Dynamic Booking Management for Shared Private Parking System

Shuo-Yan Chou
Department of Industrial Management
National Taiwan University of Science & Technology (Taiwan Tech), Taipei, Taiwan
Tel: (+886) 2 2737-6327, Email: sychou@mail.ntust.edu.tw

Anindhita Dewabharata
Department of Industrial Management
National Taiwan University of Science & Technology (Taiwan Tech), Taipei, Taiwan
Tel: (+886) 2 2737-6327, Email: dewabharata@gmail.com

Debby Rahmi Isnaeni
Department of Industrial Management
National Taiwan University of Science & Technology (Taiwan Tech), Taipei, Taiwan
Tel: (+886) 2 2737-6327, Email:debbyrahmii@gmail.com

Zaenal Arifin
Department of Industrial Management
National Taiwan University of Science & Technology (Taiwan Tech), Taipei, Taiwan
Tel: (+886) 2 2737-6327, Email:arifin.laneaz@gmail.com

Abstract. Following the trend of urbanization, the size of cities is growing continuously. Public and private transportation vehicles are subsequently increasing rapidly, resulting in traffic congestion, traffic accidents, and environmental pollution. An insufficiency of public parking spaces also causes inefficient utilization of parking. Much system planning has been devoted to public parking, mainly on helping drivers to locate and access available parking efficiently. This paper presents a proposed system for shared private parking based on a technology framework employing the Internet of Things. The system is first established so that sensing, identification, and mobile communication technologies are integrated to resolve the key issues hindering the sharing of private parking spaces. Then, an optimization engine for parking assignments and an operation management system are developed for managing free short-term private parking spaces. A dynamic booking management algorithm is proposed by using opportunity cost based on simulation to maximize revenue and space utilization. Simulation is particularly used to model a stochastic case which tracks the random arrival and different time span of each booking request. This dynamic approach shows that this methodology can improve utilization by more than 2% for 30 spaces.

Keyword: shared private parking, utilization optimization, parking reservations, order acceptance policy, revenue management

1. INTRODUCTION

Shared mobility is one of the alternative transportation trends [1] in urban areas. It is being studied with the aim of reducing congestion[2], [3], pollution[4], and resource dependency. The basic idea of shared mobility is to make it possible for individuals to share vehicles or transportation facilities when they have similar destinations in common. There are already quite a few concepts on sharing mobility which succeeded in many countries, including park sharing, bike sharing[5]–[7], car sharing [1], [5], [8]–[10] (i.e Zipcar, Car2go, Communauto, Uber and so on) or carpooling [11], [12](i.e one-way carsharing, peer-to-peer car sharing). Increases in car sharing mobility contribute to the need for more parking spaces[13]–[15].

The main problem for expanding car sharing mobility is that there is a lack of parking slots available in public areas. This problem has been accommodated through private parking sharing [16][17][18] in some countries, using applications like Parkcirca, Parkshare, and Justpark. The most common problem related to park sharing was how to develop some methods for assigning booking requests to available parking spaces. Some methods have been developed to increase utilization of parking spaces [19] by using a hybrid genetic algorithm search procedure to optimize better assignments. Another research study used mathematical formulation to optimize parking slot assignment based on scheduling and time windows [20]. The proposed algorithm in this paper [21] helps to maximize the utilization of space resources of a city and reduce unnecessary energy consumption and CO2 emission of wandering cars since it is designed to control the utilization of parking facility efficiently and reduce traffic congestion due to searches for parking spaces This research [22] also purposes an Average-reward Reinforcement Learning for Order Acceptance (ARLOA) algorithm to maximize the average revenue while quickly responding to unknown variations in order arrival rates and attributes.

The study of optimal revenue management applied to carparks whose primary objective is to maximize profit has been developed by some researchers [23][24][25]. This research developed a stochastic discrete time model [24] and proposed a rejection algorithm that makes optimal decisions (accept or reject) according to future expected revenues generated and according to the opportunity cost that arises before each sale. In this study, a Monte Carlo approach is used to derive optimal rejection policies. Some studies focus on booking reservation systems that can be used to control capacity and to maximize revenue. Based on the previous research [26] which uses OC, an acceptance policy was created which can increase utilization and expected revenue. However, the existing research did not apply optimal booking acceptance algorithms to shared private parking systems.

In this paper, we propose a model to address the possibility of using a dynamic acceptance algorithm for resource revenue management. Through this study, we want to show that by using a dynamic booking management algorithm it will be possible to maximize expected revenue and to optimally allocate the order for parking. Starting with a first come first serve policy, we combine a heuristic algorithm and approximate dynamic programming ideas to improve the expected revenue and utilization rate. Preliminary simulation experiments suggest that this approach is computationally feasible for single resource networks because the computationally demanding part of the algorithm can be carried out successfully. The simplified demand model used in this study and the relatively small number of test problems employed limit the extent of our conclusions, but the potential revenue gains warrant more extensive testing.

2. MODEL FORMULATION

The previous research note that the method has not yet reached the optimal value of utilization[26]. Ideally, for deterministic case maximum utilization of a parking capacity tends to one hundred percent, it means that entire span of time is filled with all possible reservation of orders received. In stochastic case, it is less probability. First come first serve method is fast enough to respond to incoming requests but does not guarantee the maximum utilization. Again, the OC method quite well in increasing the utilization but a less significant improvement. Based on these conditions, it is necessary an improvement method that able to increase resource utilization without compromising the computing time for a decision.

This problem shows that when a booking request arrives, the management system must decide whether to accept or reject it immediately. If a low-value booking request was accepted earlier, potentially high-value future booking requests will be rejected due to possible conflicting booking requirements for the same time slot. Figure 1 shows that in a time step there is a number of booking request[27]. We have to decide which one gives the maximum revenue if it is accepted. Figure 1 shows that the orders in a time step may greater than one. Every request has own time length to park with certain arrival and departure.

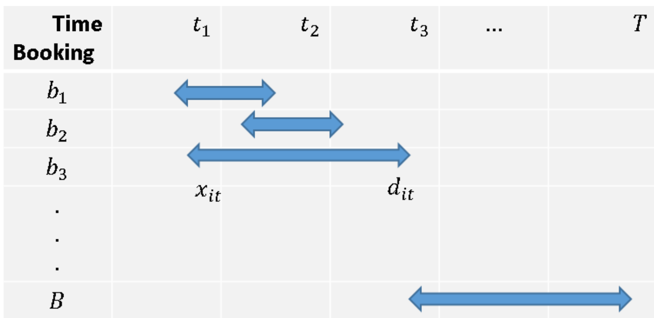

Figure 1 Order request each time horizon

We consider a parking space with same dimension and quality. The set of booking is denoted by B. The quantity b_i specifies the number of park space for order *I, i=1, 2, 3, …n*. When $b_i = 0$, all space for order i are empty, they are booked when $b_i = 1$. In this model, time is discrete by an hour time step. At the start of our study at $t = 0$, the park space is empty. The park management faces dynamic demand which can be described below. The booking come with features order time o_i, arrival time a_i, and departure time b_i. A demand request for each booking $i \in I$ is request of time slot booking i arrival, $l \in L$ request length time of each i booking where $l = b_i - a_i$.

Demand arrived following independent stochastic Poisson processes in discrete time. The instantaneous arrival rate of demand time t is λ_t. The distributions of these parameters were assumpted independent and not affected by control policies. We introduce set of parking spaces S, where $s_j = 1$ means the parking spaces is used, otherwise 0.

Price for renting space was considered homogenous or constant for each time step. The price is denoted by p, with basic price constant in time span of T. We do not make a dynamic pricing yet base on assumption. When a demand arrival at the parking slot, the management need to accept or reject of the demand based on expected revenue. Then allocation strategy is a function of $x_{ijt}(b_i, s_{jt}) = \{1,0\}$, $x_{ijt} = 1$ means the booking i is accepted, 0 is otherwise. The function x_{ijt} denote the space dedicated to booking b_i. If $x_{ijt} = 0$, booking b_i is rejected. We do not allow overbooking so that the management needs to make sure that the following space availability condition is satisfied.

For the deterministic problem, we can formulate it as a binary integer linear programming. Let there are three indexes for describe decision variable. Let x_{ijt} is booking of car i assigned to space j at period of time t. x_{ijt} = 1, if it is assigned, 0 otherwise. We also introduce y_{ijt} as decision variable describe state the car i leave space j at time t. $y_{ijt} = 0$ if the car is leaving, 0 otherwise.

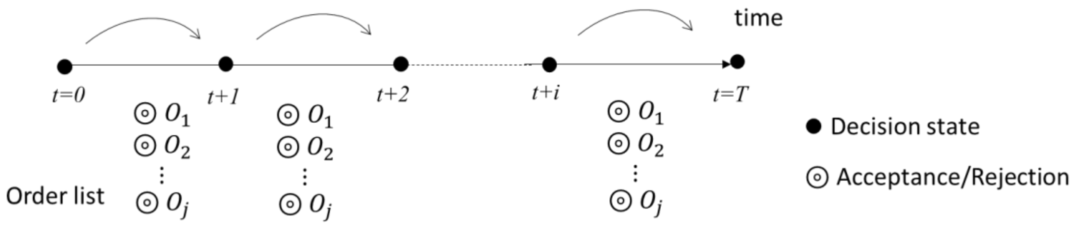

Figure 2 Order list on the time horizon

Index

i Index of number of cars ($\boldsymbol{i} = 1, \dots, \boldsymbol{I}$)
j Index of number of parking space ($\boldsymbol{i} = 1, \dots, \boldsymbol{J}$)
t Discrete time steps ($\boldsymbol{t} = t_1, t_2, \dots, \boldsymbol{T}$)

Parameters

T Planning horizon
b_i Set of booking of each i booking arrival
a_i Start time of each i booking
d_i End time of each i booking
o_i Order time of each i booking
p Basic fees per period of time
m Lower bound of available time in T
n Upper bound of available time in T
$p(a_i, d_i)$ Booking price corresponding to length of booking time i

Decision variables

$R_t(b_i, s_{jt})$ Expected Revenue of current booking in t
$y_{ijt}(b_i)$ Leaving state function of booking car i of space j at time t
$x_{ijt}(b_i, s_{jt})$ Assignment function of booking car i to space j at time t

The order acceptance policy by the first come first serve show that the system will accept a booking base on time order. In this purposed method, we try to develop acceptance policy by predicting value based on simulation that would notate as x_{it} is presented thus

$$x_{it} = \begin{cases} 1\,, if\ B_i\ are\ accepted \\ 0, \quad B_i\ are\ rejected \end{cases}$$

The objective function of this problem is to get the maximum expected revenue based on stochastic demand. Suppose that we have single resource space for a parking lot. The objective function is to maximize $V_t(m,n) = Max\ \{E[\sum_{i=1}^{I} x_{it}\, p_t]\}$ subject to $\sum_{i=1}^{I} x_{it} \le 1$ with $m, n \in [0, T]$. Where price function $p(l_i)$ is a set of price calculated by $p_i = p\ l_i$, $l_i(a_i, d_i\,) = d_i - a_i$.

Based on the single resource above we can generate the objective function for multiple resources of shared private parking spaces. Let J is an index of resources spaces, we can generate the model for multiple resources of the parking lot. This function subjects to the limited number of space in the parking area. Then we can write the problem as:

Maximize

$$R_T = Max \left\{ E \left[\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} x_{ijt}\, p_t \right] \right\}$$

Subject to:

Assignment constraint: each car i is assigned to parking lot j at period t, therefore the remaining space become zero.

$$\sum_{t=1}^{T} x_{ijt} = 1\,, for\ all\ car\ i\ and\ time\ t$$

Parking lot capacity constraint. Suppose that for shared private parking space for each customer only has one single space so the number of cars assigned to parking lot j at time t should be less or equal than one. This constraint also applied to all time period t. for example, at time t=1 assumed there is no car parked:

$$\sum_{i=1}^{I} x_{ij1} \le 1\,, for\ all\ car\ i$$

For time period t, the number of new car entering to parking lot j must be less of equal to one minus the number of spaces occupied plus the number available due to cars that have left at period t:

$$\sum_{i=1}^{I} x_{ij1} \le 1 - \sum_{i=1}^{I} \sum_{t=1}^{t-1} x_{ijt} + \sum_{i=1}^{I} \sum_{t=1}^{t} y_{ijt}\,, \forall\, i \in I, t\ \in T$$

To ensure that a car i assigned to parking lot j at period t is same as car i that leaves the parking lot j at a later time period l.

$$x_{ijt} = y_{ijl}\,, l > t, \forall\, l, t\ \in T$$

$$x_{ijt}, y_{ijt}\ \in \{0,1\}$$

The objective function above maximizes revenue for entire spaces, time horizon, and booking order. The set of constraint is subject to spaces capacity of parking spaces. The decision variable belongs to a binary number.

3. METHOD

Based on the literature review we know that there are methods used to solve parking management problem. For parking space assignment most commonly method is linear programming with the objective function is to maximize utilization of parking space. In shared private parking system, the number of customer and space provider are uncertainty. It means that the arrivals of cars to the parking spaces are dynamic (stochastic) instead of deterministic. We propose a stochastic mathematical modeling and simulation approach to solving the problem in parking reservation systems. We simulate the model by generating data instead of using real data. The simulation is used to model dynamic booking arrivals in a real time. An optimization approach is used to decide whether or not the system will accept a booking according to order acceptance policy.

First, we model the system in a mathematical formulation. We made assumptions to describe the problem. Based on these assumptions we build an objective function and constraints representative of the problem. The objective function of this model is to maximize expected revenues generated by the total number of accepted bookings which are assigned to the number of spaces during a horizon planning time. This function is subject to the limited number of space in a private parking area. Before we simulate the model, we have to obtain the data based on predefined variable and parameter. We use generated distributed random number for the simulation.

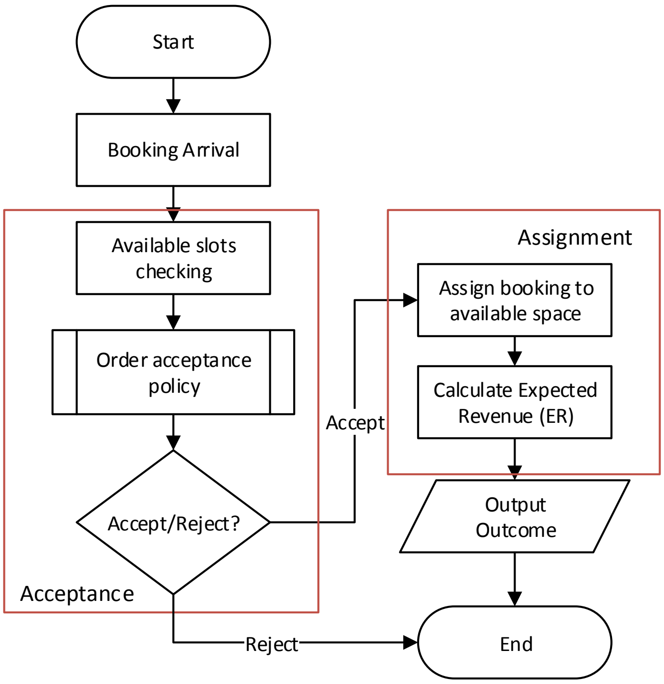

Figure 3 Dynamic Order Acceptance System Framework

Figure 3 shows the procedure used to determine how to make decisions (accepting or rejecting) to order the parking space. We simulate to model a real-time dynamic booking arrivals and resource availabilities. The availabilities of states are corresponded by state constraint for long time horizon which always need to be updated by time. This problem is computationally challenging because it creates large-scale and complex stochastic optimization (dynamic programming problem).

The next step is the study about how we conduct the simulation, starting from the initialization until collecting the results. The first come first serve (FCFS) policy is the simplest and the most widely used booking policy, but it cannot manage to book optimally. By this policy, the system will accept the earlier order when the resource is still available without any constraint considerations. For the booking assignment, the system will check further available space that matches to book and checks the gap between the available time of space and the occupied.

In this study, we will conduct opportunity cost policy to improve the utilization. Opportunity cost (OC) policy considers an opportunity cost of losing more valuable booking request in the future. In this simulation, we will first be learned from FCFS policy and improve the utilization by OC policy. The acceptance rules for OC policy: a). Request will be accepted if the booking request (time arrival a_i and time departure d_i) is match with available time of spaces from m_C to n_C, b). Request will be accept if the expected revenue (ER) of accepted booking more than rejected one ($ER_{accept(B_i)} > ER_{reject(B_i)}$), and c). Consider remaining time and space of system state. For the assignment rules***: a).*** System will assign the request in available spaces, b). If there are more than 1 spaces available, system need to compare both of the expected future revenue $ER_{accept(B_i,C_1)}$ and $ER_{accept(B_i,C_2)}$ and choose the maximum one or $ER_{reject(B_i,C_1)}$ and $ER_{reject(B_i,C_2)}$ and choose the minimum one.

The following assumptions and parameters are used in our simulation.

- No cancellations or no-show: if a booking for a particular duration is accepted, it is assumed that customer will come and pay.
- No overbooking.
- Parking reservations can be made up to 2 days in advance.
- The reservation system is open only for certain hours of weekdays.
- Reservations may be made on a half-hour basis.
- Reservation fees must be paid in advance to guarantee the reservation.
- Reserved spaces will be held for only 15 min.

Based on the assumption above we use these parameters for the simulation. Time horizon T=30 period of time, available booking time 24 period of time, max. booking time 4 period of time, max. advance booking 6 periods of time, and inter-arrival time λ=0.5;1;2. In the simulation, we compare those two methods for acceptance policy (FCFS and OC). We do a simulation by integrating these methods and compare the result. Then, we analyze the result by determining the improvement of the methods and how good solution be obtained. We also run the simulation for certain iteration to test the performance.

The last part of this simulation is an assignment of accepted booking to the available space. When the booking accepted, the system would update the state of available spaces. When there is only an available space, the system will assign the booking to space directly. Otherwise, when there are many available spaces, the system will compare them and choose the greatest expected values of the revenue. The simulation was done using MATLAB 2013.0 and a personal computer with specification Intel I7 3.8 GHz Processor, 8 GB DDRAM, Windows 10-64 Bit.

4. RESULT

Based on the simulation approach we get the expected value of any future booking order. This order may arrive later by the FCFS order acceptance and OC order acceptance. The experiment was done to compare the revenue and utilization of parking spaces based on the OC acceptance policy with the FCFS acceptance policy. Figure 4 shows the comparison of expected revenue a different number of spaces. Based the experiment we compared two acceptance policy to show the performance. As you can see that based on the simulation this two acceptance policy are quite similar in performing. The value of expected revenue has a little bit different.

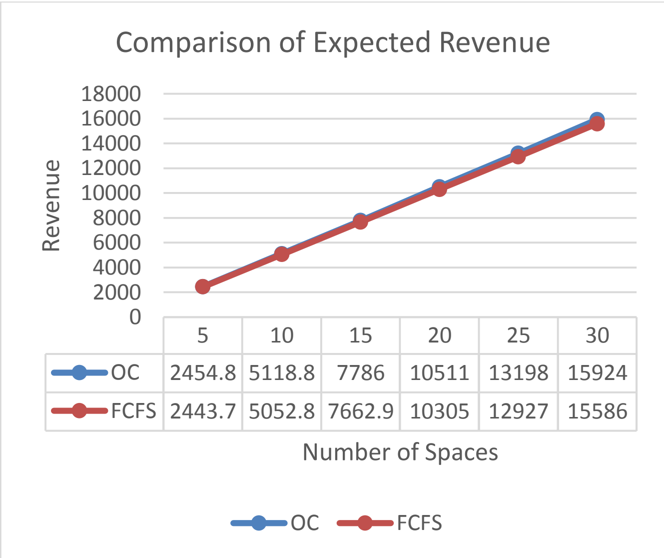

Figure 4 Comparison of expected revenue

Based on figure 4 we conclude that the OC based acceptance policy show better utilization than FCFS based. From the simulation result, we got a 2.13% maximum improvement that was done from 30 spaces case. Figure 5 shows the comparison of utilization using FSFS and OC based policy that improved the utilization using similar sets of the parameter. It is can be concluded that OC better than FSFS based policy.

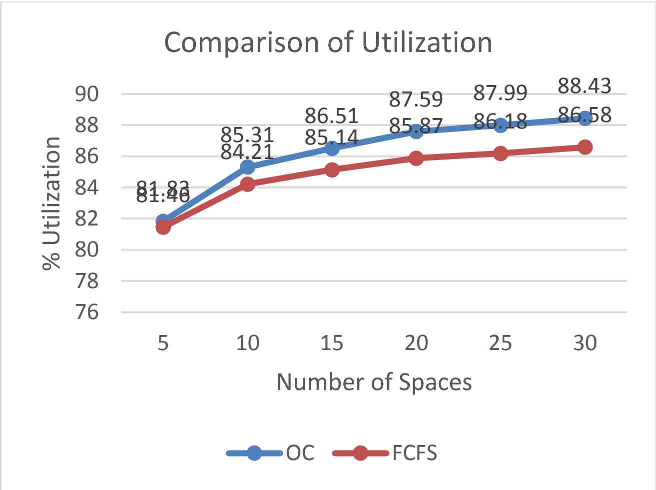

Figure 5 Comparison of utilization of spaces

This study shows that the proposed method will improve spaces utilization and expected revenue of shared private parking system. By allowing the customer to book in advance and consider the OC of order acceptance, the system could improve 2,13% of utilization for 30 spaces. By implement this order acceptance policy we also can improve the utilization of 30 spaces case until 88,43% utilized and having 2,13% improvement compared with FCFS acceptance policy.

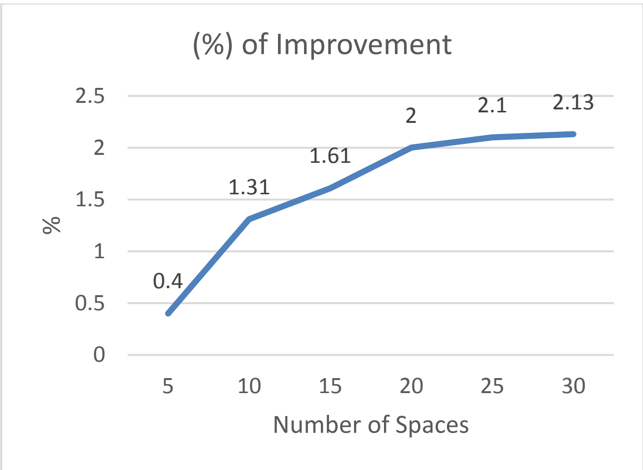

Figure 6 Improvement of utilization of spaces

Since the simulation using random data generation may create various results then the performance of the model should be measured. One of the performance indicators of the simulation model is measuring the consistency of results. Because one of the purposes of developing this systematic approach of order acceptance is to be implemented in any cases with a various number of spaces, then we chose the number of spaces equals to 30 for further analysis. Table 1 shows the consistency of simulation result for 30 spaces case in 15 runs.

Table 1 Comparison Result for several runs

Runs	Utilization (%)		Improvement (%)
	OC	FCFS	
1	88.2	86.46	1.97%
2	88.35	86.56	2.02%
3	88.31	86.48	2.07%
4	88.5	86.52	2.24%
5	88.55	86.63	2.16%
6	88.59	86.58	2.27%
7	88.31	86.56	1.98%
8	88.43	86.54	2.13%
9	88.34	86.47	2.11%
10	88.49	86.62	2.12%
11	88.5	86.59	2.15%
12	88.61	86.56	2.32%
13	88.42	86.76	1.88%
14	88.42	86.76	1.88%
15	88.39	86.61	2.02%
Mean	88.43	86.58	2.09%
Standard Deviation	0.012	0.09	0.13

By the t-statistical test to do the hypothesis test based on 15 runs in 30 spaces case, 95% levels of significance, the hypotheses are: $H0 = \mu_1 = \mu_2$, $H1 = \mu_1 \neq \mu_2$.

Based on the P value on Figure 7, the result of hypotheses test is P value<0.05, then reject the null hypothesis and accept the alternative hypothesis. This means there is a very small probability of this result occurring by chance, under the null hypothesis of no difference. The null hypothesis is rejected, since $p < 0.05$ (in fact $p = 6{,}649 \times 10^{-26}$). We also can conclude that this model is consistent because the value of standard deviation of revenue and utilization is quite small compare with the mean value.

t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	88.42733333	86.58
Variance	0.013120952	0.008057143
Observations	15	15
Hypothesized Mean Difference	0.2	
df	26	
t Stat	43.84132461	
P(T<=t) one-tail	3.32487E-26	
t Critical one-tail	1.70561792	
P(T<=t) two-tail	6.64974E-26	
t Critical two-tail	2.055529439	

Figure 7 T-statistical test

5. DISCUSSION

This study shows the design of dynamic booking management by determining order acceptance policy. The challenging part of this problem is to determine OC in advance. Because unpredictable of OC of an order acceptance, it is hard to expect the future value that may happen in the future to decide whether the system would accept or reject the arrival booking in advance. FCFS simulation result is used to approach the OC matrix which can be an optimal prediction to consider the OC of acceptance earlier booking than another booking that may arrive later. By the advantage of generated historical data simulation, we can model the real problem without using real parking systems that may have high risk.

Although this approach has many advantages, there are several limitations that may require being considered for further. One of the limitations is assuming there is no cancellation and overbooking which is assumed in ideal conditions, but it actually does not represent the real problem, because in fact there would be some number of cancellations booking per day and overbooking if the demand is really high and the management wants to get higher profit. Since this study based on simulation approach, computational time of running the simulation is also become the main issue to explore many result and conclusions related to model development.

6. REFERENCES

[1] R. Katzev, "Car Sharing: A New Approach to Urban Transportation Problems," *Anal. Soc. Issues Public Policy*, vol. 3, no. 1, pp. 65–86, 2003.

[2] M. E. Ben-Akiva, S. Gao, Z. Wei, and Y. Wen, "A dynamic traffic assignment model for highly congested urban networks," *Transp. Res. Part C Emerg. Technol.*, vol. 24, pp. 62–82, Oct. 2012.

[3] H. Yang, W. Liu, X. Wang, and X. Zhang, "On the morning commute problem with bottleneck congestion and parking space constraints," *Transp. Res. Part B Methodol.*, vol. 58, pp. 106–118, 2013.

[4] B. Boyacı, K. G. Zografos, and N. Geroliminis, "An optimization framework for the development of efficient one-way car-sharing systems," *Eur. J. Oper. Res.*, vol. 240, no. 3, pp. 718–733, Feb. 2015.

[5] M. Kaspi, T. Raviv, and M. Tzur, "Parking reservation policies in one-way vehicle sharing systems," *Transp. Res. Part B Methodol.*, vol. 62, pp. 35–50, Apr. 2014.

[6] D. Chemla, F. Meunier, and R. Wolfler Calvo, "Bike sharing systems: Solving the static rebalancing problem," *Discret. Optim.*, vol. 10, no. 2, pp. 120–146, May 2013.

[7] P. Vogel, T. Greiser, and D. C. Mattfeld, "Understanding Bike-Sharing Systems using Data Mining: Exploring Activity Patterns," *Procedia - Soc. Behav. Sci.*, vol. 20, pp. 514–523, 2011.

[8] N. T. Fellows and D. E. Pitfield, "An economic and operational evaluation of urban car-sharing," *Transp. Res. Part D Transp. Environ.*, vol. 5, no. 1, pp. 1–10, Jan. 2000.

[9] P. Baptista, S. Melo, and C. Rolim, "Energy, Environmental and Mobility Impacts of Car-sharing Systems. Empirical Results from Lisbon, Portugal," *Procedia - Soc. Behav. Sci.*, vol. 111, pp. 28–37, Feb. 2014.

[10] P. Bonsall, "Car Sharing in the United Kingdom," *Transport Economics and Policy*. 1981.

[11] R. Wolfler Calvo, F. de Luigi, P. Haastrup, and V. Maniezzo, "A distributed geographic information system for the daily car pooling problem," *Comput. Oper. Res.*, vol. 31, no. 13, pp. 2263–2278, Nov. 2004.

[12] A. Lee and M. Savelsbergh, "Dynamic ridesharing: Is there a role for dedicated drivers?," *Transp. Res. Part B Methodol.*, vol. 81, pp. 483–497, Nov. 2015.

[13] F. Nakamura, “Role of connected mobility concept for twenty-first-century cities—Trial approach for conceptualization of connected mobility through case studies,” *IATSS Res.*, vol. 38, no. 1, pp. 52–57, Jul. 2014.

[14] M. M. Berenger Vianna, L. da S. Portugal, and R. Balassiano, “Intelligent transportation systems and parking management: implementation potential in a Brazilian city,” *Cities*, vol. 21, no. 2, pp. 137–148, Apr. 2004.

[15] D. Mackowski, Y. Bai, and Y. Ouyang, “Parking space management via dynamic performance-based pricing,” *Transp. Res. Part C Emerg. Technol.*, vol. 59, pp. 66–91, 2015.

[16] A. Lee and A. March, “Recognising the economic role of bikes: sharing parking in Lygon Street, Carlton,” *Australian Planner*, vol. 47, no. 2. pp. 85–93, 2010.

[17] F. Leurent and H. Boujnah, “Traffic Equilibrium in a Network Model of Parking and Route Choice, with Search Circuits and Cruising Flows,” *Procedia - Soc. Behav. Sci.*, vol. 54, pp. 808–821, 2012.

[18] S. Parking, S. Parking, T. Approaches, and S. Parking, “8 . Shared Parking,” pp. 1–6.

[19] S. Abidi, S. Krichen, E. Alba, and J. M. Molina, “A New Heuristic for Solving the Parking Assignment Problem,” *Procedia Comput. Sci.*, vol. 60, pp. 312–321, 2015.

[20] M. Roca-Riu, E. Fernández, and M. Estrada, “Parking slot assignment for urban distribution: Models and formulations,” *Omega*, vol. 57, pp. 157–175, May 2015.

[21] J.-H. Shin and H.-B. Jun, “A study on smart parking guidance algorithm,” *Transp. Res. Part C Emerg. Technol.*, vol. 44, pp. 299–317, Jul. 2014.

[22] F. Arredondo and E. Martinez, “Learning and adaptation of a policy for dynamic order acceptance in make-to-order manufacturing,” *Comput. Ind. Eng.*, vol. 58, no. 1, pp. 70–83, 2010.

[23] J.-H. Shin and H.-B. Jun, “A study on smart parking guidance algorithm,” *Transp. Res. Part C Emerg. Technol.*, vol. 44, pp. 299–317, 2014.

[24] A. Papayiannis, P. Johnson, D. Yumashev, S. Howell, N. Proudlove, P. Duck, “Continuous-Time Revenue Management in Carparks,” no. February, 2012.

[25] D. Mackowski, Y. Bai, and Y. Ouyang, “Parking Space Management via Dynamic Performance-based Pricing,” *Transp. Res. Procedia*, vol. 7, pp. 170–191, 2015.

[26] D. R. Isnaeni and S. Chou, “Developing Acceptance Policies for a Stochastic Single-Resource Revenue Management Problem,” 2015.

[27] M. Roca-Riu, E. Fernández, and M. Estrada, “Parking slot assignment for urban distribution: Models and formulations,” *Omega*, vol. 57, Part B, pp. 157–175, 2015.