Nash Marginal Abatement Cost Estimation of Air Pollution Emissions by Stochastic Semi-Nonparametric Frontier in the Coal-Fired Power Industry

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Abstract. Emissions trading (or cap and trade) is a market-based approach used to control the emissions by providing economic incentives for achieving reductions in the emissions of pollutants. Marginal abatement costs, also termed shadow prices of air pollution emissions, provide valuable guidelines to support environmental regulatory policies for CO_2, SO_2 and NO_x, the key contributors to climate change, smog, and acid rain in China. This study estimates the marginal abatement cost of undesirable outputs with respect to Nash equilibrium on the stochastic semi-nonparametric frontier (i.e., stochastic semi-nonparametric envelopment of data, StoNED) in oligopolistic market. Considering an endogenous price function of electricity, the mixed complementarity problem (MiCP) is formulated to identify the Nash equilibrium in a production possibility set. We apply the proposed method to a case study of China coal-fired power plants in 2013 and conclude that StoNED provides a robust frontier not sensitive to the outlier and estimating the shadow prices corresponding to the Nash equilibrium is validated in non-perfectly competitive market.

Keywords: marginal abatement costs, emissions trading, Nash equilibrium, stochastic semi-nonparametric frontier, coal-fired power plant

1. INTRODUCTION

Electricity generation by burning coal usually produces several byproduct pollutants such as carbon dioxide (CO2), sulfur dioxide (SO2) and nitrogen oxide (NOx), and these pollutants cause of greenhouse effect, acid rain and smog problem. To address problem, the Greenhouse Development Rights (GDRs) was developed and formed a framework which show how the costs of rapid climate change can be shared fairly among all countries (Berk and den Elzen, 2001). Based on scientific and political understandings of GDR, it contributed to the foundation of the United Nations Framework Convention on Climate Change (UNFCCC) in 1992. UNFCCC was an international negotiation to stabilize greenhouse gas concentrations and prevent dangerous anthropogenic climate change since 1992. It has 198 parties until Jan. 2013 and the parties to the convention have met annual emission target from 1995 in Conferences of the Parties (COP) to assess progress in dealing with climate change. In 1997, the Kyoto Protocol was adopted to link UNFCCC and established legally binding obligations for developed countries to reduce their greenhouse gas emissions. In this protocol each party was regulated under the emission limitation or reduction commitment, e.g. US agreed to reduce 7% emission of the base year 1990, by 2012. However, Kyoto Protocol seemed to be a failure since the developed nations cannot meet the targets that they should reduce emissions by 5.2% below 1990 levels by 2012 in Annex 1 (King et al., 2011). To rescue the Kyoto Protocol, parties to the Kyoto Protocol have agreed to take on new carbon-cutting targets and run a second commitment period of emissions reductions from 2013 to 2020 at the 2012 Doha climate change conference.

Since climate change is a real concern of the international community. Lots of economists have discussed market approaches to environmental externalities including emissions taxes and permit trading systems. In general, the overall volume of greenhouse gases that can be emitted by the power plants and factories is limited by a 'cap' on the number of emission allowances. Each allowance gives the holder the right to emit one tonne of CO2, the main greenhouse gas or the equivalent amount of pollutants. That is, if emissions reduction fails to meet the targets, parties may participate in a permit trading market. To avoid the emission reduction causing the damage of national economy, the marginal abatement costs (MAC) or shadow prices of pollutant are investigated to represent the costs reducing one extra unit of pollutant. The emission trading mechanism is based on the seminal study regarding

Coase theorem (Coase, 1960). Cap and trade is a market-based approach used to control the emissions by providing economic incentives for achieving reductions in the emissions of pollutants. He argued that without transaction cost the bargaining will lead to an efficient outcome regardless of the initial allocation of property rights if trade in an externality. Following his work, Dales (1968) proposed the conceptual model of emission trading market with respect to pollutant. Given the MAC curve and marginal damages curve, the market will achieve the equilibrium in the presence of externalities. The emission trading builds up the market incentive and achieves cost effectiveness, that is, the benefits obtained from trading between parties will be larger than the benefit generated by emission reduction individually (Montgomery, 1972).

China is one of the CO2 major emitters. In 2012 China was the largest contributor to carbon emissions from fossil fuel burning and cement production, and responsible for 25 percent of global carbon emissions. In particular, manufacturing and power generation are the major sectors contributing to China's carbon emissions, together these sectors accounted for 85 percent of China's total carbon emissions in 2012 (Liu, 2015). In addition, in 2013-2015, China also struggled from the hazardous smog with the high concentration of PM 2.5, and its particulate matter that is small enough to lodge deep into the lungs and enter the bloodstream, causing respiratory infections, cardiovascular disease, asthma, lung cancer, etc. A vast area from Xian in central China to Harbin in the northeast would also be badly hit. In the worst case, Liaoning province reach 1,400 micrograms per cubic metre, which is 56 times than the maximum safe level defined by the World Health Organization (WHO), on Nov. 8, 2015. In fact, the two key components of urban smog and acid rain are emissions of SO2 and NOx (Zhang and Samet, 2015). Therefore, China claimed starting in 2009 to decrease its CO2 emission per unit of GDP (i.e., carbon intensity) by 40-45% by 2020 with 2005 as the reference year. Then, the government of China has set the target to reduce carbon intensity by 17% till 2015 compared to that in 2010. This paper takes the North region and the Northeast region in China as a case for estimation MACs of CO2, SO2, and NOx emissions in the coal-fired power plants.

To support emission trading policies the MACs of pollutants are used as a reference value to the allowance price in the emission trading market (Lee et al., 2002). Many previous studies have estimated the shadow prices of pollutants or undesirable outputs. Parametric and nonparametric approaches were developed to estimate shadow prices. In fact, shadow price is a differentiable characteristic of production function. Parametric method is commonly used because specified production function is differentiable everywhere. Färe et al. (1993) employed an output distance function with the translog functional form and used linear programming to solve for the combination of parameters that yields the best-fit distance function to estimate a shadow price of four pollutants generated by pulp and paper mills in Michigan and Wisconsin of 1976. Coggins and Swinton (1996) took the same approach to estimate the SO2 shadow price of Wisconsin coal-burning uiility plants in 1990–1992, and the result shows that the average shadow price was above some observed allowance prices in the trading market. Färe et al. (2005) used a quadratic directional output distance function to estimate both technical efficiency and a shadow price of SO2 for the U.S. electric utilities in 1993 and 1997. The result showed that the emission trade became difficult since the shadow price of SO2 increased from 1993 to 1997. They also measured the output elasticity of substitution between electricity and SO2 and found that inefficiency reduction (i.e., productivity improvement) benefited the emissions reduction. Marklund and Samakovlis (2007) evaluated the Burden-Sharing Agreement (BSA) which redistributes the emission reduction target in European Union (EU) under 8% reduction of 6 greenhouse gases by during 2008-1012 described in Kyoto Protocol, and used directional output distance function to estimate MAC based on the production data of EU member states for 1990–2000.

Alternatively, a nonparametric approach (e.g., data envelopment analysis, DEA) can estimate production function without specified functional form. A directional distance function is commonly used to investigate the shadow price on efficient frontier. Boyd et al. (1996) used DEA to estimate efficient frontier and the MAC of SO2 for 29 coal-burning utilities in U.S. electric power industry. The result showed some positive shadow prices against our intuition, and they claimed that this was because of the lack of uniformity in environmental regulations in 1989. Lee et al. (2002) showed that the shadow price of a pollutant is the product of the inefficiency correction factor and the slope to the frontier, where the inefficiency correction factor is the inefficiency ratio between undesirable output and desirable output, which maps an inefficient point to the corresponding point on the production frontier. They accounted for the technical inefficiency to derive the shadow prices of SO2, NOx and total suspend particulates (TSP) for Korean coal- and oil-burning power plants during the periods 1990–1995. However, DEA estimator is sensitive to outliers and shadow price values equal to zero are common; in particular, DEA does not consider statistical noise and assumes the observations are assessed without error. To address this issue, Kuosmanen (2008) proposed convex nonparametric least squares (CNLS) approach which incorporates the merits of parametric and nonparametric approaches. Kuosmanen and Johnson (2010) have showed that DEA is a special case of CNLS with sign constraints on error terms. Mekaroonreung and Johnson (2012) used CNLS approach in two ways- with random noise and without random noise, to estimate the shadow prices of SO2 and NOx in US bituminous coal power plant boilers. The result showed that both SO2 and NOx shadow price estimated by CNLS taking account of noise term are in the reasonable ranges and comparable to the allowance market prices.

The present study considers the oligopolistic market with endogenous price and proposes estimating the MAC of the Nash equilibrium benchmark (i.e., Nash-MAC hereafter) on the stochastic semi-nonparametric envelopment of data (StoNED) frontier to release some issues in the existing literatures.

2. Nash Equilibrium Identified on DEA Frontier HEADING

This section identifies the Nash equilibrium solution in production possibility set (PPS) estimated by the DEA with inputs, desirable outputs and undesirable outputs. Now consider a multiple-input and multiple-output production process. Let $x \in \mathbb{R}_+^{|I|}$ denote a vector of input variables, $y \in \mathbb{R}_+^{|J|}$ denote a vector of desirable output variables and $b \in \mathbb{R}_+^{|Q|}$ denote a vector of undesirable output variables for a production system. The PPS T is defined as $T = \{(x, y, b): x \text{ can produce } (y, b)\}$, i.e., $(x, y, b) \in T$. Assume T is a convex set. Let index $i \in I$ represent the input, index $j \in J$ the desirable output, index $q \in Q$ the undesirable output, and index $k \in K$ be the set of decision making unit (DMU) or firm index. Observations X_{ik} represent the ith input level, Y_{jk} the jth desirable output level, and B_{qk} the qth undesirable output level of firm k. In this study, we limit our model to single desirable output in power industry, i.e., electricity generation.

In the power markets, to build a price function of desirable output, we consider an inverse demand function as $P^Y(\hat{Y}) := P^{Y_0} - \kappa\hat{Y}$ where $P^Y(\cdot) \geq 0$, $\hat{Y} = \sum_k y_k$, P^{Y_0} is a positive intercept and $\kappa \geq 0$ indicates the price sensitive coefficient of desirable output. Apparently, the revenue function $P^Y(\hat{Y})y_r$ is concave. For the undesirable output of the power market, i.e., environmental externalities, cap is used to control the emissions. Cap means a legal limit on the quantity of a certain type of emission an economy can emit in a period. We limit the feasible region of the undesirable quantity $\hat{B}_q \leq \hat{B}_q^{CAP}$, where $\hat{B}_q = \sum_k b_{qk}$ and $\hat{B}_q^{CAP}$ is a constant representing the limit of total emissions. For the input of the power market, i.e., coal consumption, we assume a competitive input market and the price of input is a constant, P_i^X.

Recall that the DEA estimator assumes that the desirable outputs are freely disposable (Fried et al., 2008); however, this property cannot be directly applied to undesirable outputs. Intuitively, we can reduce the level of the desirable output which in turn will result in a proportionate reduction of the undesirable outputs. In other words, the free (or strong) disposability assumption ignores the possibility to decrease undesirable outputs by downsizing the activity level, i.e., a proportional contraction of desirable outputs and undesirable outputs is feasible simultaneously. This property is termed "weak disposability" (Shephard, 1974). The following axioms are restated regarding production when undesirable outputs are also produced:

Free (or strong) disposability of inputs and desirable outputs

Given $(x, y, b) \in T$, if $x' \geq x$ and $0 \leq y' \leq y$, then $(x', y', b) \in T$.

Weak disposability of desirable outputs and undesirable outputs

Given $(x, y, b) \in T$, if $0 \leq \rho \leq 1$, then $(x, \rho y, \rho b) \in T$.

To formulate the weak disposability, we introduce the Kuosmanen's convex technology with undesirable outputs which follows the convexity axiom and builds the minimal weakly disposable technology (Kuosmanen and Podinovski, 2009; Lee, 2015). Let λ_k be the decision variable representing the intensity weights of the convex combination between firms, μ_k the decision variable for weak disposability property of Kuosmanen's convex technology. The Kuosmanen's convex technology T can be estimated as follows.

$$\tilde{T} = \left\{ (x, y, b) \;\middle|\; \begin{array}{c} \sum_{k \in K}(\lambda_k + \mu_k)X_{ik} \leq x_i, \forall i \in I; \\ \sum_{k \in K}\lambda_k Y_k \geq y; \\ \sum_{k \in K}\lambda_k B_{qk} = b_q, \forall q \in Q; \\ \sum_{k \in K}(\lambda_k + \mu_k) = 1; \\ \lambda_k, \mu_k \geq 0, \forall k \in K \end{array} \right\} \tag{1}$$

The first, second and third constraint refer to input constraint, desirable output constraint, and undesirable output constraint, respectively. Fourth constraint presents convex combination constraint and the last one is the nonnegativity constraint.

Given index r representing one specific firm and an alias of index k, let x_{ir}, y_r, and b_{qr} represent the decision variables for input i, single desirable output, and undesirable output q of one specific firm r. Let p^Y be the decision variable representing the clearing price of single desirable output. M is a large positive constant. The cap constraint $\hat{B}_q \leq \hat{B}_q^{CAP}$ of undesirable output is added into the model. We define our maximization model of Nash profit function (NPF) with respect to each firm restricted by DEA frontier as equation (2):

$$NPF_r^{DEA*} = \max_{y_r, b_{qr}, x_{ir}} (P^{Y_0} - \kappa\hat{Y})y_r - \sum_{i \in I} P_i^X x_{ir}$$

$$\begin{aligned} \text{s.t. } & \sum_{k \in K}(\lambda_{kr} + \mu_{kr})X_{ik} \leq x_{ir}, \forall i \in I; \\ & \sum_{k \in K}\lambda_{kr} Y_k \geq y_r; \\ & \sum_{k \in K}\lambda_{kr} B_{qk} = b_{qr}, \forall q \\ & \sum_{k \in K}(\lambda_{kr} + \mu_{kr}) = 1; \\ & \hat{B}_q \leq \hat{B}_q^{CAP}, \forall q \in Q; \\ & x_{ir}, y_r, b_{qr}, \lambda_{kr}, \mu_{kr} \geq 0, \forall i \in I, q \in Q, k \in K \end{aligned} \tag{2}$$

For the PPS estimated by DEA frontier with input and desirable output case, the Nash solution exists and is unique if profit function is strictly concave (Lee and Johnson, 2015). We extend to undesirable output case, the NPF $(P^{Y_0} - \kappa\widehat{Y})y_r - \sum_{i\in I} P_i^X x_{ir}$ is strictly concave on $(x_{ir}, y_r, b_{qr}) \in \widetilde{T}$ enveloped by DEA frontier and it verifies the existence and uniqueness of Nash solution. Note that the profit function only considers the revenue of desirable output and cost of inputs since the price (i.e., MAC) of undesirable output is unknown variable we would like to estimate. That is, model (2) implies that firm's productive behavior depends on physical profit maximization and pollutant emission is just a requirement rather than an objective function. This assumption conforms with the practice.

Lemma 2.1: Consider an imperfectly competitive market with $|K|$ firms, an inverse demand function $P^Y(\cdot)$ that is strictly decreasing and continuously differentiable in y_k, and an inverse supply function $P^X(\cdot)$ that is strictly increasing (or a constant in our case) and continuously differentiable in x_k. The profit function $\pi_k(x_k, y_k)$ is concave and the variables $x_k, y_k \geq 0$, then $(x^*, y^*) = \big((x_1^*, y_1^*), (x_2^*, y_2^*), \dots, (x_{|K|}^*, y_{|K|}^*)\big)$ is a Nash equilibrium solution if and only if
$\nabla_{x_k}\pi_k(x^*, y^*) \leq 0$ and $\nabla_{y_k}\pi_k(x^*, y^*) \leq 0, \forall k$;
$x_k^*[\nabla_{x_k}\pi_k(x^*, y^*)] = 0$ and $y_k^*[\nabla_{y_k}\pi_k(x^*, y^*)] = 0, \forall k$,
where $(x_k^*, y_k^*, b_k^*) \in \widetilde{T}$ and $\widetilde{T}$ is estimated by the constraints of model (2).

Proof: ignore here due to page limit.

To solve for a Nash equilibrium associated with equation (2), the MiCP is built, where $\mu1_{ir}, \mu2_r, \mu3_{qr}, \mu4_r$ and $\mu5_q$ are Lagrange multipliers corresponding to each constraint (except nonnegativity constraint) in model (2).

Then, using the first-order conditions, the MiCP is:

$$
\begin{aligned}
&0 \leq x_{ir} \perp -P_i^X + \mu1_{ir} \leq 0, \quad \forall i, r\\
&0 \leq y_r \perp P^{Y_0} - \kappa\widehat{Y} - \kappa y_r - \mu2_r \leq 0, \quad \forall r\\
&0 \leq b_{qr} \perp \mu3_{qr} - \mu5_q \leq 0, \quad \forall q, r\\
&0 \leq \lambda_{kr} \perp -\textstyle\sum_i \mu1_{ir}X_{ik} + \mu2_r Y_k - \sum_q \mu3_{qr}B_{qk} - \mu4_r \leq 0, \quad \forall k, r\\
&0 \leq \mu_{kr} \perp -\textstyle\sum_i \mu1_{ir}X_{ik} - \mu4_r \leq 0, \quad \forall k, r\\
&0 \leq \mu1_{ir} \perp \textstyle\sum_{k\in K}(\lambda_{kr} + \mu_{kr})X_{ik} - x_{ir} \leq 0, \quad \forall i, r \qquad (3)\\
&0 \leq \mu2_r \perp y_r - \textstyle\sum_{k\in K}\lambda_{kr}Y_k \leq 0, \quad \forall r\\
&\textstyle\sum_{k\in K}\lambda_{kr}B_{qk} - b_{qr} = 0, \ (\mu3_{qr} \text{ unrestricted}), \quad \forall q, r\\
&\textstyle\sum_{k\in K}(\lambda_{kr} + \mu_{kr}) - 1 = 0, \ (\mu4_r \text{ unrestricted}), \quad \forall r\\
&0 \leq \mu5_q \perp \widehat{B}_q - \widehat{B}_q^{CAP} \leq 0, \quad \forall q
\end{aligned}
$$

The Nash equilibrium solution generated from the proposed MiCP, i.e., model (3), exists since the Nash profit function is concave for the maximization problem and PPS forms a convex set. Furthermore, we show the connection between desirable output and undesirable output based on the proposed MiCP. Therefore, we formulate Theorem 2.1 and Corollary 2.1.

Theorem 2.1: The proposed MiCP (3) generates Nash equilibrium solution $(x_{ir}, y_r, b_{qr}) \in \widetilde{T}$, where $\widetilde{T}$ is Kuosmanen technology.

Proof: ignore here due to page limit.

Corollary 2.1: The Nash solution generated by MiCP(3) must be on the frontier representing the weak disposability describing the relationship between desirable output and undesirable outputs.

Proof: ignore here due to page limit.

Corollary 2.2: The Nash solution generated by MiCP(3) suggests the same efficient benchmark to all the firms.

Proof: ignore here due to page limit.

3. Nash Marginal Abatement Costs of Pollutants

This section introduces the Nash marginal abatement cost (Nash-MAC) for jointly estimating the shadow prices of multiple pollutants with respect to Nash equilibrium benchmark.

We estimate the shadow prices of production technology with desirable outputs (i.e., electricity) and undesirable outputs (i.e., pollutants) by the profit function, which is the profit maximization problem (Lee et al., 2002; Mekaroonreung and Johnson, 2012). Model (4) defines the profit maximization problem:

$$
\begin{aligned}
&\pi(p_y, p_b, p_x) = \max_{y,b,x} p_y y - p_b' b - p_x' x\\
&\text{s.t.} \quad F(x, y, b) = 0 \qquad (4)
\end{aligned}
$$

where p_y, $p_b = (p_{b_1}, \dots, p_{b_{|Q|}})$, and $p_x = (p_{x_1}, \dots, p_{x_{|I|}})$ represent the price vectors of single desirable output, multiple undesirable outputs and multiple inputs. $F(x, y, b)$ is the transformation function corresponding to a multi-output production technology. Because we want to estimate the shadow prices of undesirable outputs, we impose the constraint $F(x, y, b) = 0$ so that only the frontier of the production possibility set is considered. Let τ be a Lagrange multiplier of the constraint. We use the method of Lagrange multipliers to transform the above production to the following Lagrange function:

$$
\max_{y,b,x} \ p_y y - p_b' b - p_x' x + \tau F(x, y, b) \qquad (5)
$$

To solve the Lagrange function, we apply the first-order conditions (FOCs):

$$p_{y_j} + \tau\frac{\partial F(x,y,b)}{\partial y_j} = 0$$
$$-p_{b_q} + \tau\frac{\partial F(x,y,b)}{\partial b_q} = 0 \quad (6)$$
$$-p_{x_i} + \tau\frac{\partial F(x,y,b)}{\partial x_i} = 0$$
$$F(x,y,b) = 0$$

We derive the shadow price of a pollutant from the FOCs, i.e. equations (6), and write them as:

$$p_{b_q} = p_y\left(\frac{\partial F(x,y,b)}{\partial b_q} / \frac{\partial F(x,y,b)}{\partial y}\right) = p_{y_j}\left(\frac{\partial y}{\partial x_i} / \frac{\partial b_q}{\partial x_i}\right) \quad (7)$$

In equation (7), $\frac{\partial y}{\partial x_i}$ and $\frac{\partial b_q}{\partial x_i}$ are indeed the marginal productivity of electricity and marginal productivity of pollutants by increasing one extra unit of one specific input, respectively. Thus, we can use the price of electricity and the ratio $\left(\frac{\partial y}{\partial x_i} / \frac{\partial b_q}{\partial x_i}\right)$ to estimate the MAC of pollutants.

Lee and Zhou (2015) claim that estimating the shadow prices of each pollutant separately may lead to an underestimation, and thus propose the directional shadow prices (DSPs) (i.e., directional MACs) of multiple undesirable outputs given a pre-determined direction. Let $g_x \in \Re_+^{|I|}$, $g_y \in \Re_+$, and $g_b \in \Re_+^{|Q|}$ be the pre-determined directions of inputs, single desirable output and undesirable outputs, respectively. Define $g_y + \sum_{q\in Q} g_{b_q} = 1$ for unit simplex. Let v_i, u, w_q and u_0 be the decision variables representing the dual multipliers of the input constraint, the desirable output constraint, the undesirable output constraint, and the convex-combination constraint of formulation (1). Let $X_i^{max} = \max\{X_{ik}\}$, $Y^{max} = \max\{Y_k\}$ and $B_q^{max} = \max\{B_{qk}\}$ to eliminate the unit of factors. Thus, model (8) estimates the directional marginal productivity (DMP) (Lee, 2014) of desirable output and undesirable outputs by increasing one extra unit of input i^* with respect to Nash solution (x_i^*, y^*, b_q^*) generated from model (3), as follows.

$$\begin{aligned}
&\text{Min } v_{i^*} \\
&\text{s.t. } \sum_{i\in I} v_i \frac{x_i^*}{X_i^{max}} - u\frac{y^*}{Y^{max}} + \sum_{q\in Q} w_q \frac{b_q^*}{B_q^{max}} + u_0 = 0 \\
&\quad \sum_{i\in I} v_i \frac{X_{ik}}{X_i^{max}} - u\frac{Y_k}{Y^{max}} + \sum_{q\in Q} w_q \frac{B_{qk}}{B_q^{max}} + u_0 \geq 0, \ \forall k \\
&\quad \sum_{i\in I} v_i \frac{X_{ik}}{X_i^{max}} + u_0 \geq 0, \ \forall k \qquad (8) \\
&\quad u g_y + \sum_{q\in Q} w_q g_{b_q} = 1 \\
&\quad v_i, u \geq 0, \ w_q, \ u_0 \text{ are free}
\end{aligned}$$

Therefore, given direction $g_y = 1$ and $\sum_{q\in Q} g_{b_q} = 0$, the DMP of electricity $\frac{\partial y}{\partial x_i}$ is calculated by $\frac{g_y Y^{max} v_{i^*}}{X_i^{max}}$; while given direction $g_y = 0$ and $\sum_{q\in Q} g_{b_q} = 1$, the DMP of pollutant $\frac{\partial b_q}{\partial x_i}$ is $\frac{g_{b_q} B_q^{max} v_{i^*}}{X_i^{max}}$, $\forall q$. In particular, the direction vector (g_y, g_{b_q}) can be regarded as the "weightings" between investigated outputs. The higher the weight the closer the direction of the DMP to the output with the higher weight. Note that, it is invalid to estimate the DMP if the assigned direction towards the portion of free disposability about the inputs (Lee and Zhou, 2015).

Based on the Nash solution on DEA frontier, the DMP generated by model (8) can be used for estimating the Nash-MAC by plugging into the equation (7). To compare with the Nash-MAC generated by a deterministic production frontier (i.e., DEA), next section introduces the Nash-MAC estimated by a stochastic production frontier (i.e., StoNED).

4. Nash Equilibrium Identified on StoNED Frontier

We have known that DEA is a mathematical programming technique to estimate the production function and does not assume any particular functional form for the frontier or the distribution of inefficiency. However, without take the noise into account, the main shortcoming of DEA is that it attributes all deviations from the frontier to inefficiency. To address the issue, one technique called convex nonparametric least squares (CNLS) was developed (Kuosmanen and Johnson, 2010) and then have led to integrate the noise term into the DEA framework, referring to stochastic semi-nonparametric envelopment of data (StoNED) (Kuosmanen and Kortelainen, 2012).

StoNED is a semi-parametric regression technique that considers the noise and inefficiency, and does not specify the functional form a priori while maintaining the standard regularity conditions from microeconomic theory for production functions, namely continuity, monotonicity, and concavity. Consider a regression function, $y = f(x) + \varepsilon$, with shape restrictions that is estimated via CNLS, where y is the dependent variable, x is a vector of independent (explanatory) variables, and ε is a composite error equal to noise v plus asymmetric inefficiency u, i.e., $\varepsilon = v - u$; in particular, the zero-mean assumption is violated, i.e., $E(\varepsilon) = E(v - u) = -E(u) < 0$. The regression function $f(\cdot)$ is assumed to satisfy the monotonicity and concavity. If the inefficiency term u has a constant variance (i.e., inefficiency term u is homoscedastic), then the expected value of the inefficiency term u is a constant, denoted as μ. The CNLS provides a consistent estimator of the frontier f minus a constant (Kuosmanen and Kortelainen, 2012).

CNLS is formulated as below.

$$\begin{aligned}
&\min_{\alpha,\beta,\gamma,\varepsilon} \quad \sum_{k\in K} \varepsilon_k^2 \\
&\text{s.t.} \\
&Y_k = \alpha_k + \sum_{i\in I} \beta_{ik} X_{ik} + \sum_{q\in Q} \gamma_{qk} B_{qk} + \varepsilon_k \quad \forall \ k \in K \\
&\alpha_k + \sum_{i\in I} \beta_{ik} X_{ik} + \sum_{q\in Q} \gamma_{qk} B_{qk} \leq \alpha_h + \sum_{i\in I} \beta_{ih} X_{ik} + \\
&\sum_{q\in Q} \gamma_{qh} B_{qk} \quad \forall \ k, h \in K \text{ and } k \neq h \qquad (9) \\
&\alpha_k + \sum_{i\in I} \beta_{ik} X_{ih} \geq 0 \quad \forall \ k, h \in K \text{ and } k \neq h \\
&\beta_{ik}, \gamma_{qk} \geq 0 \quad \forall \ k \in K, i \in I, q \in Q
\end{aligned}$$

where ε_k is the composite error that represents the deviation of firm k from the estimated function. Decision variables α_k, β_{ik} and γ_{qk} characterize the intercept and slope parameters regarding marginal products of inputs and undesirable outputs for each observation. The objective function minimizes the sum of the square with respect to the disturbance terms. First equality constraints represent a basic linear regression formulation. Second inequality constraints impose concavity using Afriat's inequalities (Afriat, 1972). The third inequality constraints impose the weak disposability between desirable and undesirable outputs. The last constraints impose monotonicity of both inputs and the costs associated with additional undesirable outputs on the underlying unknown function.

Given the CNLS residual $\hat{\varepsilon}_k$ obtained from (9), the expected value of inefficiency, $\mu = E(u_k)$, can be estimated by the method of moments (Aigner et al., 1977) and then the CNLS frontier (i.e., the lower bounds of concave envelope) is shifted upwards by adding the expected inefficiency to estimate the StoNED frontier. The method of moments requires some additional parametric distributional assumptions, and this study assumes half-normal inefficiency $u_k \sim N^+(0, \sigma_u^2)$ and normal noise $v_k \sim N(0, \sigma_v^2)$. The second central moment of the residual distribution as $\widehat{M}_2 = \sum_{k=1}^{|K|}(\hat{\varepsilon}_k)^2/(|K|-1)$. The third central moment of the residual distribution as $\widehat{M}_3 = \sum_{k=1}^{|K|}(\hat{\varepsilon}_k)^3/(|K|-1)$. The hat on top of the third central moment indicates the true estimator but unknown values of the central moments. In fact, based on half-normal inefficiency assumption, the third central moment is equal to $M_3 = \left(\sqrt{\frac{2}{\pi}}\right)\left(1-\frac{4}{\pi}\right)\sigma_u^3$. Thus, given the estimated $\widehat{M}_3$, we can estimate σ_u as $\hat{\sigma}_u = \sqrt[3]{\frac{\widehat{M}_3}{\left(\sqrt{\frac{2}{\pi}}\right)\left(1-\frac{4}{\pi}\right)}}$. Finally, the expected value of inefficiency can be estimated as $\hat{\mu} = \hat{\sigma}_u\sqrt{\frac{2}{\pi}}$, and StoNED frontier is the CNLS frontier obtained from model (9) plus estimated expected inefficiency $\hat{\mu}$.

Given the optimal coefficients $\hat{\alpha}_k$, $\hat{\beta}_{ik}$ and $\hat{\gamma}_{qk}$ obtained from model (9), now, we can define our maximization model of NPF with respect to StoNED frontier as equation (10). Similar to model (2), the profit function only considers the inputs and the desirable output.

$$
\begin{aligned}
&NPF_r^{StoNED*} = \max_{y_r, b_{qr}, x_{ir}} (P^{Y_0} - \kappa\hat{Y})y_r - \textstyle\sum_{i\in I} P_i^X x_{ir} \\
&\text{s.t. } y_r \le \hat{\alpha}_k + \textstyle\sum_{i\in I}\hat{\beta}_{ik}x_{ir} + \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} + \hat{\mu}, \forall k \in K; \\
&\quad \hat{\alpha}_k + \textstyle\sum_{i\in I}\hat{\beta}_{ik}x_{ir} + \hat{\mu} \ge 0,\ \forall\ k \in K \qquad (10) \\
&\quad \widehat{B}_q \le \widehat{B}_q^{CAP}, \forall q \in Q; \\
&\quad x_{ir}, y_r, b_{qr} \ge 0, \forall i \in I, q \in Q
\end{aligned}
$$

Similar to model (2), the NPF $(P^{Y_0} - \kappa\hat{Y})y_r - P_i^X x_{ir}$ is strictly concave on the convex PPS by StoNED frontier.

Lemma 4.1: Model (10) provides a convex PPS restricted by StoNED frontier.

Proof: ignore here due to page limit.

To solve for a Nash equilibrium associated with equation (10), let $\varphi1_{kr}, \varphi2_{kr}$ and $\varphi3_{qr}$ are Lagrange multipliers corresponding to each constraint (except nonnegativity constraint) in model (10). Based on the first-order conditions, the MiCP is:

$$
\begin{aligned}
&0 \le x_{ir} \perp -P_i^X + \textstyle\sum_k \varphi1_{kr}\hat{\beta}_{ik} + \sum_k \varphi2_{kr}\hat{\beta}_{ik} \le 0, \qquad \forall i, r \\
&0 \le y_r \perp P^{Y_0} - \kappa\hat{Y} - \kappa y_r - \textstyle\sum_k \varphi1_{kr} \le 0, \qquad \forall r \\
&0 \le b_{qr} \perp \textstyle\sum_k \varphi1_{kr}\hat{\gamma}_{qk} - \varphi3_q \le 0, \qquad \forall q, r \qquad (11) \\
&0 \le \varphi1_{kr} \perp y_r - \hat{\alpha}_k - \textstyle\sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \sum_{q\in Q}\hat{\gamma}_{qk}b_{qr} - \hat{\mu} \le 0, \qquad \forall k, r \\
&0 \le \varphi2_{kr} \perp -\hat{\alpha}_k - \textstyle\sum_{i\in I}\hat{\beta}_{ik}x_{ir} - \hat{\mu} \le 0, \qquad \forall k, r \\
&0 \le \varphi3_q \perp \widehat{B}_q - \widehat{B}_q^{CAP} \le 0, \qquad \forall q
\end{aligned}
$$

The Nash equilibrium solution generated from the proposed MiCP, i.e., model (11), exists since the Nash profit function is concave for the maximization problem and PPS forms a convex set.

Theorem 4.1: The proposed MiCP (11) generates a Nash equilibrium solution $(x_{ir}^*, y_r^*, b_{qr}^*) \in \tilde{T}$, where $\tilde{T}$ is PPS estimated by StoNED frontier model (10).

Proof: ignore here due to page limit.

Corollary 4.1: Based on the MiCP (11), the larger the κ value, the lower desirable output and undesirable output generated and close to zero.

Proof: ignore here due to page limit.

Then, we can estimate DSP by a variant of model (8).

5. Empirical Study

We conduct an empirical study to estimate the Nash-MAC of CO_2, SO_2 and NO_x in China plant-level coal-fired power plants operating in 2013. We focus on the North region and the Northeast region in China. According to the Natural Resources Defense Council in China, sixty percent of PM2.5 (airborne particles with a diameter of less than 2.5 microns) is directly generated from the coal burning and PM2.5 shows a high concentration in North region and the Northeast region (Yang, 2014). In fact, these two regions show significant abatement potential for emission reduction and have a greater influence on the national goal (Wei et al., 2012). In particular, North region includes province Beijing, Tianjin, Hebei, Shanxi, Shandong, and Inner Mongolia while Northeast region includes Liaoning, Jilin, and Heilongjiang. In 2013, the State Council issued the Air Pollution Prevention and Control Action Plan in

September to control PM2.5 and reduce the number of smoggy days. The 10 point plan includes limits to pollutant emissions, optimization of energy use and upgrades to technology. Along with the plan, Hebei claimed a reduction of 40 million tonne coal consumption, Beijing claimed 13 million tonne, and Shandong claimed a reduction of 20 million tonne comparing to annual consumption in 2012.

In this study, all the plants we investigated have nameplate capacity larger than one million kilowatts since they show large scale to affect the market price.

5.1 Data Set

Our balance plant-level data set comprises the 33 coal-burning power plants from in 2013 (EIA, 2011). Since nameplate capacity is a fixed asset and does not affect the undesirable outputs directly, we consider coal consumption as single input. One desirable output is the annual amount of coal-fired electricity generation, and the three pollutants are the annual amount of CO2, SO2 and NOx. The data is collected from the China Electric Power Yearbook (CEPP, 2014). The plant-level pollutant emissions are estimated by plant's proportion of coal consumption to the multiplication of the province's total emissions and average emission factor (IPCC, 2013; EEA, 2013). The total emission of the two regions is assigned to the Cap regarding each pollutant.

We estimate a linear price function (i.e., inverse demand function) $P^Y(\hat{Y}) := 4.5 \times 10^7 - 123\hat{Y}$ (unit: CNY$ per 108 kWh) based on the China average on-grid electricity price CNY$427.01 per MWh and total electricity generation 15,103 (108 kWh) in these two regions in 2013. Note that, $\hat{Y} = \sum_k y_k + 10{,}975$ and the constant $10{,}975$ represents the electricity generated by coal-fired plants whose nameplate capacity less than one million kilowatts. The price of coal is CNY$590 per tonne. US Dollar (USD) to Chinese Yuan (CNY) exchange rate is 6.0394.

5.2 Nash MAC Estimation

The section estimates the Nash-MAC of CO2, SO2 and NOx via DEA and StoNED frontier respectively and shows a comparison of previous studies for MAC estimation in electric power sectors. Due to the three simultaneously emitted pollutants, we estimate the Nash-MAC based on the direction vector ($g^{b_{CO_2}}$, $g^{b_{SO_2}}$, $g^{b_{NOx}}$) =(0.048, 0.508, 0.444), which is the literature-based direction vector suggested by Lee and Zhou (2015). In previous studies, the MAC range for CO2 is US$16.1 to $476.3 per tonne; our estimate, $257 per tonne for DEA and $78.5 for StoNED, is inside this range. In previous studies, the MAC range for SO2 is between US$165.3 and $8881.3 per tonne; our estimate, $7157.9 per tonne for DEA and $2185.8 for StoNED, is inside in this range. In previous studies, the MAC range for NOx is between US$450.8 and $40335.5 per tonne; our estimate, $8457.7 per tonne for DEA and $2582.8 for StoNED, is inside this range. Table 1 summarizes the previous studies.

Table 1 Studies for MAC in power sectors

Study	Country	Year	Level	Sample size	Price of electricity (US$/MWh)	Frontier estimation	Direction of inefficient unit projected to frontier*
Atkinson & Dorfman (2003)	U.S.	1980, 1985, 1990, 1995	Firm	43	Not applicable	Translog	$g^Y>0$
Boyd et al. (1996)	U.S.	1989	Plant	62	50.00	DEA	$g^Y>0$, $g^B<0$
Coggins & Swinton (1996)	U.S.	1990 -1992	Plant	42	36.38-65.87	Translog	$g^Y>0$, $g^B>0$
Färe et al. (2005)	U.S.	1993, 1997	Boiler	209	10.39-100.42	Quadratic DDF	$g^Y>0$, $g^B<0$
Gupta (2006)	India	1990 -2000	Plant	76	Not available	Translog	$g^Y>0$, $g^B>0$
Harkness (2006)	U.S.	2000	Plant	518	51.58-52.96	Translog	$g^Y>0$, $g^B>0$
Lee et al. (2002)	Korea	1990 -1995	Plant	43	66.67	DEA	$g^Y<0$, $g^B<0$
Matsushita & Yamane (2012)	Japan	2000 -2009	Firm	76	161.41	Quadratic DDF	$g^Y>0$, $g^B<0$
Mekaroonreung & Johnson (2012)	U.S.	2000 -2008	Boiler	336	17.26-165.70	CNLS	$g^Y>0$
Park & Lim (2009)	Korea	2001 -2004	Plant	80	56.07	Translog	$g^Y>0$, $g^B>0$
Rezek & Campbell (2007)	U.S.	1998	Plant	260	41.6-119	Translog	$g^Y>0$
Turner (1995)	U.S.	1985 -1987	Plant	147	32.98-93.47	DEA	$g^Y>0$
Zhou et al. (2015)	China	2009 -2011	Sector	30	Not applicable	DEA	$g^Y>0$, $g^B<0$
Lee & Zhou (2015)	U.S.	1990 -2010	State	48	33.7-180.6	DEA	Only efficient unit
This Study	China	2013	Plant	33	Endogenous price 71.74	DEA StoNED	Only Nash

Figure 1, 2 and 3 illustrates the comparisons of the MACs of CO2, SO2 and NOx from 1980 to 2015. First, we compare the DEA methods in different studies. Figure 1 shows that the proposed DEA method provides a relatively higher MAC of CO2 US$257 per tonne. Though our estimate is within a limited range of MAC fluctuations with an average of around US$288 suggested by Lee and Zhou (2015), but much higher than US$92.5 suggested by Zhou et al. (2015). The similar result is also shown in Figure 2 of MAC of SO2. The proposed DEA method provides a higher MAC of SO2 US$$7157.9 per tonne, which is more larger than an average MAC US$3492 by Lee and Zhou (2015) and US$1072 by Mekaroonreung & Johnson (2012). Thus, the MAC result of CO2 and SO2 implies that the abatement technology and policy of CO2 and SO2 in coal-fired power industry is urgent in the North and the Northeast regions of China; nevertheless, a higher MAC suggested by the proposed DEA method implies the development of abatement technology on reducing CO2 and SO2 is not affordable. That is, in the short run, the province should tend to purchase emission allowances of CO2 and SO2 from the market if the allowance price is much lower than MAC.

On the other hand, Figure 3 shows the proposed DEA method provides a relatively reasonable MAC of NOx US$ 8458 per tonne within a limited range of MAC fluctuations with an average of around US$7929 suggested by Mekaroonreung & Johnson (2012) and much lower than US$15572 suggested by Lee and Zhou (2015). The MAC result implies that the plant is encouraged to invest the development of the NOx abatement techniques at the present stage. Though the allowance price is lower than MAC, in the long run, when carbon regulation becomes more and more stringent, the MAC and allowance price is likely to rise.

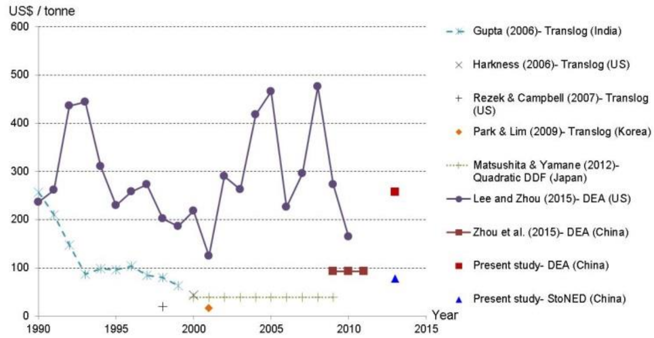

Figure 1 Comparison of the MAC of CO2

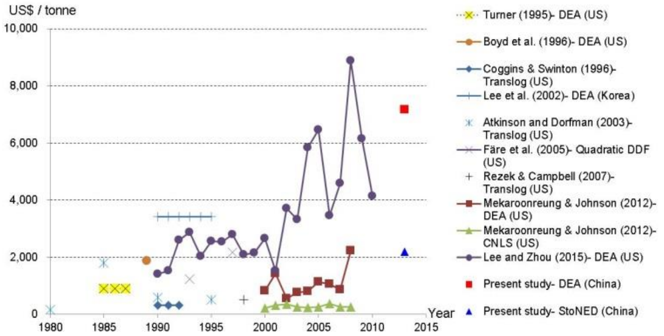

Figure 2 Comparison of the MAC of SO2

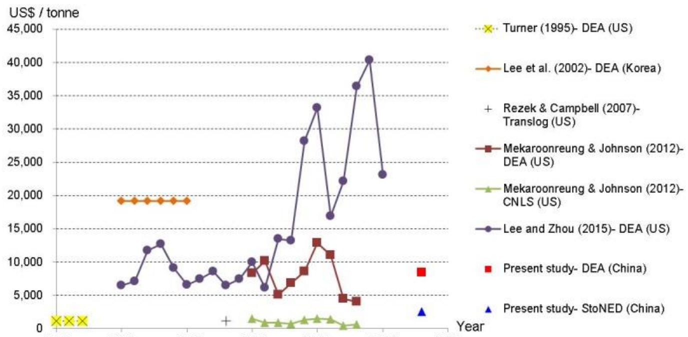

Figure 3 Comparison of the MAC of NOx

In addition, a comparison between the Nash-MAC estimated by different approaches is conducted to investigate whether model choice has a significant impact on the MAC. The choice between nonparametric DEA model and semi-parametric StoNED model would significantly affect the MAC of emission. As a whole, Nash-MAC estimated by StoNED frontier is much lower than that estimated by DEA frontier and close to the emission allowance. The main reason is that StoNED frontier considers the composite error term including noise and inefficiency. StoNED technique can estimate a production function more robust than DEA frontier, and thus reduce the outlier effect where DEA cannot address well. Thus, the present study suggests the StoNED approach for MAC estimation when the collected data (i.e., observations) shows a large (or unknown) variation.

6. Conclusion

Knowing the MAC (or shadow price) of pollutants provides environmental policy guidelines, such as the allowance price in emission trading markets and the penalty rates for pollutant emission. This study provides a new model for Nash-MAC estimation via StoNED frontier.

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