Optimal Routing Problem for Responders in Emergency Situations

Sungwoo Kim
Department of Industrial Engineering
Seoul National University, Seoul, Korea
Tel: (+82) 2-880-7152, Email: asdf1358@gmail.com

Youngchul Shin
Department of Industrial Engineering
Seoul National University, Seoul, Korea
Tel: (+82) 2-880-7152, Email: chulbee@snu.ac.kr

Seohyeon Choi
Department of Industrial Engineering
Seoul National University, Seoul, Korea
Tel: (+82) 2-880-7152, Email: miuh102@snu.ac.kr

Ilkyeong Moon †
Department of Industrial Engineering
Seoul National University, Seoul, Korea
Tel: (+82) 2-880-7151, Email: ikmoon@snu.ac.kr

Abstract. Events such as the attacks in Paris in 2015 and Afghanistan in 2016 have shown glimpses of calamity caused by terrorism. Therefore, research on evacuation plans for reducing damages and casualties has been conducted to defend against similar threats. However, despite the attention drawn to this research, emergency response management designed to neutralize risks has been frequently undermined. Therefore, we propose the response problem in which unique characteristics of the time period, such as limited resources and time-varying risk components, are considered. An integer programming model is developed to find the optimal routing plan for responders to prevent and eliminate further damage. The result shows that the model is applicable in the real world problems.

Keywords: Integer program, Macroscopic model, Security management, Terrorism

1. INTRODUCTION

Forests of buildings are commonly observed in modern society and the structures have become more complex in architectural design and increasingly capable of holding denser populations. Such congested public areas would experience critical damage from internal attacks or accidents. Due to the structural disadvantages, these areas have become vulnerable targets for terrorism. Recent terrorist attacks, such as those in Paris in 2015, the bombings in Brussels 2016, and the Iskandariya suicide bombing in 2016, share similarities in that the events occurred in populated, public areas. Therefore, establishment of emergency procedures is essential to prevent additional disastrous results of attacks in modern urban areas.

The growth of technology contributes to real-time emergency planning for evacuating buildings, which has been improved over past efforts. Various smart devices are used to share information during emergency situations. Numerous researchers have conducted studies to mitigate risks during emergencies by grafting increased awareness of public safety and technology development. According to Hamacher and Tjandra (2001), the evacuation research models can be divided into microscopic and macroscopic categories. From the microscopic perspective, physical and mental characteristics of involved individuals and their actions as well as knowledge of evacuation are considered as major factors contributing to an attack. However, the microscopic approach does not give an optimal solution for preventing damage. Using social force models, Dong *et al*. (2014) and Hou *et al.* (2014) analyzed effects of decision-making individuals who have information of evacuation. Pelechano and Badler (2006) considered interactions between different types of people. Tan *et al*. (2015) suggested an agent-based model that incorporates the location of fire-fighting facilities and building structures.

The macroscopic model assumes the population to be a homogeneous group. Although mathematical models for decision making have been developed, they are of limited use in the consideration of bottleneck problems. Chen and Miller-Hooks (2008) suggested a mixed integer programming formulation and implemented an exact algorithm based on Benders decomposition. Kang *et al*. (2015) proposed both integer and linear programming models for evacuation. Lin *et al*. (2008) established a multi-stage time-varying quickest flow approach, which assigns a priority value to each occupant group and thus takes into account heterogeneous characteristics.

Previous studies emphasized perspectives of emergency management. However, an effective response to problematic situations is as important as evacuation planning. The Brussels bombings in 2016 serve as a valid example for the importance of efficient response. The incident would have been worse if the first-response agencies failed to detect the unexploded bombs.

We propose an optimal routing problem for responders (ORPR) as a solution. The aim of this study is to provide an appropriate suppression plan during emergency situations. The integer programming model concentrates on the response problems with a dynamic network flow model. The objective is to maximize prevention of risks in a given time period. The remainder of this paper is organized as follows: Section 2 introduces the suppression plan which was formulated as an integer program. Section 3 describes the case study, and Section 4 shows results. Section 5 presents the discussion and Section 6 features conclusions.

2. Mathematical Model

The ORPR is a special type of emergency problems. When an urgent situation occurs, responders have a golden timeframe in which to react with appropriate equipment to treat the situation from the outside of the building. Throughout this critical time, the number of available responders who trained to treat emergency problems is limited, and the response is responsible for deciding optimal positions to place subordinates to mitigate risks in the building.

2.1 Model Description

The model assumes that general information about the building is available when the emergency occurs. As state-of-the-art technologies enhance the efficiency of emergency management, high quality cameras, iBeacons, and other sensors offer real-time information of in the building (Chen and Miller-Hooks, 2008).

The model is based on a macroscopic model. As responders are trained, their physical characteristics are relatively homogeneous. Furthermore, they are expected to follow commands of their leader. Thus, the decision making of a response team leader is important. These conditions comport with the main characteristics of macroscopic models. The proposed model is related to the *dynamic evacuation model* of Hamacher and Tjandra (2001).

Building structures, targets, and responders are considered as the three major components of the emergency situation. Rooms, lobbies, and any intersections are presented as nodes whereas corridors, halls, and links between intersections are described as arcs. The condition of the building structure varies depending on time. Thus, the structure of a building can be represented as a time-dependent and directed dynamic network $G(N, A, T)$, where N is the set of nodes and A is the set of directed arcs. T is a discrete time horizon $\{0, \ldots, t_{max}\}$, where t_{max} indicates the upper bound of time when additional responders from the outside of the building arrive.

Multiple sources (entrances) and sinks (exits) are located in the building. The nodes in the network are defined by a predetermined time horizon. The model uses super source node s and super sink node d to handle the complexity. A super source node connects all source nodes at time 0, and the super sink node connects all sink nodes excluding those at time 0. Through the provided notations, we can transform the model into a single source node and a single sink node network model.

During the emergency situation, the initial location of responders is provided. Responders can either move from node i to node j or stay at node i at time t. The behavior 'stay' can be interpreted as neutralizing risky target i or simply remaining at node i. Responders also make decisions on target selection and the required duration for elimination. During the golden time, every responder should be located in specific areas, defined as sink (waiting) node $i \in D$, to support additional responders from the outside of the building. The area that contains risk is labeled as the target. Risk includes fire spread, gas leak, explosions, and related harm. We assume that these risks do not spread to other areas because ORPR is established for only considering a brief time.

In the model, the cumulative and discrete time function $p_i(t)$ is used to compare priority. To eliminate future damage, corresponding work time $f_i(t)$ must be considered. Both risks and work time vary. Their quantitative values will be derived by emergency management experts. Observations made during the situation provide crucial supplementary information during the process. When a target is neutralized at certain time t, the amount of risk is the risk during the golden time minus the risk at time t is prevented (Figure 1) and is defined as *prevented risk*. Thus, the objective of the model is to maximize prevented risk.

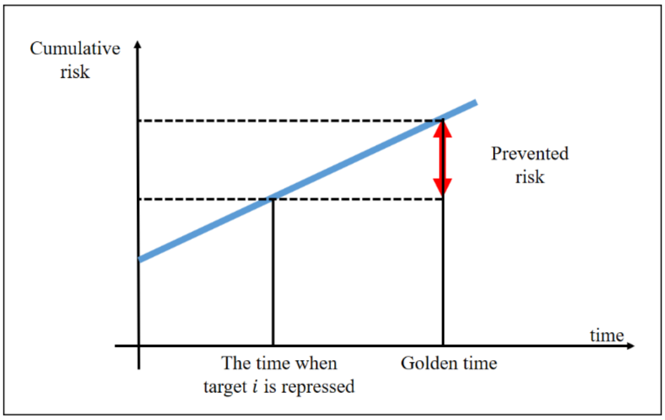

Figure 1: Graph of prevented risk

2.2 Model formulation

The indices are defined as follows:

s	Super source node
d	Super sink node
i, j, k	Node indices
t	Time index
t_{max}	Upper bound of the time period

The definitions of the sets are as follows:

N	Node set
S	Set of source nodes
D	Set of sink (waiting) nodes
R	Set of targets
T	Time period $(0, \ldots, t_{max})$
A	Pairs of (i, j) nodes set, $(i, j) \in N$

The parameters are defined as follows:

q_i	Number of responders in node $i \in S$
$p_i(t)$	Risk in node $i \in R$ at $t \in T$
$f_i(t)$	Required time to neutralize target $i \in R$ at time $t \in T$
$a_i(t)$	Capacity of node $i \in N$ at time $t \in T$
$b_{ij}(t)$	Capacity of arc (i, j) between node $i \in N$ and node $j \in N$
λ_{ij}	Travel time of arc (i, j) from node $i \in N$ to node $j \in N$

The decision variables are defined as follows:

$z_{ij}(t)$	Number of responders who move from node $i \in N$ to node $j \in N$ at time $t \in T$
$w_i(t)$	Number of responders who work in node $i \in N$ at time $t - 1 \in T$
$c_i(t)$	1 if responders neutralize target $i \in R$ at time $t \in T$, 0 otherwise

The formulation of the mathematical model is as follows:

$$\textit{Maximize} \sum_{i \in R, t \in T \setminus \{0\}} \{p_i(t_{max}) - p_i(t)\} \qquad (1)$$

Subject to

$$\sum_{t \in T} c_i(t) \le 1 \qquad \forall i \in R \qquad (2)$$

$$\sum_{t_1=0}^{t} w_i(t_1) \ge f_i(t) \cdot c_i(t) \qquad \forall t \in T, \quad \forall i \in R \qquad (3)$$

$$z_{si}(0) = q_i \qquad \forall i \in D \qquad (4)$$

$$\sum_{t \in T, i \in D} z_{id}(t) = \sum_{i \in S} q_i \qquad (5)$$

$$w_i(t) \le a_i(t) \qquad \forall t \in T, \quad \forall i \in N \qquad (6)$$

$$z_{ij}(t) \le b_{ij}(t) \qquad \forall t \in T, \quad \forall (i, j) \in A \qquad (7)$$

$$w_i(t+1) - w_i(t) = \sum_{k \in pred(i)} z_{ki}(t - \lambda_{ki}) - \sum_{j \in succ(i)} z_{ij}(t) \qquad \forall t \in T, \quad \forall (i, j) \in A \qquad (8)$$

$$w_i(0) = 0 \qquad \forall i \in N \qquad (9)$$

$$z_{di}(t) = 0 \qquad \forall t \in T \setminus \{0\}, \quad \forall i \in D \qquad (10)$$

$$z_{ij}(t) \in \mathbb{Z}_+ \qquad \forall t \in T, \quad \forall (i, j) \in A \qquad (11)$$

$$w_i(t) \in \mathbb{Z}_+ \qquad \forall t \in T, \quad \forall i \in N \qquad (12)$$

$$c_i(t) \in \{0,1\} \qquad \forall t \in T, \quad \forall i \in R \qquad (13)$$

The objective function (1) maximizes prevented risks in the golden time period. Constraints (2) and (3) restrict the time allotted to neutralize targets. Constraints (4) through (8) enforce network flows of responders. Constraints (9) and (10) eliminate infeasible solutions. Lastly, Constraints (11), (12), and (13) explain the definition of variables.

3. Case study

The data of Central City in Seoul, Korea, is used to evaluate the model. To improve reliability, we modified the experiment data of Kang *et al.* (2015) (Figure 2), and obtained the dimensions of halls and corridors by direct measurement.

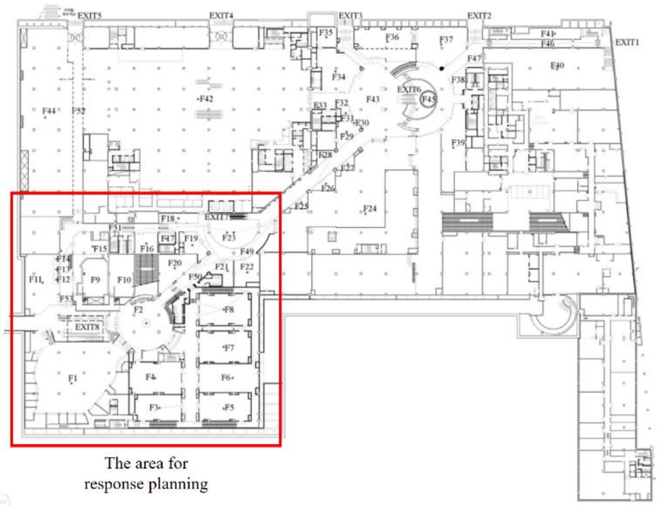

Figure 2: Layout of Central City, Seoul, Korea

The assumed case is comprised of multiple instances of suicide bombing attacks and information of additional bombs within the target area. More specifically, three areas were attacked by terrorists and fire is spreading in these areas. The four areas are searched for hidden bombs (Table 1).

Table 1: Experimental data

Node	$f_i(t)$	Type of target
1	$5 + 2^{0.3t}$	Variable
2	$15 + 2^{0.2t}$	Variable
3	15	Stationary
4	20	Stationary
5	30	Stationary
6	$45 + 2^{0.2t}$	Variable
7	50	Stationary

In this example, we define threats in fire-spreading areas as variable targets, while those in search areas are defined as stationary targets. Five responders are located in various areas. They will try to neutralize targets in nodes during the golden time. At the end of the golden time, they are expected to be located in the waiting zones to support additional responders who are equipped with proper tools to ameliorate hazards. Additional responders are expected to arrive within 180 seconds and 3 seconds as the basic time unit. We assume that suppressing the risks of variable targets receive a higher priority than handling of stationary targets. However, the risk posed by each type of targets are the same.

To evaluate efficiency of the ORPR, we conducted experiments with a sequential algorithm. In the algorithm, all variable targets are neutralized before the stationary targets are suppressed.

The model was coded in *XPRESS-IVE 7.9* with the *XPRESS-MP* mathematical programming solver and implemented in *Java 1.8.071* language with the *XPRESS-MP library*. The model was tested with an Intel(R) Core(TM) i5-3570 CPU 3.4GHz with 8.00GB of RAM in Windows 10.

4. Results of the experiment

The ORPR found a better way than the sequential algorithm to prevent risks during the response period (Table 2). The model was able to eliminate 6 targets while the algorithm was able to suppress 2 targets, respectively, in the corresponding period.

Table 2: Comparison of ORPR and the Sequential Algorithm for the Amount of Each Suppressed Target

Type of targets	Number of targets	Number of neutralized targets	
		ORPR	Sequential algorithm
Stationary	4	4	0
Variable	3	2	2
Total	7	6	2

Responders of the sequential algorithm tried to eliminate all risks from variable targets. Because ORPR places priority on variable targets over stationary targets, the initial response was similar. However, it was impossible to suppress all variable targets in the golden time, which led the inability to address other stationary targets which could have been eliminated.

5. Discussion

The main contribution of the study is the description of emergency situations. A limited number of responders and restricted time periods reflect realistic problems. Additionally, the model considers time-varying components in areas presenting risks. The fast growth of risks and the required time to eliminate them can dramatically change emergency situations, especially during the initial stages of the threat.

However, the mathematical model has limited ability to solve real-sized problems within a desirable time. Therefore, development of a heuristic or meta-heuristic algorithm is necessary. Different types of responses can be considered in the future research as well. In addition, relief works in problem situations can be as important as suppression of risks. In such cases, the formulation on methods for reaching victims is required.

6. Conclusions

We defined the optimal routing problem for responders and provided an integer programming model based on a dynamic network flow model. Along with the model, a sequential algorithm is considered to evaluate the performance of the formulation, data came from the Central City in Seoul, Korea. The result shows that the ORPR is more appropriate than other methods, for application in real situations.

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