An optimization procedure to solve forward-reserve allocation problem with (s, S) policy

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Abstract. To reduce order picking activities, many distribution centers include a forward area, where items are stored in a unit of piece for easy retrieval by an order picker, and a reserve area, where items are stored in a unit of case for picking and replenishing stocks in the forward area. Reducing inventory space of the forward area leads to more frequent replenishment from the reserve area to the forward area. To tackle the tradeoff between replenishment frequency and inventory space, forward-reserve problems with (s, S) policy determine order-up-to and reorder levels of each stock keeping unit (SKU) in the forward area. Furthermore, this study proposes an optimization procedure, which has two phases. Based on actual data sets in Japan, we show significance of the proposed two phase method and the tradeoff.

Keywords: Warehousing, Forward-reserve allocation problem, (s,S) policy, Optimization, Logistics

1. INTRODUCTION

In the supply chain, a distribution center (DC) is one of the most important facilities. A company considered in this paper is the largest toy manufacture in Japan. As shown in Figure 1, products made in China or Southeast Asia are carried to DC in Chiba prefecture in Japan and temporarily stored there. When an order of some products arrives from a store in Japan, the products are order picked, packed and shipped to the store. As a facility to keep balance of demand and supply, a DC has a major role to improve profitability.

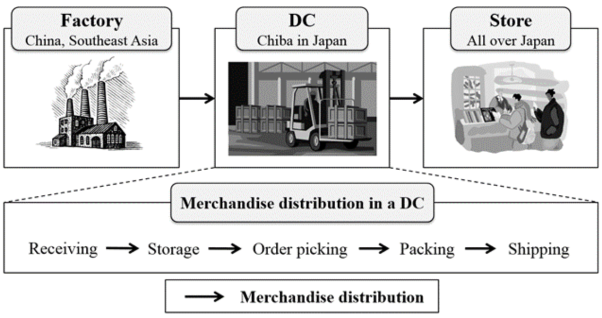

Figure 1. Merchandise distribution in the targeted DC

Major works in a DC are receiving, storage, order picking, packing and shipping. In these works, the cost of order picking is estimated to be as much as 55% of the total warehouse operating expense (De Koster et al., 2007). Since an efficiency of order picking will be changed by where products are stored and how many quantities of product are stored, some reflections of inventory management can lead to cost saving.

In recent years, the number of different stock keeping units (SKUs) that must be delivered is exploding. Due to the spread of electronic commerce, filling customer orders within a 24-hour period is becoming the new standard in many industries, which means that an increasing number of SKUs must be delivered more frequently and faster (Van den Berg and Zijm, 1999). Namely, shipping form changes to high frequency and small lot delivery. Needless to say, distribution centers (DCs) have no choice but to improve their order fulfillment operations through better storage, item locating, replenishing, picking and routing strategies (Petersen and Aase, 2004).

In order to correspond to high frequency and small lot delivery, there are many DCs configured with a forward area (or a fast pick area) and a reserve area (or a bulk area). The former is an area, where items are stored in a unit of piece for easy retrieval by an order picker (Rouwenhorst et al., 2000), and the latter is an area, where items are stored in a unit of case for picking and replenishing stocks in the forward area. The picked quantity in the forward area is generally less than the number of pieces in a case, otherwise separate forward and reserve areas would not be effective. An SKU in the forward area is replenished with one or more cases from the reserve area when the inventory level decreases.

In the targeted DC, inventories in the forward area are controlled by (s, S) policy as shown in Figure 2. Under the (s, S) policy, a replenishment order is released if the inventory level is below the reorder level s. When a SKU is replenished, it is raised by the least number of cases over the order-up-to level S. This replenishment strategy is called regular replenishment (RR). RR usually occurs 3 times after the all picking operation in a week in the targeted DC.

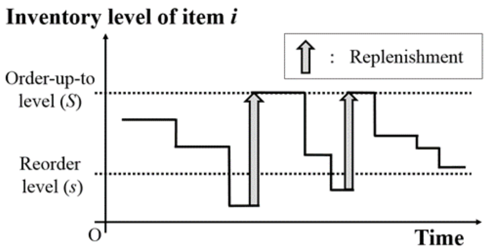

Figure 2. (s, S) policy

In the forward area of the company, there are many SKUs that have stock-outs or excessive stock. If an inventory level of an item is below picked amount, the picking operation must be stopped until that item can be replenished by a batch replenishment (BR). While waiting for the supply to be replenished, the order picker is unproductive. On the other hand, excessive stock leads to decrease space of another items since the space all over warehouse is limited.

Based on an actual data, this paper studies the forward-reserve allocation problem with (s, S) policy. The forward-reserve allocation problem with (s, S) policy is the problem based on the forward reserve allocation problem (FRAP) and the problem of determining the number of SKUs to be stored in the forward area and the space to be allocated to each SKU by setting order-up-to and reorder levels of each SKU.

The FRAP has the following form;

$$\text{minimize} \quad \sum_{i\in I}\sum_{r=1}^{n_i} c_i \frac{d_i}{S_{ir}} x_{ir} \tag{1}$$

subject to

$$\sum_{r=1}^{n_i} x_{i,r} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{i\in I}\sum_{r=1}^{n_i} w_{ir} \cdot x_{i,r} \leq IS \tag{3}$$

$$x_{i,r} \in \{0,1\} \qquad \forall i \in I,\ r = 1, \dots, n_i \tag{4}$$

where I is a predefined set of items, n_i, c_i, d_i, S_{ir}, w_{ir}, IS are parameters which means the number of possible storage modes of item i , replenishment unit price of item i, total demand of item i per planning period, the number of item i that can be stored in storage mode r, the space required by storing item i in storage mode r and inventory space, respectively, and x_{ir} is a decision variable which will be 1 when storage mode j of item i is chosen, otherwise which will be 0. We assume that the storage modes of each item i are labeled so that $w_{i1} \geq w_{i2} \geq \ldots \geq w_{in_i}$ throughout this paper.

The objective function (1) is to minimize the replenishment cost per planning period. As formulated in the mathematical model (1) through (4), the FRAP decides on the model in which each SKU is stored in the forward area (constraint (2)). Equivalently, it decides on the number of items to be stored per SKU without exceeding inventory space IS (constraint (3)) which is assumed to be at least as large as $\sum_{i\in I} w_{i1}$. Otherwise, not all SKUs of the predefined set could be stored in the forward area, namely the model is infeasible. (4) indicates that x_{ir} is binary variable. Clearly, the more space is associated with each SKU, the less replenishments are required. Note that the underlying assumptions with regard to replenishments are discussed in detail by Bartholdi and Hackman (2008).

The FRAP is mathematically equivalent to the well-known multiple-choice knapsack problem (MCKP), which becomes obvious when interpreting the parameters of the MCKP as follows (Walter *et al*, 2013):

- SKUs correspond to classes and modes correspond to items.
- The profit of item r of class i is p_{ir}=- $c_i \cdot d_i / S_{ir}$ (minimizing $\sum_i \sum_r p_{ir} x_{ir}$ is equivalent to maximizing $\sum_i \sum_r -p_{ir} \cdot x_{ir}$).
- The weight of item r of class i equals the space w_{ir} required by storing item i in mode r.
- The capacity of the knapsack is c=IS.
- The size of class i is n_i.

The purpose of this study is to assist DC managers to judge how much inventory space should be allocated to each SKU in the forward area in the limited space by giving a tradeoff between replenishment frequency and inventory space.

The remainder of this paper is organized as follows. Section 2 reviews important researches related to the problem addressed in this paper. Section 3 describes the proposed model. Section 4 presents a procedure for optimizing the proposed model. Detailed computational results are given in Section 5, and our conclusions are offered in Section 6.

2. LITERATURE REVIEW

2.1 Forward area and reserve area

The next level of heading is boldface with upper and lower case letters. The heading is flushed left with the left margin. There are several papers whose target is a distribution center configured with a forward and a reserve area. At first Hackman et al. (1990) formulated a mathematical model to allocate space to items in a forward area and proposed a greedy heuristic. Their work gave an incentive for the paper of Hackman and Platzman (1990) who proposed a generic model for deciding which SKUs to pick from forward area and how much space to allocate on which storage shelf to each selected SKU. They also developed a heuristic procedure with a good performance whenever each allocation is a small fraction of storage space. Further contributions raised from Van den Berg et al. (1998), who optimized unit-load replenishments that take place during busy and idle periods and Bartholdi and Hackman (2008), who analyzed two wide-spread real-world stocking strategies for small parts in a forward area. Gu et al. (2010) provided a branch-and-bound algorithm for solving the joint assignment and allocation problem. And Walter et al. (2013) summarized three kinds of discrete forward-reserve allocation problems, the discrete forward-reserve allocation problem (DFRAP), the discrete forward-reserve assignment and allocation problem (DFRAAP) and the discrete forward-reserve allocation and sizing problem (DFRASP), and contributed to avoid continuous space allocated to each SKU, namely consider discrete unit space allocated to each SKU. DFRAP is the most basic problem, where the given space of a forward area is to be partitioned among a predetermined set of SKUs. DFRAAP combines the space allocation problem with the assignment problem of selecting the products to be stored in the forward area. Finally, DFRASP treats the allocation problem jointly with the sizing problem, i.e., for a given set of products a forward area of variable size is to be allocated.

Gagliardi et al. (2008) considered a warehouse with a forward area, a reserve area and a pick-to-belt system. The warehouse faces stock-outs in the forward area during picking and only one technician is responsible for a continuous replenishment. The authors proposed four heuristic replenishment policies. Two are based on long-term demand information, while the other two also consider short-term demand information by checking incoming picking orders. They showed that selecting the right locate in and replenishment methods can significantly reduce the number of stock outs in the forward area. However, they consider only the next product to be replenished by only one technician. De Vries et al. (2014) considered wave-picking and set priorities for all products to be replenished by several people. They presented three new internal stock replenishment policies in order to minimize the problem of 0-picks, which are stock-outs in the forward area. The unique feature of these policies is that they assign priorities to replenishment orders based on short-term demand information that is available because of the wave-picking strategy used in the warehouse.

Osumi et al. (2015) also considers a warehouse with a forward area where inventories are controlled by (s, S) policy and a reserve area. They propose a model to determine order-up-to and reorder levels of each item in the forward area. Its objective is to minimize the inventory space to improve excessive stocks. And it has a constraint of prohibiting stock-outs to improve stock-outs in the forward area. Their experiments of an actual data showed a drastic reduction of the number of stock-outs and the inventory space.

2.2 (*s,S*) policy

There are several papers whose target is (s, S) policy. Under (s, S) policy, the inventory level of an item is reviewed regularly and if it is found to be below a reorder level s, an order is placed to bring the inventory position up to the order-up-to level S. The choice of s and S is made to minimize the expected cost while taking into account the fixed ordering cost, inventory holding cost, and the cost associated with stock outs. The fundamental work on (s, S) policies is due to Arrow et al. (1951). However, the optimality of such a policy, under general conditions, for the finite horizon case with backorders was first established by Scarf (1960). Scarf(1960) showed that (s, S) policy is the optimal policy when the inventory holding cost and stock out cost are linear. Federgruen and Zipkin (1984) considered backorders and proposed an algorithm which determined order-up-to and reorder levels when total cost is the lowest by enumerating combination choices of order-up-to and reorder levels. Xu et al. (2010) considered lost sales and proposed an algorithm which get optimal order-up-to and reorder levels.

Arrow et al.(1951), Scarf(1960), Federgruen and Zipkin(1984) and Xu et al.(2010) considered (s, S) policy under the situation where the replenishing units are same as the shipping units. Osumi et al. (2015) considered (s, S) policy under differences between replenishing and shipping units. They targeted a distribution center configured with a forward area where inventories are controlled by (s, S) policy and a reserve area. Shipping units of the forward area are pieces, on the other hands, replenishing units from the reserve area to the forward area are cases.

3. PROPOSED MODEL

In this section, we present the model design, the proposed formulation and notation. The objective function of this model is to minimize replenishment frequencies.

3.1 Model design

There are several papers whose target is (s, S) policy. Under (s, S) The problem discussed in this paper is the forward-reserve allocation problem with (s, S) policy. By determining order-up-to level S and reorder level s, we will determine the number of SKUs to be stored in the forward area and the space to be allocated to each SKU. This model has some assumptions described below:

1: An initial stock of each item is equal to order-up-to level.
2: A replenishment unit from a reserve area to a forward area is different from a shipping unit. The former is a case and the latter is a piece.
3: Demand quantity is less than the number of pieces per case.
4: Inventories in a forward area are controlled by (s, S) policy. As a replenishment unit is a case, an inventory level after replenishment can be over order-up-to level S and the region is

[order-up-to level, order-up-to level + the number of pieces per case).

5: Each item can be replenished by two kinds of replenishment, batch replenishment (BR) and regular replenishment (RR). BR occurs when an inventory level of a demanded item is below picked amount. RR occurs when an inventory level is below the reorder level s after all picking operations on RR dates.

6: Each item has enough inventories in a reserve area. Namely if a replenishment order is released, the replenishment is invariably accomplished.

3.2 Model Formulation

3.2.1 Definition of Sets and Parameters

I : Set of items

T : Set of dates

T_R : Set of dates when RR occurs

B_t : Set of batches on date t

v_t : Piece volume of item i

$d_{i,t,b}$: Quantity of item i demaded in batch b on date t

C_i : The number of pieces per case of item i

$|b_t|$: Last batch on date t

IS : Inventory Space

α : Weight on the frequency of BR

β : Weight on the frequency of RR

M : Sufficiently large value

3.2.2 Definition of Decision Variables

s_i : Reorder level of item i

S_i : Order-up-to level of item i

$I_{i,t,b}$: Inventory quantity of item i before batch b on date t

$m^p_{i,t,b}$: The number of BR cases of item i at batch b on date t

$m^r_{i,t}$: The numberof RR cases of item i on date t

$$p_{i,t,b}\begin{cases}=1: \text{if BR of item } i \text{ occurs in batch } b \text{ on date } t\\ =0: \text{otherwise}\end{cases}$$

$$r_{i,t}\begin{cases}=1: \text{if RR of item } i \text{ occurs on date } t\\ =0: \text{otherwise}\end{cases}$$

3.2.3 Objective function and constraints

As formulated in the mathematical model (5) through (20), the forward reserve allocation problem with (s, S) policy decides on determining the number of SKUs to be stored in the forward area and the space to be allocated to each SKU by determining order-up-to and reorder levels.

The objective function is given as (5), and it specifies the replenishment frequencies; the first summation represents the batch replenishment frequencies (BRF) and the second summation represents the regular replenishment frequencies (RRF). The space constraint is given as (6). (7) is a constraint on the initial stock. (8) determines whether the reorder level s is less than or equal to order-up-to level S. (9) and (10) determine whether BR occurs, and (11) and (12) determine whether RR occurs. (13) and (14) indicate whether BR has raised the inventory level to the order-up-to level S and over, and (15) and (16) indicate whether BR has raised the inventory level to the order-up-to level S and over. (17) and (18) are constraints on integrity and are related to BR and RR. (19) denotes the inventory inherited between two batches. (20) and (21) indicate the change in inventory from one day to the next.

$$\text{minimize } \alpha\sum_{i\in I}\sum_{t\in T}\sum_{b\in B_t}p_{i,t,b}+\beta\sum_{i\in I}\sum_{t\in T_R}r_{i,t} \quad (5)$$

subject to

$$\sum_{i\in I}v_i(S_i+C_i\text{-}1)\le IS \quad (6)$$

$$I_{i,0,0}=S_i \quad \forall i\in I \quad (7)$$

$$s_i\le S_i \quad \forall i\in I \quad (8)$$

$$I_{i,t,b}\text{-}d_{i,t,b}+M\cdot p_{i,t,b}\ge 0 \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (9)$$

$$I_{i,t,b}\text{-}d_{i,t,b}\text{-}M\left(1\text{-}p_{i,t,b}\right)<0 \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (10)$$

$$I_{i,t,|b_t|}+M\cdot r_{i,t}\ge s_i \quad \forall i\in I, \forall t\in T_R \quad (11)$$

$$I_{i,t,|b_t|}\text{-}M\left(1\text{-}r_{i,t}\right)<s_i \quad \forall i\in I, \forall t\in T_R \quad (12)$$

$$S_i\le I_{i,t,b}\text{-}d_{i,t,b}+C_i\cdot m^p{}_{i,t,b}\text{-}M\left(1\text{-}p_{i,t,b}\right) \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (13)$$

$$I_{i,t,b}\text{-}d_{i,t,b}+C_i\cdot m^p{}_{i,t,b}\le S_i+C_i\text{-}1 \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (14)$$

$$S_i\le I_{i,t,|b_t|}+C_i\cdot m^r{}_{i,t}\text{-}M\left(1\text{-}r_{i,t}\right) \quad \forall i\in I, \forall t\in T_R \quad (15)$$

$$I_{i,t,|b_t|}+C_i\cdot m^r{}_{i,t}\le S_i+C_i\text{-}1 \quad \forall i\in I, \forall t\in T_R \quad (16)$$

$$m^p{}_{i,t,b}\le M\cdot p_{i,t,b} \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (17)$$

$$m^r{}_{i,t}\le M\cdot r_{i,t} \quad \forall i\in I, \forall t\in T_R \quad (18)$$

$$I_{i,t,b+1}=I_{i,t,b}\text{-}d_{i,t,b}+C_i\cdot m^p{}_{i,t,b} \quad \forall i\in I,\ \forall t\in T,\ \forall b\in B_t \quad (19)$$

$$I_{i,t+1,0}=I_{i,t,|b_t|}+C_i\cdot m^r{}_{i,t} \quad \forall i\in I, \forall t\in T_R \quad (20)$$

$$I_{i,t+1,0}=I_{i,t,|b_t|} \quad \forall t\in T\setminus T_R \quad (21)$$

3.3 Differences between previous studies and the proposed model

The differences between FRAP and our proposed model are with regard to the following: (i) the number of kinds of replenishments (one or two); (ii) whether the quantities of each SKU assigned to forward area are included in decision variables; (iii) replenishment unit (piece or case); and (iv) replenishment quantity (fixed or the integer multiple of a case). The difference (iv) comes from a difference of inventory control policy. In FRAP, inventories are controlled by fixed order quantity system. On the other hands, in the proposed model, inventories are controlled by (s, S) policy.

We will get solutions of FRAP by optimizing our proposed model if some requirements are met as follows:

- Addition to a constraint; reorder level $s_i = 0, \forall i$. This leads to decrease the number of kinds of replenishments from two to one because no regular replenishment of item *i* occurs when s_i is equal to zero.
- All possible quantities of each SKU assigned to a forward area as a parameter.
- Regarding the number of pieces per case C_i of all SKUs as 1.
- Changing (*s*, *S*) policy into fixed order quantity system.

However the forth requirement is not realistic requirement because the structure of the proposed model would change if we try to meet the requirement.

Additionally, our proposed model includes the model which is proposed by Osumi *et al.* (2015). The objective of their model is to minimize inventory space which is defined as $\sum_{i\in I} v_i(S_i+C_i\text{-}1)$ with a constraint that all inventory level of each item on each day after completing picking operation is equal to or higher than zero, namely no BR happens. We will get optimal solutions of their model by optimizing our model if two requirements are met as follows;

- Regarding the weight on the frequency of BR α as infinity results in no frequency of BR.
- Searching for minimum *IS* can be realized by moving the value of *IS*, which is a parameter in the proposed model.

4. OPTIMIZATION PROCEDURE

We explain a procedure for optimizing the proposed model. This procedure has 2 phases: enumerating all the choices and solving a multi-choice knapsack problem. In the first phase, all of the objective function values are enumerated for each item. In the second phase, from those enumerated in the first phase, a single objective function value is chosen for each item.

4.1 Phase 1: Enumerating all the choices

All objective function values of each item i which exist in $[\,0,\ \max\{\alpha, \beta\} \times \left\lceil \sum_{t\in T} \sum_{b\in B_t} d_{i,t,b}/C_i \right\rceil +1\,]$ are evaluated as follows, where $\lceil X \rceil$ means rounding up X. $S_{i,r}$ is a minimal order-up-to level of item *i* when replenishment frequency is equal to or less than *r*. By moving r, all objective function values of each item i are enumerated. Decision variables are the same as in the proposed model. The other constraints (7) through (21) are needed.

minimize $\quad S_{i,r}$ (22)

subject to

$$\alpha \sum_{i\in I} \sum_{t\in T} \sum_{b\in B_t} p_{i,t,b} + \beta \sum_{i\in I} \sum_{t\in T_R} r_{i,t} \le r \qquad (23)$$

4.2 Phase 2: Solving multi-choice knapsack problem

We will solve the multi-choice knapsack problem discussed above. If there is an optimal solution to this problem, it will be the optimal solution of the proposed model. The other constraints (2) through (4) are needed. R_i is a set of possible objective function values for item *i*. w_{ir} is a parameter which is equal to $v_i\left(S_{i,r}+C_i\text{-}1\right)$. The decision variable $x_{i,r}$ will be 1 when replenishment frequency of item *i* is *r*, and otherwise, it will be 0.

minimize $\sum_{i\in I} \sum_{r\in R_i} r\, x_{i,r}$ (24)

5. EXPERIMENTAL RESULTS

In this section, we detail computational results. Firstly, we clarify the difference between forward reserve allocation problem (FRAP) and the proposed model. Secondly, we show experiments with an actual data. We used the Gurobi Optimizer version 6.0.0 to solve the mixed integer programming problem.

5.1 Comparative experiments

We show the comparison between FRAP and the proposed model in this subsection. In order to compare it appropriately, we regard the number of pieces per case C_i as 1 at this experiments, which leads to consider replenishment unit piece. In this experiment, only Assumption 3 that demand quantity is less than the number of pieces per case is not considered. Also, we show the results of optimizing the proposed model with $s_i = 0$, which results in changing the number of the kinds of replenishment from two to one, or only BR.

5.1.1 Data set

Table 1 implies the data set of comparative experiments of the proposed model, the optimization procedure and the FRAP. To lose the differences of the replenishment unit and unit of objective function, the number of pieces per case of each item is set 1, and the weight on the frequency of BR α and the weight on the frequency of RR β is considered as replenishment unit price of BR and RR respectively.

There are 25%, 50% or 75% chances to happen a demand event of each item in a day by selection with equal probability. The quantity of demand is 1,2,3,4 or 5 by selection with equal probability. Because d_i which is a parameter of FRAP is defined as the total demand of item *i* per planning period, it is equal to $\sum_{t\in T} \sum_{b\in B_t} d_{i,t,b}$. Furthermore, the number of possible storage modes n_i which is a parameter of FRAP is set d_i, which means all possible assignment quantity can be chosen. The space required by storing item *i* in storage mode *r*, w_{ir} is defined as $v_i \cdot S_{ir}$.

Data set 1 is used for small scale test and Data set 2 for large scale test.

Table 1. Data set of comparative experiments

	Data set 1	Data set 2
$\#I$	10, 20	1000
$\#T(\Sigma\#B_t)$	20(20)	40(40)
$\#T_R$	12 (3 times in 5 days)	24 (3 times in 5 days)
IS	50, 100, 200	0, 1486, 5000, …, 75000 (5000 increments), 75122
v_i	1 or 2	1 or 2
C_i	1	1
c_i	1	1
α, β	1, 0.25	1, 0.25
Demand event probability$_i$	0.25, 0.5, 0.75	0.25, 0.5, 0.75
$d_{i,t,b}$	1, 2, 3, 4 or 5	1, 2, 3, 4 or 5

5.1.2 Results of small scale test

We report comparative results of small scale test obtained by optimizing the proposed model with the time limit of 10,800 seconds, executing optimization procedure and optimizing FRAP by using Data set 1 showed Table 1. Table 2 implies replenishment cost of PM, OP, OP* and FRAP and Table 3 shows the execution time with the time limit of 10,800 seconds. PM represents the results of optimizing the proposed model (5) - (21) previously explained in 3.2.3, OP represents the results of executing optimization procedure explained in section 4, OP* represents the results of executing optimization procedure with a constraint $s_i = 0$ at Phase1 and FRAP represents the results of optimizing the forward reserve allocation problem explained in section 1.

We assume that each item *i* has an initial stock which is equal to S_i in PM and OP (See Assumption 1), however there are no initial stock of each item *i* in FRAP. In order to consider that each item *i* has initial stock in FRAP, objective function is redefined as follows:

$$\text{minimize} \quad \sum_{i\in I}\sum_{r=1}^{n_i} c_i \frac{d_i}{S_{ir}} x_{ir} \ - (\#I) \qquad (1)^*$$

The form of FRAP is described as (1)* subject to (2) - (4). Note that we can get the following observations from these tables:

- Replenishment cost of PM is same as OP.
- Replenishment cost fulfills "PM = OP < OP* and FRAP" because inventory space is used more effectively in order-up-to system than fixed order quantity system, and PM and OP have RR whose cost is lower than BR.
- As shown in Table 3, execution time of PM decreases once it increases as *IS* increases. Also execution time of PM is much longer as experiments are larger scale. On the other hands, OP can get the optimal solution relatively rapidly.

Replenishment cost fulfills "FRAP < OP*" when (Item, *IS*) = (10, 200), however it fulfills "FRAP > OP*" besides (Item, *IS*) = (10, 200). We will reveal the reason in observations of next experiments.

Table 2. Replenishment cost of small scale test

Item	*IS* =50				*IS* =100				*IS* =200			
	PM	OP	OP*	FRAP	PM	OP	OP*	FRAP	PM	OP	OP*	FRAP
10	26.50	26.50	36.00	56.23	9.25	9.25	19.00	22.94	2.50	2.50	7.00	6.59
20	117.00	117.00	127.00	295.50	64.50	64.50	83.00	131.60	21.75	21.75	46.00	55.42

Table 3. Execution time (sec)

Item	*IS* =50				*IS* =100				*IS* =200			
	PM	OP	OP*	FRAP	PM	OP	OP*	FRAP	PM	OP	OP*	FRAP
10	76.43	17.98	4.74	0.11	73.64	17.97	4.74	0.05	4.65	18.02	4.68	0.05
20	244.11	52.29	32.89	0.09	10800 (3.88%)	52.30	32.80	0.08	10800 (12.64%)	52.29	32.81	0.11

5.1.3 Results of large scale test

We report comparative results of large scale by using Data set 2 showed in Table 1. In this experiments, we compare the results of OP, OP* and FRAP because scale of test is too large to get solutions of PM.

Table 4 depicts replenishment cost of large scale test of OP, OP* and FRAP and gap between OP and OP*, and between OP* and FRAP, where gap between A and B is defined as $100\times(\text{A-B})/\text{A}$ (%) . Table 5 shows the execution time of large scale test. A hyphen means indefinable.

We can get the following observations from these tables:

- The results of OP is the best in the others, the results of OP* and FRAP.
- Replenishment cost of OP* is smaller than replenishment cost of FRAP below *IS* = 35000 because total replenishment quantity of OP* is nearly same as that of FRAP and replenishment quantity of OP* is more than that of FRAP.
- Replenishment cost of FRAP is smaller than replenishment cost of OP* above *IS* = 40000 because replenishment frequency of FRAP permits a decimal fraction.
- The gap of OP and OP* is equal to zero at *IS* = 0 and *IS*= 75122 because there are no items replenished by RR. Except for them, the gap of OP and OP* increases as *IS* is larger because reorder level of each item is set to decrease replenishment cost.
- A defect of OP is the execution time at Phase1. As the scale of test is larger, the number of enumeration at Phase1 increases, which results in long execution time.

5.2 Case study

5.2.1 Data set

Table 6 indicates the data set of case study. Regular replenishment is occurred once, …, 5 times in a week after completing all picking operations. Because the targeted DC operates 5 days in a week, 5 times in a week means RR is occurred every day after picking operation.

Figure 3 depicts the Pareto chart about piece demand of 5,000 SKUs in 60 days. From the chart, we can observe a concentration of over 80% piece demand quantity on 946 SKUs in 5,000 SKUs and scattering of demand on the other SKUs, 4,054 SKUs.

Table 4. Replenishment cost of large scale test and gap

IS	OP	OP*	FRAP	Gap(OP,OP*)	Gap(OP*,FRAP)
0	20244.00	20244.00	-	0%	-
1486	13340.00	14079.00	50453.00	6%	258%
5000	5973.50	8084.00	12911.75	35%	60%
10000	2271.00	4685.00	5916.64	106%	26%
15000	1207.00	3108.00	3605.56	157%	16%
20000	776.50	2215.00	2452.91	185%	11%
25000	545.00	1651.00	1761.75	203%	7%
30000	399.25	1252.00	1301.38	214%	4%
35000	296.75	960.00	972.79	224%	1%
40000	223.00	746.00	727.17	235%	-3%
45000	168.00	569.00	537.98	239%	-5%
50000	123.50	422.00	389.08	242%	-8%
55000	87.00	300.00	270.20	245%	-10%
60000	57.25	202.00	174.78	253%	-13%
65000	34.50	125.00	98.94	262%	-21%
70000	16.00	59.00	41.05	269%	-30%
75000	0.50	2.00	0.67	300%	-67%
75122	0.00	0.00	0.00	0%	0%

Table 5. Execution time of large scale test (sec)

OP	Phase1	14145.15
	Phase2	< 10.00
OP*	Phase1	5357.37
	Phase2	< 10.00
FRAP		< 10.00

Table 6. Data set of case study

$\#I$	5000
$\#T(\Sigma_t \#B_t)$	60(656)
RR in a week	once,…5 times
IS	1357, 1500, 1750, 2000, 2250, 2500, 2762 (Sai*)
α	4
β	1

5.2.2 Results

Table 7 summarizes the results of executing the optimization procedure. It was observed that the replenishment frequencies decrease as the IS increases. And they either decrease or the same if RR in a week increases because opportunities for RR increase.

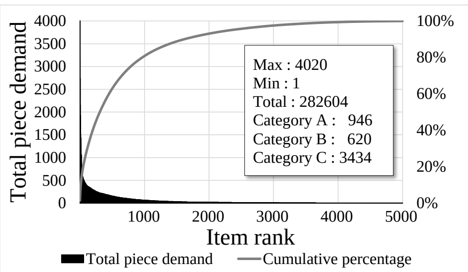

Figure 3. Pareto chart about total piece demand of 5000 SKUs in 60 days

Table 7. Objective function value (replenishment frequency)

IS	Regular replenishment (RR) in a week				
	Once	Twice	3 times	4 times	5 times
1357	49,736	49,736	49,736	49,736	49,736
1500	11,365	9,986	9,624	9,495	9,144
1700	4,103	3,730	3,619	3,599	3,484
1900	1,875	1,756	1,720	1,714	1,667
2100	886	847	827	827	814
2300	386	368	366	366	362
2500	133	127	126	126	125
2700	17	17	16	16	16
2762	0	0	0	0	0

Figure 4 depicts the tradeoff between the total replenishment frequency (TRF) which is defined as the sum of BRF and RRF, and the inventory space when RR in a week is 3 times. If the proposed model is executed with IS below 1357, there is no feasible solution, because this violates the space constraint (2). RRF is zero when IS is equal to 1357, because all order-up-to and reorder levels are 0. As IS increases, BRF decreases, because "α" is bigger than "β" .

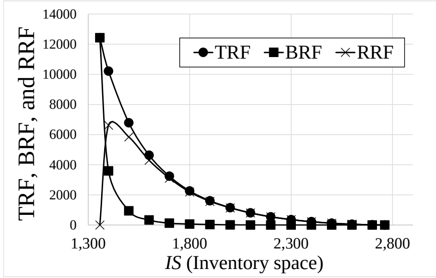

Figure 4. Tradeoff between TRF and inventory space (RR: 3 times in a week)

Table 8 signifies the execution time of optimization procedure. The execution time of Phase1 increases as RR in a week increases because the number of constraints and decision variables increases. The execution time of Phase2 is within 3.0 seconds regardless of IS.

Table 8. Execution time of optimization procedure (sec)

	Regular replenishment (RR) in a week				
	Once	Twice	3 times	4 times	5 times
Phase1	7613.51	11275.24	14959.38	22013.80	32312.11
Phase2	<3.00	<3.00	<3.00	<3.00	<3.00

Table 9 depicts the average order-up-to and reorder level of 5,000 SKUs. Average of order-up-to level Si is larger as IS is larger. On one hand, as IS is larger, average of reorder level si decreases once it increases. The reason is to increase items replenished by RR. The increase of RR is observed at IS from 1,357 to 1,400 in Figure 4. The reason it decreases is to decrease RR. In general, possibility of RR occurring is higher when reorder level si is bigger. Also, as RR in a week increases, the average of order-up-to level almost unchanged, on the other hand, the average of reorder level decreases.

Table 9. Average of $\boldsymbol{S_i}$ and $\boldsymbol{s_i}$ of 5,000 SKUs

IS	Regular replenishment (RR) in a week									
	Once		Twice		3 times		4 times		5 times	
	S_i	s_i	S_i	s_i	S_i	s_i	S_i	s_i	S_i	s_i
1357	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1500	9.33	2.21	9.15	2.38	9.33	2.21	9.36	2.12	9.38	1.94
1700	19.47	1.92	19.47	2.23	19.47	1.92	19.55	1.71	19.57	1.54
1900	30.69	1.32	30.34	1.72	30.69	1.32	30.82	1.13	30.80	1.12
2100	41.13	0.89	41.14	1.26	41.13	0.89	41.16	0.73	41.15	0.71
2300	47.78	0.66	47.47	0.93	47.78	0.66	47.62	0.35	47.63	0.41
2500	52.89	0.42	52.78	0.69	52.89	0.42	53.00	0.12	52.94	0.09
2700	56.22	0.23	56.25	0.53	56.22	0.23	56.22	0.02	56.23	0.02
2762	56.52	0.21	56.52	0.52	56.52	0.21	56.52	0.00	56.52	0.00

6. CONCLUSTIONS

We considered the forward-reserve allocation problem with (s, S) policy. Existing researches focus on forward reserve allocation problem which allocates the storage space among a given set of SKUs. This study assumes two kinds of replenishment BR, RR, the difference between replenishing unit and shipping unit, and (s, S) policy. These assumptions are not considered in the previous papers. In these assumptions, we proposed a model for determining order-up-to level S and reorder level s of each item in the limited space. Our experiments showed the tradeoff between total replenishment frequencies and inventory space. Experimental results will assist managers to judge how much inventory space should be allocated to each SKU in the forward area in the limited space.

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