Byrka and Aardal (2010), Li (2013) presented a 1.488-approximation factor for the metric UFLP which is the currently best ratio. Guha and Khuller (1999) proved that there is no possible to get a 1.463-approximation algorithm for UFLP, unless $NP \subseteq DTIME(n^{O(\log \log n)})$.

Charilar and Guha (1999) introduced a local search heuristic algorithm which allows inserting a facility and deleting more than one facility, and showed that it achieved an approximation factor of 3. Korupolu et al. (2000) showed that a local search heuristic algorithm which allows inserting, deleting or moving a facility had an approximation factor of 5. Arya et al. (2004) improved the approximation factor of 5 given by Korupolu et al. (2000). Arya et al. (2004) proved an approximation factor of 3 for a local search heuristic algorithm which allows inserting, deleting or moving a facility.

2.3 DISCUSSION OF LOCAL SEARCH HEURIS TIC ALGORITHMS

Arya et al. (2004) analyzed local search heuristic algorithm for the facility location problem. Arya et al. (2004) defined the locality gap of a local search procedure as follows: for a minimization problem, the maximum ratio of a locally optimum solution which obtained using the local search algorithm to the global optimum.

Based on the concept of Arya et al. (2004), we propose three similar local search operations for the multi-service facility location problem in this study. This paper is organized as follows. In Section 3, we consider the multi-service facility location problem, and give detailed explanations of the multi-service local search heuristic algorithm. We show that for the multi-service facility location problem the locality gap of the multi-service local search heuristic algorithm is at most 3. In Section 4, we implement the algorithm proposed. In Section 5, we conclude the paper with some future work.

3.THE MULTI-SERVICE FACILITY LOCATION PROBLEM

We are given some sets: $F = \{f_1, f_2, \dots, f_m\}$, the set of facilities ; $S = \{s_1, s_2, \dots, s_p\}$ the set of services; and $C = \{c_1, c_2, \dots, c_n\}$, the set of clients which is associated with a subset of services $D_j \subseteq S$ representing the type of services client c_j need. We define d(i,j) as the connection cost between facility $f_i \in F$ and client $c_j \in C$ and that the transportation cost between facility and client can be considered the connection cost times the number of services transferred in the connection.

The goal of the multi-service facility location problem is to select a subset of facility $F' \subseteq F$ and determine the subset of services $S_i' \subseteq S$ for each $f_i \in F'$ which represent the services provided in f_i. For each client $c_j \in C$, we need to find a subset $F^c \subseteq F'$ which is the assignment of clients to get the specific services such that the different requirements of each clients can be satisfied, and the total cost, including the facility set-up cost, service cost and connection cost, which is usually measured by the metric distance between facilities and clients, is minimized. When p=1, each facility provides a single type of service. Without the different types of services, it is the same problem as the traditional facility location problem, which aims at locating facilities in potential sites and connecting clients to the closest facility.

The multi-service facility location problem (MSFLP). For each facility $f_i \in F'$, the costs of opening a facility is denoted by cost(f_i), and for each service provided in $f_i \in F'$, the costs of providing service s_k is denoted by cost($s_{k,i}$). The cost of connection between facility f_i and client c_j is defined by d(i,j). The goal is to find out a set of facility $F' \subseteq F$ and identify its corresponding service provided to serve all clients such that the total cost including facility set-up cost, service cost and connection cost is minimized.

$$\text{cost(MSFLP)} = \sum_{f_i \in F'} \text{cost}(f_i) + \sum_{f_i \in F'} \sum_{s_k \in S_i} cost(s_{k,i}) + \sum_{f_i \in F'} \sum_{c_j \in C} \sum_{s_k \in D_j} d(i,j)$$

3.1 FIND AN INITIAL SOLUTION

To solve this problem, we decompose this problem into p sub-problems according to the given p services. In each sub-problem, there is only one type of service provided. Since the service cost di□ers from the types, without the di□erent types of services, it is the same problem as the traditional facility location problem, then each sub-problem can be approximated within a factor of the currently best ratio (Li, 2013) to the traditional facility location problem.

We solve each sub-problem by a local search heuristic algorithm for the traditional facility location problem. Combining the result of each sub-problem, we can get a feasible solution for the multi-service facility location problem. Based on the feasible solution, we apply the multi-service local search heuristic algorithm to solve this problem.

We use the example shown in Figure1 to illustrate how to get the initial solution for the multi-service facility location problem. In this example, there are only two kinds of services, and each client has its own need for the two services. As shown in Figure 1, the triangle represents a client, and the circle represents a facility. The symbols s_a and s_b next to each client represent the specific service that the client needs. Clients c_1, c_2, c_4, c_5, c_6, c_8, c_{10} have a single service, and clients c_3, c_7, c_9 have two services to be satisfied. Next, we find an arbitrary feasible solution of this example to illustrate a solution of the multi-service facility location problem (see Figure 2).

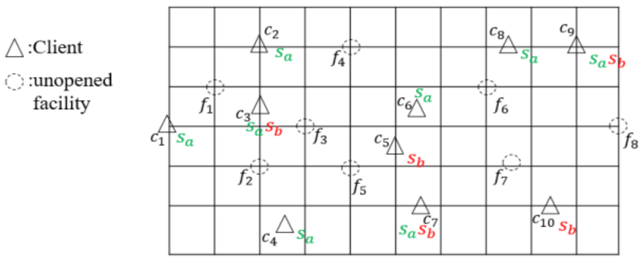

Figure 1: A simple example of this problem

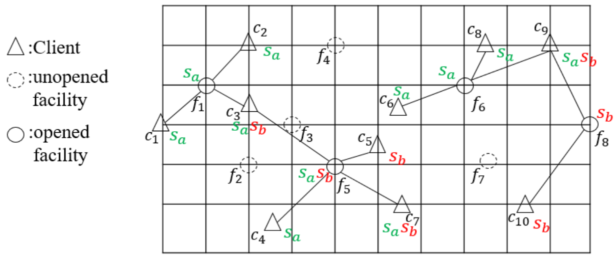

Figure 2: A feasible solution of the example

We decompose the example into two sub-problems according to the demand of services of each client (see Figure 3), and use local search steps to find a feasible solution as shown in Figure 4. Combining the solution of each sub-problem, we can get the initial feasible solution.

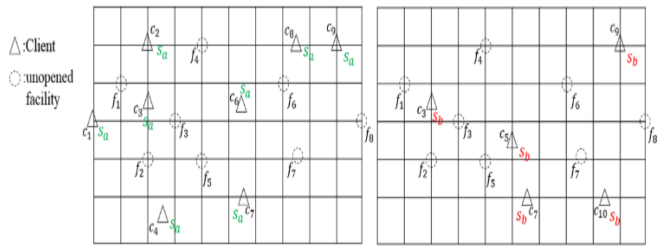

Figure 3: Decompose the feasible solution (shown in Figure 1) into two sub-problems

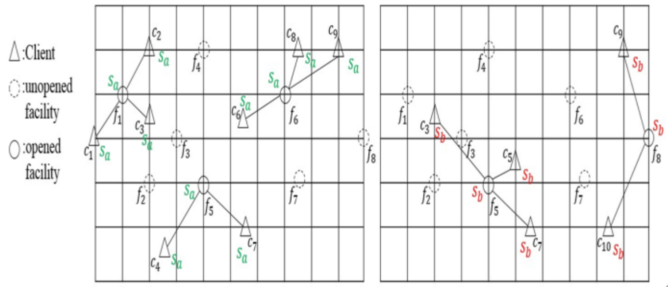

Figure 4: Local search solution for each sub-problem

3.2 MULTI-SERVICE LOCAL SEARCH HEURISTIC ALGORITHM

Based on local search heuristic algorithms for the facility location problem, we propose an algorithm called Multi-service local search heuristic algorithm to solve this problem. There are three operations allowed in Multi-service local search: inserting a service, deleting a service, and moving services. Multi-service local search starts with the feasible solution obtained by solving each sub-problem and conducts the three operations to improve the solution. Once the solution cannot be improved by the three operations, we get the local optimum. These operations described as follows:

1. Insertion: add a service s_k with cost(s_k) to an arbitrarily facility $f_i \in F$. Before insertion, if f_i is closed, then we need to open it with cost(f_i). This operation is denoted by $I(s_{k,i})$.

2. Deletion: delete a service s_k which has provided in $f_i \in F'$ and save cost(s_k). Before deletion, if f_i only provide s_k, then we can delete facility f_i and save cost(f_i). This operation is denoted by $D(s_{k,i})$.

3. Move: delete a service s_k that provide in $f_i \in F'$, and open this service in another facility f_j which does not provide service s_k. Before move, if f_i provides s_k only, then we can delete facility fi and save cost(f_i), and if f_j is closed, then we need to open it with cost(f_i). It is denoted by $M(s_k, f_i, f_j)$.

We give three easy examples to show the details of these operations.

For insertion, consider that we do $I(s_{b,1})$ in the feasible solution shown in Figure 2. Since f_1 is already opened, we only need to spend cost(s_b) to add s_b in f_1. After insertion, all the clients will be reassigned to the closest facility that has provided the specific service, then as shown in Figure 5, c_3 will be reassigned to f_1 to get s_b with lower connection cost. If the total cost increased after insertion, then we will revoke the operation $I(s_{b,1})$, otherwise, remain it.

For deletion, consider that we do $D(s_{a,5})$ in the feasible solution shown in Figure2. Since f_5 has provided s_a and s_b, we can delete s_a from f_5, and save cost(s_a). After deletion, all the clients will be reassigned to the closest facility that has provided the specific service, then as shown in Figure 6, c_4, c_7 will be reassigned to f_1 and f_6 respectively to get s_a. If the total cost increased after deletion, then we will revoke the operation $D(s_{a,5})$, otherwise, remain it.

For move, consider that we do $M(s_b, f_5, f_6)$ in the feasible solution shown in Figure2. Since f_5 has provided s_a and s_b, and f_6 has provided s_a, we can move s_b from f_5 to f_6 without an additional cost. After move, all the clients will be reassigned to the closest facility that has provided the specific service, then as shown in Figure 7, c_3, c_5, c_7 will be reassigned to f_6 to get s_b. If the total cost increased after move, then we will revoke the operation M (s_b, f_5, f_6), otherwise, remain it.

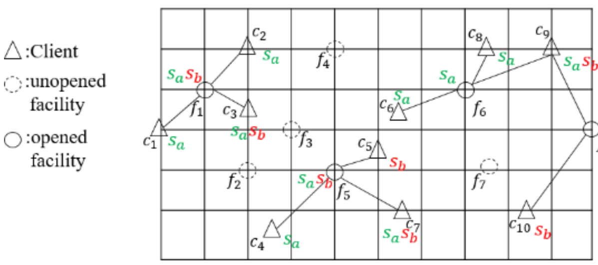

Figure 5: Do $I(s_{b,1})$ in the feasible solution shown in Figure2

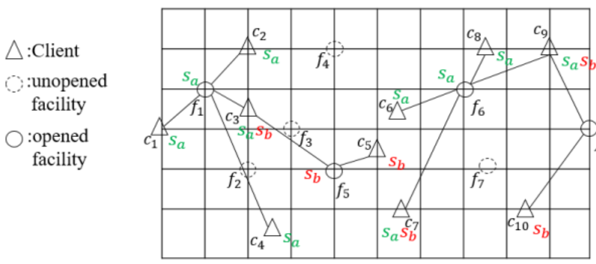

Figure6: Do $D(s_{a,5})$ in the feasible solution shown in Figure2

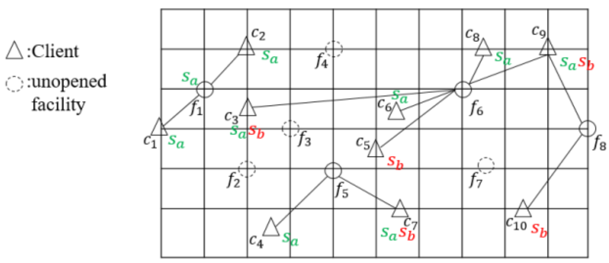

Figure7: Do $M(s_b, f_5, f_6)$ in the feasible solution shown in Figure2

3.3 THE LOCALITY GAP

Similar to the proof in (Arya et al. 2004), we propose an upper bound for the multi-service facility location problem when we use the multi-service local search heuristic algorithm. We first define some notations. Let L be the solution returned by the local search procedure, O be the optimal solution and A be an arbitrary feasible solution of this problem. Let costA(f), costA(s), costA(d) be the facility cost, service cost and connection cost of solution A. Thus

$$cost(A)=cost_A(f)+cost_A(s)+cost_A(d).$$

For every service s_k provided in $\mathbf{f_i}$, $N_A(s_{k,i})$ denotes as the set of clients that get service s_k by $\mathbf{f_i}$ in solution A. Given a service $s_{k,i} \in O$, we can partition $N_O(s_{k,i})$ into subsets $N^{s_{k,i}}_{s_{k,l}} = N_O(s_{k,i}) \cap N_L(s_{k,l})$ as shown in Figure 16.

Definition 1. *If $N_L(s_{k,l}) \cap N_O(s_{k,i}) > \frac{1}{2} N_O(s_{k,i})$, we say that a service s_k provided in $f_l \in L$ captures a service s_k provide in $f_i \in O$.*

It is obvious that service s_k provided in any $f_i \in O$ can be captured by at most one service s_k provided in any $f_l \in L$. If a service $s_{k,l} \in L$ captures some service $s_{k,i} \in O$, then we call $s_{k,l} \in L$ *bad*, and otherwise *good*. We define a 1-1 and onto function π: $N_O(s_{k,i}) \longrightarrow N_O(s_{k,i})$, and it satisfies property 1.

Property 1. *If $s_{k,l} \in L$ does not capture any $s_{k,i} \in O$, that is $N_{s_{k,l}} < \frac{1}{2} N_O(s_{k,i})$*, then $\pi(N^{s_{k,i}}_{s_{k,l}}) \cap N^{s_{k,i}}_{s_{k,l}} = \emptyset$

Based on the property, the next two lemmas follow.

Lemma 1. (connection cost)

$$cost_L(d) \leq cost_O(f)+cost_O(s)+cost_O(d) \leq cost(O)$$

Lemma 2. (facility and service cost)

$$cost_L(f)+cost_L(s) \leq cost_O(f)+cost_O(s)+2cost_O(d) \leq 2cost(O)$$

Combining Lemma 1. and Lemma 2. we get the following result.

Theorem 1. *For the multi-service facility location problem, the locality gap of the multi-service local search heuristic algorithm which allowed insertion, deletion and move is at most 3.*

4. EMPIRICAL STUDY

The facility location problem in previous works involved merely the cost of distances of setting up a factory facility, whereas when incorporated with the element of “service”, not only added another consideration into site selection, but can also vitally affect warehousing and logistics. Note that online location setting was made effortlessly in this study.

Due to the nature of this study and corporate privacy policies, no private business has yet to provide related information. This study however utilizes public information provided by the government on the recycling industry in T aipei, Taiwan. The algorithm used in this study is conducted based on a single enterprise; hence, all twenty recycling companies in Taipei can be considered to be branches of a single corporation. Possible locations are used to verify the feasibility of the algorithm proposed in this study. Owing to Taiwan’s continuously falling birthrate, elementary schools face risks of mergers and closings in the near future. Hence, 12 elementary schools around Taipei are set as possible site selections for future branch locations of the recycling industry. Clients’ choices will be set as one village per unit, for a total of 456 villages, therefore, 456 clients will be used for this study. Service types are categorized into 34 types as listed by the EPA, however, only 16 frequently used services will be utilized in this study. Their location can be identified on Google Map given the above conditions (see Figures 8 and 9).

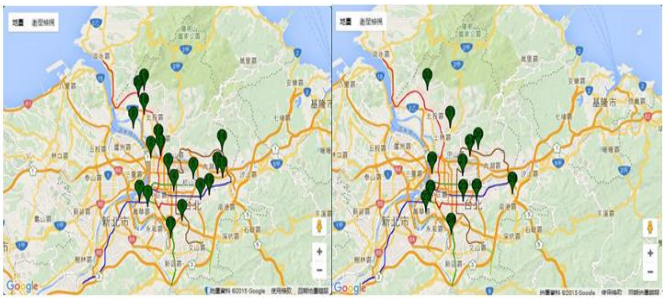

Figure 8: The location of 20 recycling companies (left side) and 12 candidates (right side)

Figure 9: The location of 456 clients

Having established the final algorithm and results, we used Google Maps as a visualization interface for this study. As shown on the left of Figure 10, 20 recycling locations along with their types of recyclable waste services, along with the respective associated village centers. The interface also shows an option of 12 “candidate recycling locations”, each of which can be selected by users to add to the former 20 recycling locations, of which will be shown on the list alike. Users can compare their chosen locations. After calculation, the interface displays further information of each recycling location.

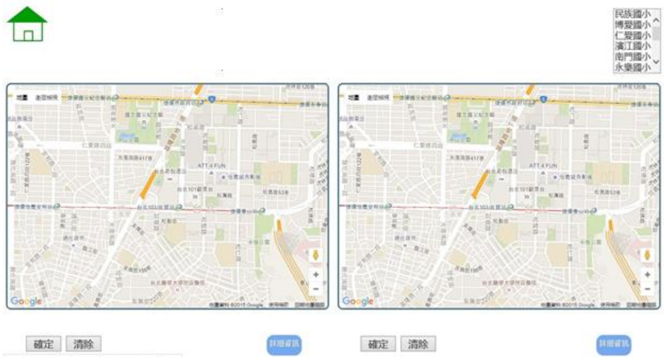

Figure 10: The interface of Experiments

The left side of Figure 11 shows the result of original 20 recycling facilities after calculation. The right side is the local optimal result of adding all 12 candidates into the algorithm.

Precisely, we find out that the total cost can be cut down if there are more potential locations. The cost was 152,341 units when there were only 20 facilities, but after adding the other 12 candidates into the calculation, the cost became 135,306 units. The 11.18 percent reduction of cost can be said as the profit that the company earns after using the algorithm, which is a significant improvement.

To look closer into the example, No.5 facility provided No.84 village services in Figure 12. After adding the other 12 candidates, No.84 village gets services from No.30 facility in Figure 13 because the distance between them is shorter. The reduction of distance can decrease the company's total cost.

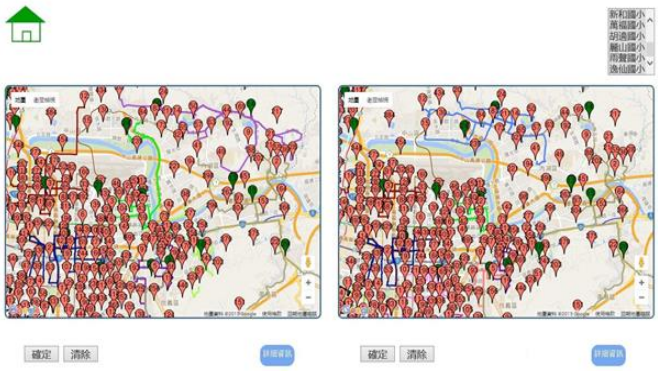

Figure 11: The near optimal solution before and after adding the candidates

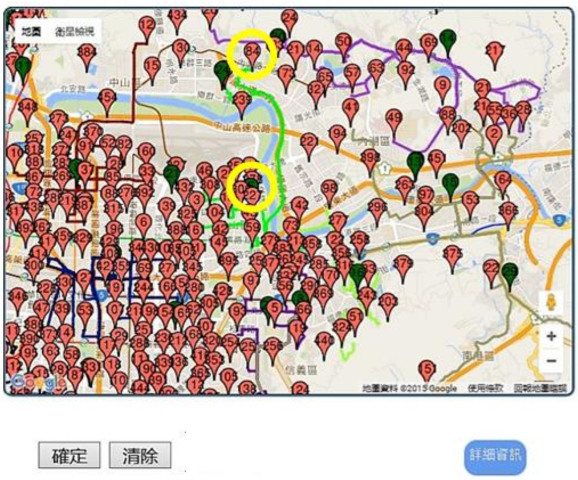

Figure 12: The near optimal solution of the original 20 facilities

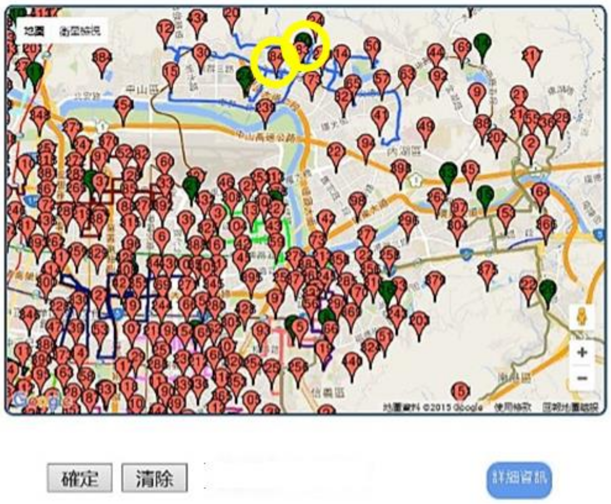

Figure 13: The near optimal solution of the original 20 facilities plus 12 candidates

5. CONCLUSIONS AND FUTURE WORK

In this study, we have proposed a new problem called the multi-service facility location problem. This research has attempted to create a new thinking of the facility location problem that not only consider location selection, but also determine what kind of service that one should provide and fulfill the demand of the market simultaneously.

We have modified the local search heuristic algorithm, and have presented an algorithm called the *multi-service local search heuristic algorithm* to solve this problem. We have shown that the multi-service local search heuristic algorithm has a *locality gap* of 3. We have demonstrated the outcome on Google Map that allows users to understand the result easily. This algorithm has better fit the condition in the real world than other traditional facility location problems. It can be applied to not only recycling industry but also other industries, such as logistics industry and retail industry.

In the future work, it would be of interest to consider the model of taking off the redundant connection cost which means that once a connection is built, facility can transfer multiple services through the connection without additional cost. The other interesting research direction is to study the capacitated version of the multi-service facility location problem so as to meet more real-world requirements.

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