Reliability with Limited Buffer Capacity for a Multistate Manufacturing Network with Two Production Lines

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Abstract. This paper evaluates the system reliability of a manufacturing system with two production lines. The manufacturing system is constructed as a multistate manufacturing network (MMN). The system reliability is defined as the probability of demand satisfaction, as well as all buffers are not running out of storage. In the MMN, buffers with limited capacities are considered to avoid the MMN from blockage and starvation. First, the amount of input materials, workload, and minimal capacity that each workstation has to provide to satisfy the demand, are studied in flow analysis. Second, a 'buffer usage matrix (BUM)' is proposed to calculate the buffer usage and buffer reliability. Third, system reliability with limited buffer capacities is derived. An example is utilized to illustrate the proposed method.

Keywords: buffer reliability, limited buffer capacity, buffer usage matrix (BUM), system reliability.

1. INTRODUCTION

Multistate manufacturing network (MMN) is a genre of multistate network model. In the MMN model, each workstation consists of k identical or similar machines, meaning that $(k + 1)$ performance states are exhibited. The highest level k corresponds to all machines are normally operating, while zero is the lowest level of operation. Earlier studies (Lin, 2007; Yeh, 2008, 2011; Azadeh et al., 2015) have been devoted to constructing a manufacturing system as a conventional multistate network to evaluate its system reliability. Generally speaking, the system reliability is defined as the probability of demand satisfaction. This system reliability indicates the probability that the current capacity state of an MMN can successfully process a specific demand amount. However, the abovementioned studies are assumed to follow the so-called flow conservation law (Ford and Fulkerson, 1962), meaning that no flow increases or decreases during production. In the real-world, input flow processed by a workstation may not equal output flow due to the possibility of defect and scrap. Hence, the conventional multistate network is incapable to deal with practical characteristic of defects because the limitation of flow-conservation law. Recently, Lin and Chang (2012, 2013) develop a more practical MMN model with defect consideration. The primary characteristic of this MMN model is violation of flow conservation law. A graphical technique, integrating transformation and decomposition, is developed to deal with defect and scrap in a practical manufacturing system. This MMN model is further applied to the footwear manufacturing system (Lin et al., 2013). However, all of the above works assume the buffer capacity between workstations is unlimited. In other words, those studies have no capability to deal with the blockage and starvation of workstations.

The unlimited buffer capacity assumption in the MMN model can simplify the flow analysis and reliability evaluation. However, because all workstations in an MMN operate at a distinct production rate, work-in-process (WIP) may have to wait before entering the downstream workstation; or downstream workstations may be idle when they have to wait for the next WIP (Xiaobo et al., 2001; Becker and Scholl, 2006). Therefore, a workstation may be blocking or starving while the production rates of workstations are different. In order to eliminate the situation of blockage or starvation, setting buffer between workstations allows sequential workstations to operate more independent of each other (Demir et al., 2014). In light of practical needs, to model buffers with limited capacity is a crucial issue when studying the system reliability of an MMN.

2. MODEL CONSRUCTION

In this paper, a flow-shop manufacturing system with two production lines, is constructed as an MMN for reliability evaluation. This sections utilizes an activity-on-arc (AOA) diagram to represent an MMN. Each arc is regarded as a workstation consisting of identical machines; each vertex denotes an inspection station following the workstation. Different from the previous studies (Lin and Chang, 2012, 2013; Lin et al., 2013), this paper extends the MMN model to consider limited buffer capacity between workstations. The MMN is studied according to the following assumptions.

1) All inspections (vertices) are perfectly reliable. The inspections do not damage any WIP/products.
2) The capacity of each workstation (arrow) is a random variable which takes possible values according to a historical probability distribution.
3) The capacity of each workstation is independent from the one for any other workstation.

2.1 MMN with Buffer

Let (**V**, **A**, **B**) represent an MMN, in which **V** is the set of vertices (inspections) and $\mathbf{A} = \{a_{j,i} | j = 1, 2;\ i = 1, 2, \ldots, n\}$ is the set of arcs (workstations). The notation $a_{j,i}$ denotes the ith workstation (operation/process) in the jth production line L_j. Because the capacity $x_{j,i}$ of each workstation $a_{j,i}$ is a random variable according to assumption (2), the MMN is stochastic (i.e. multistate). For each $a_{j,i}$, $x_{j,i}$ takes multiple possible values $0 = x_{j,i(1)} < x_{j,i(2)} < \ldots < x_{j,i(c_{j,i})} = k_{j,i}$. The set of buffers is denoted by $\mathbf{B} = \{b_{j,i/i+1} | j, i$: a buffer is set between $a_{j,i}$ and $a_{j,i+1}\}$. Note that, it is not necessary to model buffers between every pairs of workstations; the buffer capacity is zero if there is no buffer set between workstations. For any pair of workstations in the same production line L_j, the buffer (circled part) is represented as Figure 1.

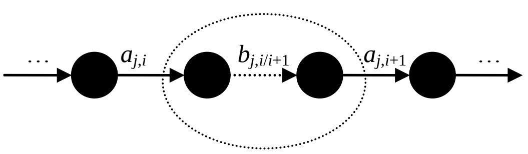

Figure 1: A buffer between two workstations.

The flow is affected by the defect rate of each workstation; the defect rates of workstations affect the output of a production line and lead to defective products. Hence, for a demand pair (d_1, d_2) assigned to production lines (L_1, L_2), the amount of raw materials should be determined backward in advanced. Let I_j be the amount of raw materials to produce O_j units of product in L_j; it is intended to determine the relationship between I_j and O_j such that $O_j \geq d_j$. Given the defect rate $q_{j,i}$ of $a_{j,i}$ and thus the success rate $p_{j,i} = 1 - q_{j,i}$, where $0 \leq p_{j,i} \leq 1$ The maximum capacity of L_j, denoted by K_j, is calculated by

$$K_j = \min(k_{j,1}p_{j,1}p_{j,2}\ldots p_{j,n},\ k_{j,2}p_{j,2}\ldots p_{j,n},\ \ldots,\ k_{j,n}p_{j,n}) \quad (1)$$

The expected input amount of raw materials for L_j is

$$I_j = d_j \Big/ \prod_{i=1}^{n} p_{j,i}\,. \quad (2)$$

Because the amount of raw materials is obtained by Eq. (2), the workload entering each $a_{j,i}$ can be calculated in terms of the workstation defect rate. The workload of workstation $a_{j,i}$, denoted by $w_{j,i}$, is calculated by

$$w_{j,i} = \begin{cases} I_j & \text{if } i = 1 \\ I_j \prod_{t=1}^{i-1} p_{j,t} & \text{if } i > 1 \end{cases} \quad (3)$$

For all workstations, the workload vector $W = (w_{1,1}, w_{1,2}, \ldots, w_{1,n}, w_{2,1}, w_{2,2}, \ldots, w_{2,n})$. In most situations, the capacities provided by workstations may not exactly equal the workloads. To satisfy the workload of each workstation, the following transformation finds the minimal capacity that should be provided.

$$y_{j,i} = \begin{cases} x_{j,i(\theta)} & \text{if } x_{j,i(\theta)} \geq w_{j,i} > x_{j,i(\theta-1)} \\ \text{doesn't exists} & \text{else} \end{cases}, \quad (4)$$

where $\theta = 1, 2, \ldots, c_{j,i}$. Therefore, the minimal capacity vector $Y = (y_{1,1}, y_{1,2}, \ldots, y_{1,n}, y_{2,1}, y_{2,2}, \ldots, y_{2,n})$.

2.2 Buffer Usage

For each possible value of $x_{j,i}$, the corresponding probability is denoted by $\Pr(x_{j,i} = x_{j,i(\theta)})$ for $\theta = 1, 2, \ldots, c_{j,i}$. For the sake of compact representation, let $\pi_{j,i(\theta)}$ denote $\Pr(x_{j,i} = x_{j,i(\theta)})$. The multistate of a workstation affects the capacity usage of a buffer capacity.

Consider a buffer $b_{j,i/i+1}$ with the capacity $k_{j,i/i+1}$ set between an upstream workstation $a_{j,i}$ and a downstream workstation $a_{j,i+1}$. Given $a_{j,i}$ with a success rate $p_{j,i}$, the expected output from $a_{j,i}$ is $x_{j,i} \times p_{j,i}$. The possible capacity of $a_{j,i}$ takes value from $\{x_{j,i(1)}, x_{j,i(2)}, \ldots, x_{j,i(c_{j,i})}\}$ with corresponding probability $\{\pi_{j,i(1)}, \pi_{j,i(2)}, \ldots, \pi_{j,i(c_{j,i})}\}$. Similarly, the possible capacity of $a_{j,i+1}$ takes value from $\{x_{j,i+1(1)}, x_{j,i+1(2)}, \ldots, x_{j,i(c_{j,i})}\}$ with corresponding probability

$\{\pi_{j,i+1(1)}, \pi_{j,i+1(2)}, \ldots, \pi_{j,i(c_{j,i})}\}$. The difference between output from $a_{j,i}$ and the capacity state of $a_{j,i+1}$ is $x_{j,i} \times p_{j,i} - x_{j,i+1}$. The extra amount of WIP is stored temporally in the buffer $b_{j,i/i+1}$ if $x_{j,i} \times p_{j,i} - x_{j,i+1} > 0$.

To obtain the used capacity of a buffer, the buffer usage matrix (BUM) is proposed to state all the possible values of the difference between upstream and downstream workstations. The value of $x_{j,i} \times p_{j,i} - x_{j,i+1}$ indicates the amount of buffer capacity needed. If $x_{j,i} \times p_{j,i} - x_{j,i+1} \le 0$, it means that no buffer capacity is used and thus the buffer usage is zero.

$$\begin{bmatrix} p_{j,i}x_{j,i(1)} - x_{j,i+1(1)} & p_{j,i}x_{j,i(1)} - x_{j,i+1(2)} & \cdots & p_{j,i}x_{j,i(1)} - x_{j,i+1(c_{j,i+1})} \\ p_{j,i}x_{j,i(2)} - x_{j,i+1(1)} & p_{j,i}x_{j,i(2)} - x_{j,i+1(2)} & \cdots & p_{j,i}x_{j,i(2)} - x_{j,i+1(c_{j,i+1})} \\ \vdots & \vdots & \ddots & \vdots \\ p_{j,i}x_{j,i(c_{j,i})} - x_{j,i+1(1)} & p_{j,i}x_{j,i(c_{j,i})} - x_{j,i+1(2)} & \cdots & p_{j,i}x_{j,i(c_{j,i})} - x_{j,i+1(c_{j,i+1})} \end{bmatrix} \quad (5)$$

The corresponding probability distribution matrix is

$$\begin{bmatrix} \pi_{j,i(1)}\pi_{j,i+1(1)} & \pi_{j,i(1)}\pi_{j,i+1(2)} & \cdots & \pi_{j,i(1)}\pi_{j,i+1(c_{j,i+1})} \\ \pi_{j,i(2)}\pi_{j,i+1(1)} & \pi_{j,i(2)}\pi_{j,i+1(2)} & \cdots & \pi_{j,i(2)}\pi_{j,i+1(c_{j,i+1})} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{j,i(c_{j,i})}\pi_{j,i+1(1)} & \pi_{j,i(c_{j,i})}\pi_{j,i+1(2)} & \cdots & \pi_{j,i(c_{j,i})}\pi_{j,i+1(c_{j,i+1})} \end{bmatrix} \quad (6)$$

3. RELIABILITY EVALUATION

Under the capacity vector $X = (x_{1,1}, x_{1,2}, \ldots, x_{1,n}, x_{2,1}, x_{2,2}, \ldots, x_{2,n})$, the system reliability with unlimited buffer for demand pair (d_1, d_2) is defined as $R(d_1, d_2)$.

$$R(d_1, d_2) = \prod_{i=1}^{n} \Pr(x_{1,i} \ge y_{1,i}) \cdot \prod_{i=1}^{n} \Pr(x_{2,i} \ge y_{2,i}) \quad (7)$$

Considering Matrix (5), a buffer $b_{j,i/i+1}$ is reliable when $x_{j,i} \times p_{j,i} x_{j,i+1} \le k_{j,i/i+1}$; that is, the used amount is less than the maximum capacity of the buffer. Let **S** be the event that the capacities $(x_{j,i}, x_{j,i+1})$ in L_j make the buffer $b_{j,i/i+1}$ empty or partially empty. Therefore, $(x_{j,i}, x_{j,i+1}) \in$ **S** if and only if $x_{j,i} \times p_{j,i} - x_{j,i+1} \le k_{j,i/i+1}$. On the other hand, let **F** be the event that $(x_{j,i}, x_{j,i+1})$ make the buffer blocked. That is, $(x_{j,i}, x_{j,i+1}) \in$ **F** if and only if $x_{j,i} \times p_{j,i} - x_{j,i+1} > k_{j,i/i+1}$. By defining the buffer reliability of $b_{j,i/i+1}$ as $\Pr(\mathbf{S} | x_{j,i} \ge y_{j,i}$ and $x_{j,i+1} \ge y_{j,i+1})$, the reliability can be evaluated to get rid of dependency calculation according to Matrix (6).

Table 1: Data of the MMN.

L_1	Workstation	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$
	Success rate	0.980	0.985	0.996	0.993	0.990
	Capacity (Probability)	0 (0.010)	0 (0.005)	0 (0.002)	0 (0.001)	0 (0.001)
		10 (0.010)	15 (0.010)	5 (0.003)	5 (0.001)	6 (0.002)
		20 (0.010)	30 (0.010)	10 (0.005)	10 (0.001)	12 (0.002)
		30 (0.020)	45 (0.015)	15 (0.010)	15 (0.002)	18 (0.005)
		40 (0.950)	60 (0.960)	20 (0.010)	20 (0.002)	24 (0.010)
				25 (0.015)	25 (0.003)	30 (0.980)
				30 (0.955)	30 (0.010)	
					35 (0.010)	
					40 (0.970)	
L_2	Workstation	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$
	Success rate	0.985	0.980	0.996	0.993	0.995
	Capacity (Probability)	0 (0.010)	0 (0.005)	0 (0.001)	0 (0.001)	0 (0.002)
		10 (0.010)	15 (0.005)	5 (0.002)	5 (0.001)	6 (0.003)
		20 (0.010)	30 (0.015)	10 (0.002)	10 (0.001)	12 (0.005)
		30 (0.020)	45 (0.015)	15 (0.005)	15 (0.002)	18 (0.005)
		40 (0.020)	60 (0.960)	20 (0.010)	20 (0.002)	24 (0.010)
		50 (0.930)		25 (0.015)	25 (0.003)	30 (0.010)
				30 (0.965)	30 (0.010)	36 (0.965)
					35 (0.010)	
					40 (0.970)	

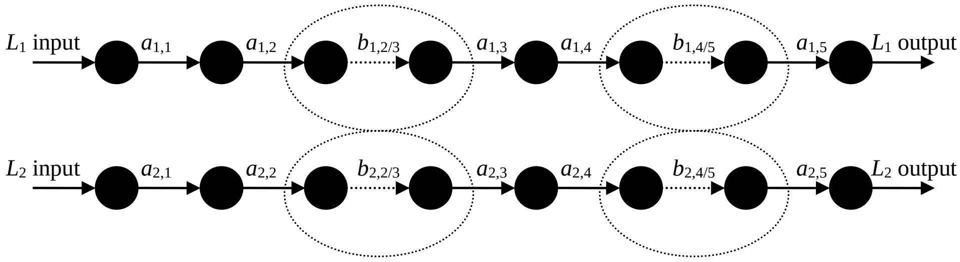

Figure 2: Manufacturing system with two production lines.

For the case of multiple buffers in production lines, an aggregated value, namely aggregated buffer reliability R_b, is defined by

$$R_b = \prod_{i=1}^{n} \Pr(\mathbf{S} \mid x_{j,i} \geq y_{j,i} \text{ and } x_{j,i+1} \geq y_{j,i+1} \text{ for } j = 1, 2) \quad (8)$$

Hence, the system reliability to consider limited buffer capacity is

$$R^b(d_1, d_2) = R(d_1, d_2) \times R_b \quad (9)$$

4. EXAMPLE

A manufacturing system with two production lines, L_1 and L_2, is used to demonstrate the proposed method (see Figure 2). There are five workstations in each production line, $\{a_{1,1}, a_{1,2}, a_{1,3}, a_{1,4}, a_{1,5}\}$ in L_1 and $\{a_{2,1}, a_{2,2}, a_{2,3}, a_{2,4}, a_{2,5}\}$ in L_2, and the buffers are set in front of the bottleneck workstations (i.e. $a_{1,3}$, $a_{1,5}$, $a_{2,3}$ and $a_{2,5}$).

To satisfy the demand of $d_1 + d_2 = 45$ (items per unit time), the production policy is to produce $(d_1, d_2) = (23, 22)$ by L_1 and L_2, respectively. The minimal capacity vector for $(d_1, d_2) = (23, 22)$ is $Y = (y_{1,1}, y_{1,2}, y_{1,3}, y_{1,4}, y_{1,5}, y_{2,1}, y_{2,2}, y_{2,3}, y_{2,4}, y_{2,5}) = (30, 30, 25, 25, 24, 30, 30, 25, 25, 24)$. Before further analysis on buffer reliability, the buffer is assumed to be unlimited. Thus, the system reliability $R(23, 22)$ is derived by Eq. (7).

$R(23, 22)$
$= \Pr(X|X \geq (30, 30, ..., 24))$
$= \Pr(x_{1,1} \geq 30) \times \Pr(x_{1,2} \geq 30) \times \ldots \times \Pr(x_{2,5} \geq 24)$
$= 0.970 \times 0.985 \times \ldots \times 0.985$
$= 0.838654.$

That is, the probability that the demand can be satisfied is 83.87%. Please note that buffers are not considered so far. Furthermore, four buffers, $\{b_{1,2/3}, b_{1,4/5}, b_{2,2/3}, b_{2,4/5}\}$, are set in the manufacturing system. The buffer capacities are $k_{1,2/3} = k_{2,2/3} = 30$ and $k_{1,4/5} = k_{2,4/5} = 10$. The minimal capacity $(y_{1,4}, y_{1,5}) = (25, 24)$ for a_4 and a_5, respectively. Two sets are obtained, $\{\mathbf{S}|x_{1,4} \geq 25$ and $x_{1,5} \geq 24\} = \{(25, 24), (30, 24), (25, 30), (30, 30), (35, 30), (40, 30)\}$ and $\{\mathbf{F}|x_{1,4} \geq 25$ and $x_{1,5} \geq 24\} = \{(35, 24), (40, 24)\}$. Hence, the buffer reliability of $b_{1,4/5}$ is $\Pr(\mathbf{S}|x_{1,4} \geq 25$ and $x_{1,5} \geq 24) = (0.000030 + 0.000100 + 0.002940 + 0.009800 + 0.009800 + 0.950600) / (0.000030 + 0.000100 + \underline{0.000100 + 0.009700} + 0.002940 + 0.009800 + 0.009800 + 0.950600) = 0.990031$. The underlined values are the probabilities of $(x_{1,4}, x_{1,5}) \in \mathbf{F}$.

The other buffer reliabilities can be calculated in a similar manner. Note that, exception for $\{b_{1,2/3}, b_{1,4/5}, b_{2,2/3}, b_{2,4/5}\}$, there is no other buffer between the other pairs of workstations. This implies that their buffer capacities should be zero. The buffer reliabilities are provided in Table 2 and thus the aggregated buffer reliability $R_b = 0.908478$. By apply Eq. (9), the system reliability is $R^b(23, 22) = R(23, 22) \times R_b = 0.838654 \times 0.908478 = 0.761898$. That is, the probability that the demand can be satisfied as well as all buffers are not running out of storage is 76.19%.

Table 2: Buffer reliabilities of the MMN.

Production line	Buffer station	Buffer capacity	Buffer reliability
L_1	$b_{1,1/2}$	0	0.990057
	$b_{1,2/3}$	300	0.984929
	$b_{1,3/4}$	0	0.997026
	$b_{1,4/5}$	100	0.990031
L_2	$b_{2,1/2}$	0	0.970634
	$b_{2,2/3}$	300	0.985158
	$b_{2,3/4}$	0	0.997025
	$b_{2,4/5}$	100	0.989981

5. DISCUSSION

In the MMN, it is reasonable and intuitive that the system reliability decreases while the total demand $(d_1 + d_2)$ increases. Moreover, the production manager can assign demand to both production lines by using the system reliability as a reference. For example, if the total demand is $d_1 + d_2 = 40$, a suggested demand pair is $(d_1, d_2) = (14, 26)$ with the optimal system reliability $R^b(14, 26) = 0.856597$ when consider limited buffer capacity. Table 3 shows the system reliability with limited buffer capacities for $d_1 + d_2 = 40$. One may notice that the system reliability does not change from (14, 26) to (12, 28). It is because that the minimal capacity vector is fixed as (20, 15, 15, 15, 18, 30, 30, 30, 30, 30) within these demand pairs.

Table 3: System reliability of the MMN.

Demand pair (d_1, d_2)	Buffer capacity $R(d_1, d_2)$
(22, 18)	0.857669
(20, 20)	0.838654
(18, 22)	0.857759
(16, 24)	0.853339
(14, 26)	0.856597
(12, 28)	0.856597

6. CONCLUSION

The main contribution of this paper is to evaluate the system reliability of an MMN with two production lines considering limited buffer capacities. In particular, the buffer usage matrix (BUM) is proposed to calculate the capacity usage of a buffer. Subsequently, the corresponding probability of BUM is used to obtain the buffer reliability. The buffer reliability affects the system reliability evaluation while considering limited buffer capacity. The case demonstration indicates that the assumption of unlimited buffer capacity buffer reliability may overestimate the system reliability. Therefore, it is necessary to model buffers into a network-structured MMN.

Future works can be devoted to studying the case of multiple production lines. In the case of multiple production lines, the buffers may be located in parallel or be jointed. It would be a possible issue to compare the result of both allocations.

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