An Optimal Model for Integrating Robust Parameter and Tolerance Designs for Asymmetric Quality Characteristic

Wildan Trusaji, Muhammad Akbar and Dradjad Irianto

Manufacturing Systems Research Group, Bandung Institute of Technology

Jl. Ganesha 10, Bandung 40132, INDONESIA

E-mail: dradjad@mail.ti.itb.ac.id

Abstract. Quality improvement can be done through system design, robust parameter design and robust tolerance design. To perform parameter and robust tolerance design, Taguchi promotes a sequential step approach by (i) performing optimized experimental design involving related parameter to find a set of mean and variance values, and then (ii) setting tolerance based on given mean and variance values. Accordingly, the optimized robust tolerance design is restricted by the result of robust parameter design. This research is aimed at optimizing these sequential steps procedure simultaneously by developing a total cost model involving quality loss with an asymmetric loss function, rework cost, and scrap cost. Instead of using a given value of mean and variance from robust parameter design, this model considers reiterating the optimization process by giving alternative sets of mean and variance in robust tolerance design

Keywords: quality improvement, robust parameter design, robust tolerance design, optimization

1. INTRODUCTION

Along with manufacturing environment changes from mass production to mass customization, nowadays, the off-line method is more often used to ensure the quality improvement (Irianto 1995). Inspired by Taguchi, the off-line method consists of three phase, namely robust system design, robust parameter design (RPD), and robust tolerance design (RTD). Robust system design is the phase where the functional prototypes are developed. RPD is a cost-effective methodology for determining the best settings that make product performance robust and to ensure the uniformity of the products. RTD is the process to determine the tolerance limits that minimize the total cost incurred by both the customer and the manufacturer. (Cho et al. 2000).

Taguchi showed the importance of robust design and systematic study of noise factors and introduction of quadratic loss function (Meng et al. 2010). To perform robust parameter design and tolerance design, Taguchi promotes a sequential steps approach by (i) performing optimized experimental design involving related parameter to find a set of mean and variance values, and then (ii) setting tolerance based on given mean and variance values.

Taguchi's philosophy about robust design is sound, but his tactics have some particular shortcomings (Pignatiello and Ramberg 1991). Some of the statistical methods were proposed to enhance Taguchi's tactic using more statistically and efficient approaches. In an early attempt, Vining and Myers (1990) proposed the dual response approach, based on response surface methodology to tackle RPD problems to achieve the same goals of the Taguchi philosophy within a more rigorous statistical methodology.

However, Del Castillo and Montgomery (1993) showed that the solution technique used by Vining and Myers (1990) does not always guarantee optimal RD solutions, and proposed a standard nonlinear programming techniques, specifically, the generalized reduced gradient (GRG) algorithm and the Nelder-Mead simplex method, and in some cases may provide better robust parameter design solutions.

Del Castillo, Fan, and Semple (1999) developed an effective heuristic for computing global (or near-global) optimal solutions for dual response system which arising in response surface modeling. Tang and Xu (2002) proposed a goal programming approach to optimize a dual response system. Quesada and Del Castillo (2004) proposed an extension to the dual-response approach to robust parameter design for the case of multiple responses and the methodology provides unbiased estimates of the process covariance matrix and of the vector of expected values using parameter estimates from a multivariate regression. Wu and Chyu (2004) developed an approach to optimizing correlated multiple quality characteristics by using a proportion of quality loss reduction and principal component analysis. The goal is minimizing the total average quality loss of experiments.

Accordingly, the optimized tolerance design is restricted by the result of robust parameter design. Thus, this method can't guarantee that its solution will fall into optimum global solution. To find the global optimum solution, RPD and RTD optimization must be integrated solved. Cho, et al. (2000) proposed an integrated RPD-RTD optimization procedure with an objective function to minimize expected total cost. Quality loss, rework cost, and scrap cost are the total cost component. The model proposed by Cho assume that loss function below and above targets are the same or symmetric.

This research is aimed to develop Cho, et al. (2000) works at integrated optimization of RPD and RTD procedure. Instead of using a given value of mean and variance from robust parameter design, this model considers to giving alternative sets of mean and variance in tolerance design. This model also uses more general assumption, that the quality loss function can be asymmetric.

2. INTEGRATED PARAMETER AND TOLERANCE DESIGN

Chan and Xiao (1995) was the first author who researched the integrated robust parameter design and tolerance design. In their model, optimization was done in two phase, first, (i) robust parameter design was optimized by using a signal to noise ratio with a zero-bias solution. Then (ii) optimization was done to tolerance design.

By setting mean μ equal to target τ (zero-bias solution), the phase (i) can't guarantee that loss function will be minimum. As an alternative, Cho, et al. (2000) proposed the new model which in the phase (i) optimization, the solution can be non-zero and its objective function is to minimize loss function, dissimilar to Chan and Xiao (1995) where the phase (i) objective function is to minimize variation. And to find the optimal solution in phase (i), Cho , et al. (2000) use response surface methodology (RSM) that proposed by Lin and Tu (1995) where they proposed the mean-squared error (MSE) model. A new robust design method from an inverse-problem perspective by relaxing the zero-bias assumption.

This study uses non-zero bias solution with minimizing loss function model to find the robust parameter design. To find the optimal parameter, RSM is utilized. And to find the optimal tolerance, this study develops an asymmetric loss function model that developed from Cho's symmetric loss function model. After that, an isocost plot is developed to find a list of alternative mean and variance sets.

3. THE PROPOSED METHOD

Tolerance design that has been developed in this study is asymmetric tolerance model where t_1 and t_2 are lower and upper tolerance limit. In this model, the asymmetric loss function is used so $k_1 \neq k_2$, where k_1 and k_2 are constant for quality characteristic for below and above target, and $C_r \neq C_s$. C_s and C_r are scrapping cost and reworking cost respectively. Continuous normal distribution is used to modeling the distribution of output quality characteristic.

A product that falls below t_1 causes scrapping cost C_S and that falls above or t_2 causes reworking cost C_r. A product that falls in between t_1 and t_2 is delivered to a customer. The delivered product causes loss to the customer as a result of product bias and variance. Thus, expected total cost $E[TC]$ among producer and customer is an addition of loss that occurs, scrapping cost, and reworking cost. As of $E[TC]$ can be formulated as

$$E[TC] = E[L(y)] + P(Y \leq t_1).C_s + P(Y \geq t_2).C_r \quad (1)$$

Where $E[L(y)]$ is expected *quality loss*, $P(Y \leq t_1)$ is a probability that a product fall below tolerance lower limit t_1, $P(Y \geq t_2)$ is a probability that a product fall above tolerance upper limit t_2. Target τ is assumed to fall in between tolerance limit. The amount of product that falls below tolerance lower limit t_1 is $F_y(t_1)$ and that falls above tolerance upper limit t_2 is $[1 - F_y(t_2)]$, where $F_y(.)$ and $f_y(.)$ are cumulative distribution function and density function for the quality characteristic.

Furthermore, optimization for equation (1) can be formulated as a total cost optimization function as

$$Min\ \ E[TC] = \int_{t_1}^{t_2} L(y) f_y dy + F_y(t_1) C_s + [1 - F_y(t_2)] C_r \quad (2)$$

$$\text{Subject to } t_1 < \tau < t_2. \quad (3)$$

With asymmetric loss function, expected loss can be formulated as:

$$\int_{t_1}^{t_2} L(y) f_y dy = k_1 \int_{t_1}^{\tau} (y - \tau)^2 f_y(y) dy + k_2 \int_{\tau}^{t_2} (y - \tau)^2 f_y(y) dy \quad (4)$$

If equation (4) is transformed so that $\mu(y) = \mu(x), \sigma(y) = \sigma(x), z = \frac{y - \mu(x)}{\sigma(x)}, y = z\sigma(x) + \mu(x)$ and $dz = dy$ where $\mu(x)$ and $\sigma(x)$ are obtained from robust parameter design and z is standard normal value from y, then equation (4) become

$$\int_{t_1}^{t_2} L(y) f_y dy = k_1\left[\int_{t_1}^{\tau}\big(z\sigma(x)+\mu(x)\big)^2 f(z)dz - 2\tau\int_{t_1}^{\tau}\big(z\sigma(x)+\mu(x)\big)f(z)dz + \tau^2\int_{t_1}^{\tau} f(z)dz\right] + k_2\left[\int_{\tau}^{t_2}\big(z\sigma(x)+\mu(x)\big)^2 f(z)dz - 2\tau\int_{\tau}^{t_2}\big(z\sigma(x)+\mu(x)\big)f(z)dz + \tau^2\int_{\tau}^{t_2} f(z)dz\right] \quad (5)$$

For normal distribution is known that:

$$\int_{z}^{\infty} f(z)dz = 1 - F(z);\quad \int_{z}^{\infty} z f(z)dz = f(z);\quad \int_{z}^{\infty} z^2 f(z)dz = 1 - F(z) + z f(z) \quad (6)$$

Thus if define that $d_1 = \frac{t_1-\mu(x)}{\sigma(x)}$ and $d_2 = \frac{t_2-\mu(x)}{\sigma(x)}$ and $d = \frac{\tau-\mu(x)}{\sigma(x)}$ then we obtain:

$$E\big(L(y)\big) = E\big(L(X)\big) = k_1\{\sigma^2(x)[F(d) - df(d) - F(d_1) + d_1 f(d_1)] - 2\sigma(x)\big(\tau-\mu(x)\big)[f(d_1) - f(d)] + [\tau^2 + \mu^2(x) - 2\tau\mu(x)][F(d) - F(d_1)]\} + k_2\{\sigma^2(x)[F(d_2) - d_2 f(d_2) - F(d) + df(d)] - 2\sigma(x)\big(\tau-\mu(x)\big)[f(d) - f(d_2)] + [\tau^2 + \mu^2(x) - 2\tau\mu(x)][F(d_2) - F(d)]\} \quad (7)$$

And $F(t_1) = F(d_1);\ F(t_2) = F(d_2).$ (8)

By using equation 7 and 8, equation no 2 become

$$E(TC) = k_1 F(d_1)\left[-\sigma^2(x) - \mu^2(x) - \tau^2 + 2\tau\mu(x) + \frac{C_s}{k_1}\right] + k_1 f(d_1)[\sigma^2(x)d_1 + 2\sigma(x)\mu(x) - 2\tau\mu(x)] + k_1 F(d)[\sigma^2(x) + \tau^2 + \mu^2(x) - 2\tau\mu(x)] + k_1 f(d)[\sigma^2 d - 2\sigma(x)\mu(x) + 2\sigma(x)\tau] + k_2 F(d_2)\left[\sigma^2(x) + \mu^2(x) + \tau^2 - 2\tau\mu(x) - \frac{C_r}{k_2}\right] + C_r + k_2 f(d_2)[-\sigma^2(x)d_2 - 2\sigma(x)\mu(x) + 2\tau\sigma(x)] - k_2 F(d)[\sigma^2(x) + \tau^2 + \mu^2(x) - 2\tau\mu(x)] + k_2 f(d)[\sigma^2 d + 2\sigma(x)\mu(x) - 2\sigma(x)\tau] \quad (9)$$

Optimized value of $E[TC]$ can be obtain by making derivative function of equation (10) partially to d_1, d_2 and d, with sufficient condition $\frac{\partial E(TC)}{\partial d_1} = 0$, and $\frac{\partial E(TC)}{\partial d_2} = 0$ where d_1, d_2, are lower limit and upper limit respectively on normal standard value. The optimum d_1 value can be obtained by this method. If

$$\frac{\partial E(TC)}{\partial d_1} = -k_1 f(d_1)\left[(\sigma(x)(d_1) + \mu(x) - \tau)^2 - \frac{C_s}{k_1}\right] \quad (10)$$

Then optimization can be done by

$$(\sigma(x)(d_1) + \mu(x) - \tau) = \pm\sqrt{\frac{C_s}{k_1}} \quad (11)$$

Considering the constraint at equation 3, optimum value for d1 is

$$d_1 = \frac{-\big(\tau - \mu(x)\big) + \sqrt{\frac{C_s}{k_1}}}{\sigma(x)} \quad (12)$$

Furthermore, with same method, optimum value for d2 can be obtained by

$$\frac{\partial E(TC)}{\partial d_2} = k_2 f(d_2)\left[(\sigma(x)(d_2) + \mu(x) - \tau)^2 - \frac{C_r}{k_2}\right] \quad (13)$$

Then optimization can be done by

$$(\sigma(x)(d_2) + \mu(x) - \tau) = \pm\sqrt{\frac{C_r}{k_2}} \quad (14)$$

Considering the constraint at equation 3, optimum value for d2 is

$$d_2 = \frac{(\tau - \mu(x)) + \sqrt{\frac{C_r}{k_2}}}{\sigma(x)} \quad (15)$$

If calculated from the mean, the lower limit and upper limit tolerance will be:

$$t_1 = \mu(x) + d_1\sigma(x) \quad (16)$$

$$t_2 = \mu(x) + d_2\sigma(x) \quad (17)$$

4. DISCUSSION AND ANALYSIS

To see how the proposed model perform, a case that had been used in Vining and Myers (1990), Del Castillo and Montgomery (1993), and Cho et al. (2000), is used. $C_r = 30, C_s = 100, \tau = 500, k_1 = k_2 = 1$. Table 2 shows the result of the proposed model and from the other model. Although Vining and Myers (1990) and Del Castillo and Montgomery (1993) didn't develop the tolerance design model, Cho's tolerance design model is embedded in their model to see how their model performs integrally. As we can see, the proposed model performs as better as Cho's model when the loss quality characteristic for below and above mean are symmetric, $k_1 = k_2$. And the proposed model has more generality when it comes to solving an asymmetric quality characteristic problem, as in clearance fit or interference fit problem.

From table 2, we can see that when quality characteristic k is higher, then the tolerance will converge to its mean, as shown in $\boldsymbol{d_1}$ or $\boldsymbol{d_2}$ that converging to zero. It means that we have a tighter lower tolerance limit when the quality characteristic below target is higher than above target, as in interference fit problem. Vice versa when a clearance fit problem is faced.

In reality, sometimes we can't achieve desired mean and variance due the production process or supplier limitation. Thus we need, a list of mean-variance sets that can produce same total cost, or isocost, as the desired mean-variance. To generate a list of mean-variance sets, an isocost plot is built from equation 1 with d_1 and d_2 optimal from equation 12 and 15.

From figure 1 we can see that we have a list a mean-variance sets that have a same total cost. This finding helps designer and procurer to find an alternative when the desired mean-variance set can't be achieved due the production process or supplier limitation. Besides that, a zero bias solution is not a necessary condition that must be met, because a same total cost can be achieved when an alternative process has a mean that is slightly off the target but has a smaller variance or more robust

5. CONCLUSION

This research is aimed to develop an integrated optimization of RPD and RTD procedure that can give alternative sets of mean and variance in tolerance design. This model assumes that quality loss function can be asymmetric. A non-zero solution with minimizes loss function model is used to find the robust parameter design with utilizing RSM as in Cho et al. (2000). And to find the optimal tolerance, this study develops an asymmetric loss function model with total cost minimization objective function. When the quality characteristic is asymmetric, the optimal tolerance will differ from a symmetric problem. The higher a quality characteristic is, the more converge the tolerances to its mean. Thus we have a tighter tolerance when the quality characteristic is higher. And to generate alternative sets of mean and variance in tolerance design, an isocost plot is built. The plot gives a list a mean-variance sets that have a same total cost. Thus designer and procurer can find an alternative when the desired mean-variance set can't be achieved. Thus, this plot shows that a zero bias solution is not a necessary condition that must be met to find the optimal solution.

Table 1: Model Result Comparison

	Robust parameter design						Tolerance Design					
	x_1	x_2	x_3	μ	σ	$E(L)$	d_1	d_2	$E(L(y))$	$E(C_r)$	$E(C_S)$	$E(TC)$
The Proposed Model	1.00	0.07	-0.25	494.70	44.46	2005.10	-0.11	0.24	6.93	13.74	40.42	61.09
Cho's Model	1.00	0.07	-0.25	494.70	44.46	2005.10	-0.11	0.24	6.93	13.74	40.42	61.09
VM Model	0.61	0.23	0.10	500.00	51.78	2680.96	-0.19	0.11	5.92	12.70	45.79	64.41
DM Model	1.00	0.12	-0.26	500.00	45.10	2033.74	-0.22	0.12	6.78	12.37	45.17	64.31

*VM model is Vining and Myers (1990) and DM model is Del Castillo and Montgomery (1993) model

Table 2: Asymmetric Quality Characteristic Result Comparison

k_1	k_2	d_1	d_2	$E(L(y))$	$E(C_r)$	$E(C_S)$	$E(TC)$
1.000	**1.000**	-0.106	0.242	6.930	13.739	40.420	61.088
1.500	**1.000**	-0.064	0.242	5.395	14.231	40.420	60.045
2.000	**1.000**	-0.040	0.242	4.296	14.525	40.420	59.241
3.000	**1.000**	-0.011	0.242	2.628	14.874	40.420	57.921
1.000	**1.500**	-0.106	0.220	7.202	13.739	41.298	62.238
1.000	**2.000**	-0.106	0.206	7.544	13.739	41.823	63.106
1.000	**3.000**	-0.106	0.190	8.313	13.739	42.448	64.500

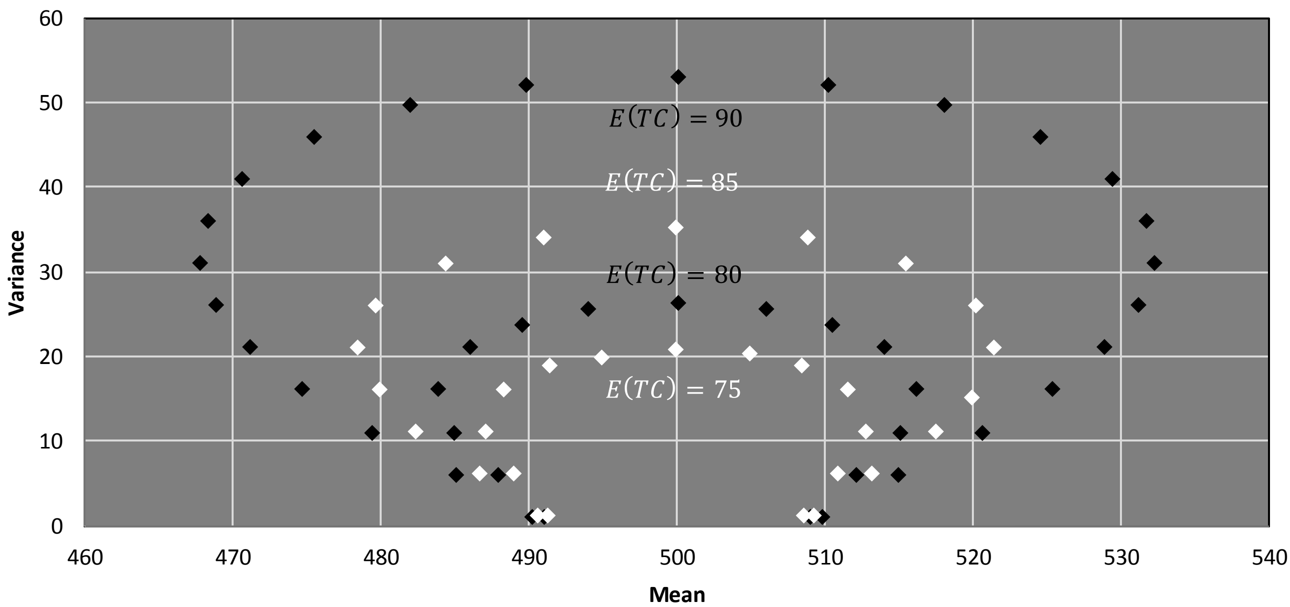

Figure 1: Isocost plot for $E(TC) =$ 75, 80, 85, and 90

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