# Integration of Bucket Brigades and Worker Collaboration on Discrete Work Stations 

Aditya Tirta Pratama ${ }^{1}$, Katsuhiko Takahashi ${ }^{2}$, Katsumi Morikawa ${ }^{\mathbf{3}}$, Keisuke Nagasawa ${ }^{4}$<br>Department of System Cybernetics, Graduate School of Engineering<br>Hiroshima University, Higashi-Hiroshima, 739-8527, Japan, Tel: (+81) 82-424-7705, Email: aditya.tp@gmail.com ${ }^{1}$; takahasi@hiroshima-u.ac.jp ${ }^{2}$; mkatsumi@hiroshima-u.ac.jp ${ }^{3}$; nagasa-kei@hiroshima-u.ac.jp ${ }^{4}$<br>Daisuke Hirotani<br>Faculty of Management and Information Systems, Prefectural University of Hiroshima, Hiroshima, 734-8558, Japan, Tel: (+81) 82-251-9737, Email: dhiro@pu-hiroshima.ac.jp


#### Abstract

The purpose of this paper is to confirm other possibility to improve the performance of bucket brigades on discrete work stations by using worker collaboration. Bucket brigades are self-balancing production line where worker can move from one station to the next station to continue working on given part. For maximizing throughput in the bucket brigades, the best policy is by sequencing workers from slowest to fastest. Meanwhile, the throughput of bucket brigades on discrete work stations may decrease due to blocking/starvation even if workers are sequenced from slowest to fastest. To suppress the throughput decrease, workers can work collaboratively at same task so the idle time can be minimized. This paper describes the dynamical behavior between bucket brigades and worker collaboration on discrete work station based on simple model of worker coordination that has fewer number of workers than number of work stations. We examine production line with 3 -work station, 2 -worker both slowest-to-fastest and fastest-toslowest sequencing by considering the distribution of work-content on each work station. Workers can work together at same task, in which the combined velocity of team is proportional to the sum of velocity of individual workers. The velocity of team is influenced by worker's collaboration coefficient.


Keywords: bucket brigades, worker collaboration, collaboration coefficient, discrete work station, dynamical behavior

## 1. INTRODUCTION

When bucket brigades (BB) are formed, each worker simultaneously assembles a single item along the line. The worker carries an item from work station to work station until either hands over the item to successor or completes the work of the item. Then worker walks back to get another item from predecessor or introduce item in beginning of the line. BB are a way to organizing workers on a production line so that the line balances itself then it can increase the production efficiency by minimize the worker's idle time. Bartholdi and Eisenstein (1996) introduce serial BB to coordinate workers along an assembly line with more stations than workers. They
consider a model of BB with deterministic work content and each worker has a deterministic, finite work velocity and an infinite walk-back velocity. In the paper, they show that if workers are sequence from slowest to fastest according to their velocities in the direction of production flow, then the hand-off item will converge into a fixed point, and each workers repeatedly works on fixed portion of the line. Based on the same model, Bartholdi et al. (1999) study the dynamics of two- and three-worker BB with workers not necessarily sequenced from slowest to fastest. Bartholdi et al. (2001) consider the stochastic work content on work stations. They find that dynamic and throughput of the stochastic system will be similar to that of the deterministic system when there is sufficient work
distributed among sufficiently many stations. Bartholdi and Eisenstein (2005) extend the basic model of BB to captures walk-back time and hand-off time. Bartholdi et al. (2006) extend the ideas of BB to a network of sub-assembly lines so that all sub-assembly lines are synchronized to produce at the same rate and items are completed at regular, predictable intervals. Hirotani et al. (2006) extend the ideas of BB by analyzing other conditions that can achieve the same self-balancing effect and characterize the line by deriving the imbalance condition and influence of initial position. They also have analyzed mathematically about the performance of self-balancing production line with $n$ workers. Lim and Yang (2009) analyze the dynamic of BB on discrete work stations and identify the best policies that can maximize the system's throughput. In the paper, they show that the policy of fully cross-trained teams with slowest-fastest sequence is not always the best policies for the system, even though it outperforms other policies for most work-content distribution.

Many modern work environments, with and without cross-training, make use of teams to accomplish some tasks. The fundamental justification is team work will ultimately improve productivity. According to Hopp and Oyen (2004), a basic measure of collaboration efficiency is the relative percentage increase in average task speed (or labor productivity) that result from assigning multiple workers to the same task. Andradóttir et al. (2001) used model assumption that when multiple servers are assigned to the same task, their combine service rate is additive. Several servers can work together on single customer, in which case the combined rate of server team is proportional to the sum of rates of the individual servers. They consider $\alpha$ as a measure of magnitude of server's collaboration/synergy. Van Oyen et al. (2001) also consider when combined rate of a set of collaborating servers is additive (i.e., $\alpha=1$ ). Their paper describes server assignment policy in which all servers work as team on a single job minimizes the cycle time per job when all servers are identical and complete collaboration of all servers are possible. Andradóttir and Ayhan (2005) study how the servers should be assigned dynamically to stations to obtain optimal long-run average throughput. They also consider that several servers can work together on a single job and travel times between stations are negligible. Andradóttir et al. (2007) also consider $\alpha=1$ (i.e., the service rates are additive). Their paper explains when the servers are synergistic and generalist, the optimal policy takes full advantages of this synergy and a non-idling policy is no longer sufficient to achieve optimal throughput.

The purpose of this paper is to examine the behavior and characterize worker collaboration methodology in a production line with 3 -work station, 2 -worker at fully cross-trained and partially cross-trained team, slowest-
fastest sequence and fastest-slowest sequence to achieve maximum system's throughput. It also analyzes how worker collaboration methodology can overcome the impact of blocking/starvation on discrete work stations.

## 2. THE PRODUCTION LINE

This section explains the model assumptions of production line and rule for collaborative work.

### 2.1 Model Assumptions

Consider a production line in which each instance of a product is progressively assembled on the same sequence of $m$ work stations. We assume that the work content of the product on work station $j$ is deterministic and is denoted as $\mathrm{s}_{j}$ and normalized the total work content to 1 so $\sum_{j=1}^{m} s_{j}=$ 1. Workers are indexed from 1 to $n$ and they remain in this sequence along the production line in the direction of production flow. Workers $i-1$ and $i+1$ are the predecessor and the successor, respectively of worker $i$. Each worker $i$ is cross-trained to work on zone $Z_{i}$-a set of contiguous stations on the line. A worker $i$ is fully cross-trained if $Z_{i}$ contains all stations on the line. Worker $i$ works forward with a constant velocity $v_{i}$ within zone $Z_{i}$. Each worker $i$ carries a single item and continues to assemble his/her item from station to station until he/she hands off his/her item to his/her successor or, if worker $i$ is the last worker on the line, he/she completes the work on his/her item at the end of the line. A worker $i<n$ will be blocked if he/she finishes his/her work on station $j$ while his/her successor is still working on the station $j+1$. The blocked worker remains idle until the next station becomes available. A worker $i<n$ will be halted if he/she finishes his/her work on all station in $Z_{i}$ before he/she can hand off his item to his/her successor. The halted worker remains idle until his/her successor takes over his/her item. A worker $i>1$ will be starved if he/she reaches the beginning of his/her zone before his/her predecessor can hand off the item to him/her. The starved worker remains idle until his/her predecessor hands over an item. After hand over an item, worker $i$ walks back upstream to take over work from his/her predecessor or, if worker $i$ is the first worker on the line, he/she initiates a new item. When an item is handed off, we assume that the work content is preemptible without any loss of work. When the last worker (worker $n$ ) finishes the work on his/her item, the line resets itself. Each worker spends negligible time to hand over the item and to walk back to his/her predecessor or to the beginning of his/her zone.

Worker collaboration begins when the successor supports the predecessor in his/her zone and worker collaboration finish the collaboration before relinquishing
the item to the successor at the end of collaborative work station. At most two workers can work together on the same task, in which case the combined velocity of a team is proportional to the sum of velocity of the individual workers. If workers are indexed from 1 to $n$ and they remain in this sequence along the production line in the direction of production flow, then the collaborative velocity ( $\mathrm{v}_{\text {coll }}$ ) is equal to $\alpha \sum_{i=1}^{n} v_{i}$ where $\alpha$ is a measure/coefficient of the worker's collaboration/synergy. Velocity of collaborative work must be higher than minimum worker velocity $\left[\mathrm{v}_{\text {coll }}>v_{\text {min }}\right.$, where $\left.v_{\text {min }}=\min \left\{v_{i}\right\}\right]$.

### 2.2 Rule and Option for Collaborative Work

Figure 1a and figure 1 b show an example of time chart of two workers with BB and worker collaboration. The horizontal axis represents the worker's position and vertical axis represents time. Zero at horizontal axis denotes the head of the line, and the one denotes the end of line. Diagonal lines can represent the processing workers according to their velocities, while straight line represents walk back and take over. At BB, when successor finishes an item in position one, he/she walks back to predecessor to takes over next item, as shown of figure 1a. For worker collaboration at figure 1b, the successor finishes an item in position one, he/she walks back to predecessor then begin the collaborative work by velocity of team and finish at the end of collaborative work station. Table 1 summarizes the rules that must be followed by workers.

Table 1: Each worker independently follows the worker collaboration rule
Forward Rule -- Work forward until one of the following events occurs:

1. You complete the collaborative work with your successor at the end of collaborative work station (subset zone);
2. You are halted, in this case wait till you pass your item to your successor;
3. You are blocked, in this case wait till next station can be occupied;
4. You complete your item at the end of the line;
then follow the Backward Rule.
Backward Rule -- Walk back until one of the following events occurs:
5. You encounter your predecessor, in this case begin to work collaboratively;
6. You are starved in your zone, in this case wait till your predecessor arrives at your zone (subset zone) and begin to work collaboratively;
7. You reach the start of the line, in this case begin a new item;
then follow the Forward Rule.

At fully cross-trained teams, workers have two conditional options to establish collaborative work:
Option 1 (minimum collaboration). Worker can collaborate only at one station at one time. If successor encounters predecessor, then they begin and finish the collaborative work only at one work station (subset work station) at one time.
Option 2 (maximum collaboration). Worker can collaborate at all work stations in their subset zone at one time. If successor encounters predecessor, then they begin and finish the collaborative work at all the work stations in their subset zone at one time.

## 3. A THREE STATIONS, TWO WORKERS LINE

Consider a production line with $m=3$ stations and $n=2$ workers. The production line is divided into two cases: 1) All workers are fully cross-trained with $\mathrm{Z}_{1}=\{\mathrm{s} 1, \mathrm{~s} 2\}$ and $\mathrm{Z}_{2}=\{\mathrm{s} 1, \mathrm{~s} 2, \mathrm{~s} 3\}$ and 2) Each worker is partially cross-trained with $Z_{1}=\{s 1, s 2\}$ and $Z_{2}=\{s 2, s 3\}$.

### 3.1 Classification and Formulation of Worker Collaboration

At the production line, there are two major events that will be experienced by workers: 1) No Collaboration and 2) Worker Collaboration

### 3.1.1 No Collaboration

The main condition of this event is when converge point located at the last work station. Successor cannot support predecessor for collaboration if halting position and relinquishing item position are located at the end of subset work station.

### 3.1.2 Worker Collaboration

The main condition for worker collaboration is when converge point located at subset work stations. It means that worker have chance to establish collaborative work at subset work stations. Other condition is chance of blocking that occur at warming up of production line. We can formulate based on processing time by successor at $2^{\text {nd }}$ work station more than predecessor's processing time at $1^{\text {st }}$ work station. Other condition is chance of halting that may occur at production line. We can formulate based on processing time of predecessor at $2^{\text {nd }}$ work station less than processing time by successor at the last work station. We have formulated reference point or meeting point for worker collaboration which influences the formulation of system's throughput.

Worker conditions before and during worker collaboration and also throughput formulation can be obtained based on time chart simulations with various work-content combination. These classifications and throughput formulations can be used as guidance to achieve the
maximum throughput. Table 2 and table 3 describes the details of classification and throughput formulation of worker collaboration under fully cross-trained and partially cross-trained team.


Figure 1. Example of time chart: (a) BB (b) Worker Collaboration
Table 2: Classification and Throughput Formulation for Fully Cross-Trained Worker Collaboration Option 1

| Class | Condition |  |  |  |  |  |  |  |  |  | Throughput Formulation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 1.0 | $\checkmark$ | - | - | - | - | - | - | - | - | - | $\frac{1}{((1-(s 1+s 2)) / v 2)}$ |
| 1.1 | - | $\checkmark$ | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | - | - | - | $\frac{2}{\left(2\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{s 2}{v 2}+\frac{\left(s 1-\frac{(1-s 1-s 2) v 1}{v 2}\right)}{\alpha(v 1+v 2)}+\frac{\left(s 1+s 2-\left(s 1+\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.2 | - | $\checkmark$ | $\checkmark$ | - | $\sqrt{ }$ | - | - | $\checkmark$ | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.3 | - | $\checkmark$ | $\checkmark$ | - |  | $\sqrt{ }$ | $\checkmark$ | - | - | - | $\frac{2}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{(1-s 1)+}{v 2}+\frac{\left(s 1-\frac{(1-s 1-s 2) v 1}{v 2}\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.4 | - | $\checkmark$ | $\checkmark$ | - | - | $\checkmark$ | - | $\sqrt{ }$ | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.5 | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\checkmark$ | - | $\frac{1}{\left(\frac{(1-s 1)}{v 2}+\frac{\left(s 1-\left(\frac{(1-s 1) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.6 | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | - | $\sqrt{ }$ | - | - | $\checkmark$ | $\frac{2}{\left(2\left(\frac{(1-s 1-s 2)}{v 2}\right)++\frac{s 2}{v 2}+\frac{\left(s 1-\frac{(1-s 1-s 2) v 1}{v 2}\right)}{\alpha(v 1+v 2)}+\frac{\left(s 1+s 2-\left(\frac{(1-s 1) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |


| 1.7 | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | - | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8 | - | $\checkmark$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\frac{1}{\left(\frac{(1-s 1)}{v 2}+\frac{\left(s 1-\left(\frac{(1-s 1) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.9 | - | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | - | - | $\sqrt{ }$ | $\frac{2}{\left(2\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{s 2}{v 2}+\frac{\left(s 1-\frac{(1-s 1-s 2) v 1}{v 2}\right)}{\alpha(v 1+v 2)}+\frac{\left(s 1+s 2-\left(\frac{(1-s 1) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 1.10 | - | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |

Table 3: Classification and Throughput Formulation for Partially and Fully Cross-Trained Worker Collaboration Option 2

| Class | Condition |  |  |  |  |  |  |  |  |  | Throughput Formulation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Fully Cross-Trained Option 2 | Partially Cross-Trained |
| 2.0 | $\sqrt{ }$ | - | - | - | - | - | - | - | - | - | $\frac{1}{((1-(s 1+s 2)) / v 2)}$ | $\frac{1}{((1-(s 1+s 2)) / v 2)}$ |
| 2.1 | - | $\sqrt{ }$ | $\sqrt{ }$ | - | - | - | $\sqrt{ }$ | - | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 2.2 | - | $\sqrt{ }$ | $\sqrt{ }$ | - | - | - | - | $\sqrt{ }$ | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ | $\frac{1}{\left(\frac{s 1-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)}{v 1}\right)+\left(\frac{s 1+s 2-s 1}{\alpha(v 1+v 2)}\right)+\left(\frac{(1-s 1-s 2)}{v 2}\right)}$ |
| 2.3 | - | $\sqrt{ }$ | - | $\checkmark$ | - | - | $\sqrt{ }$ | - | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ |
| 2.4 | - | $\sqrt{ }$ | - | $\checkmark$ | - | - | - | $\sqrt{ }$ | - | - | $\frac{1}{\left(\left(\frac{(1-s 1-s 2)}{v 2}\right)+\frac{\left(s 1+s 2-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)\right)}{\alpha(v 1+v 2)}\right)}$ | $\frac{1}{\left(\frac{s 1-\left(\frac{(1-s 1-s 2) v 1}{v 2}\right)}{v 1}\right)+\left(\frac{s 1+s 2-s 1}{\alpha(v 1+v 2)}\right)+\left(\frac{(1-s 1-s 2)}{v 2}\right)}$ |

## Legend:

$" \sqrt{"}=$ yes ; "-" = no
Condition 1: $s 3>\left[1-\frac{(v 1 / v 2)}{(1+v 1 / v 2)}\right]$; Condition 2: $s 3 \leq\left[1-\frac{(v 1 / v 2)}{(1+v 1 / v 2)}\right]$; Condition 3: $\frac{s 2}{v 2}>\frac{s 1}{v 1}$; Condition 4: $\frac{s 2}{v 2} \leq \frac{s 1}{v 1}$; Condition 5:
$\frac{s 2}{v 1} \geq \frac{(1-s 1-s 2)}{v 2}$; Condition 6: $\frac{s 2}{v 1}<\frac{(1-s 1-s 2)}{v 2}$; Condition 7: $x^{0}<s 1$; Condition 8: $x^{0} \geq s 1$; Condition 9: $x^{1}<s 1$;
Condition 10: $x^{1} \geq s 1$

### 3.2 Analysis and Discussion

For this section, we will compare the dynamic behavior of BB and worker collaboration for fully crosstrained team and partially cross-trained team, slowestfastest sequence and fastest-slowest sequence, respectively. We will also compare performance of worker collaboration for various worker's velocities under same $\alpha$.

To compare the behavior between BB and worker collaboration, we can use diagram of plotted throughput or diagram of throughput percentage difference. The $x$ and $y$ axis represent the distribution of work content on the work stations. For diagram of plotted throughput, $z$ axis represents the amount of throughput produces by the system. The color scale is used to interpret the throughput amount at the system. For diagram of throughput percentage difference, z axis represents the throughput percentage difference. Neutral plane or zero surface represents by gridline plane. The color scale is used to indicate the significant level of worker collaboration performance towards BB. We can divide it into 7 levels: highest decrease (dark blue), high decrease (blue), decrease (light blue), neutral/zero (yellow), increase (light red), high increase (red), highest increase (dark red).

### 3.2.1 Comparison for Fully Cross-Trained Workers, Slowest-Fastest Sequence

Figure 2 shows the multi-plotted throughput comparison between BB and worker with minimum collaboration by $\alpha=1.0$. BB surface/region is indicated by gridline while worker with minimum collaboration is indicated with surface/region without gridline. Based on figure 2, worker with minimum collaboration can improve almost all regions than original BB, represent by region I and region II. This happens due to ability of worker with minimum collaboration can reduce the possibility of blocking that may occur at the next cycle of the production system. But, it has limitation where collaboration cannot be established due to halting at the end of subset stations (region III) and occurrence of repetitive blocking even though $\alpha=1$ (region I).

Figure 3 (a, b, c) show the throughput percentage difference between BB and workers with minimum collaboration. If we compare figure $3 a$ and figure $3 b$, the blue and light blue region will shrink and the light red region begin to appear due to increasing of $\alpha$, while at figure 3c when $\alpha=1$, the light blue region will shrink and yellow, light red and red region will expand. These shrinkage region and expansion regions by figure $3 \mathrm{a}, 3 \mathrm{~b}$ and 3 c represent the performance of worker with minimum collaboration can be equal or better than BB due to increasing of $\alpha$ at several work-content distributions.

Figure 4 (a, b, c) show the throughput percentage difference between BB and workers with maximum collaboration. If we compare figure $4 a$, figure $4 b$, and figure 4 c , maximum collaboration will give similar region behavior compared with minimum collaboration. Based on those figures, worker minimum and maximum collaboration can reduce the impact of blocking at the system. If we compare figure 3 a and figure 4 a or figure 3 b and figure 4 b , minimum collaboration has better performance compared to worker with maximum collaboration under $\alpha<1.0$ which is indicated by domination of light blue or light red region. This explains that worker with minimum collaboration can increase the performance of production line due to collaborative work done separately at different station at different time. But, if we compare figure 3 c and figure 4 c , worker with maximum collaboration has better performance compared to worker with minimum collaboration when $\alpha=1.0$. Worker with minimum collaboration still has light blue region due to limitation of worker collaboration to deal with the occurrence of blocking every production cycle.


Figure 2: Throughput comparison of BB and worker with minimum collaboration for fully cross-trained, slowest-fastest sequence $(\mathrm{v} 1=2 / 3, \mathrm{v} 2=4 / 3) \alpha=1.0$.

### 3.2.2 Comparison for Fully Cross-Trained Workers, Fastest-Slowest Sequence

The performance of BB cannot achieve maximum throughput due to repetitive halting and blocking that experiences by predecessor at the production line. Figure 5 $(a, b)$ show the throughput percentage difference between BB and worker collaboration for fully cross-trained, fastestslowest sequence. By comparing figure 5 a and figure 5 b , maximum collaboration has better performance compared to minimum collaboration when $\alpha=1.0$. Minimum collaboration still has light blue region compared to maximum collaboration where the throughput percentage different are almost at light red to red region. This explains that minimum collaboration still has possibility of blocking occurred at every cycle of production, while worker with maximum collaboration can eliminate almost all chances of blocking by predecessor.

### 3.2.3 Comparison for Partially Cross-Trained Workers

Workers can only establish collaborative work at one subset station. At slowest-fastest sequence, most of worker collaboration will be dominated under starvation condition by successor, while at fastest-slowest sequence, most of worker collaboration will be dominated under blocking and halting condition by predecessor. Figure $6(a, b)$ show the throughput percentage difference between BB and worker collaboration at partially cross-trained. Figure 6a and figure 6 b show that worker collaboration has better performance under fastest-slowest sequence. Under fastest-slowest sequence, we can see at figure 6 b , the expansion region of red to dark red region. In these regions, worker collaboration can effectively eliminate the blocking condition. But form figure $6 b$, we can see the limitation of worker collaboration that it cannot overcome the impact of
starvation by successor which represent by light blue region.

### 3.2.4 Comparison for Same $\alpha$, Various Velocity

The maximum throughput of production line can be defined by sum of worker's velocity. Figure 7 ( $a, ~ b, c$ ) show the plotted throughput of worker with minimum collaboration with different velocity configuration. If we compare figure 7a, figure 7 b and figure 7 c , there will be correlation between worker's velocity and expansion or shrinkage of regions. Based on those figures, by increasing predecessor velocity and constant successor velocity, we can see the shrinkage of region III (region with maximum throughput) and expand of region I and region II (region with below maximum throughput). Region II indicates the region with repetitive blocking and region I indicates the region with halting condition by predecessor at the end of subset stations under worker with minimum collaboration.


Figure 3: Throughput percentage difference of BB \& workers with collaboration for fully cross-trained, slowest-fastest sequence ( $\mathrm{v} 1=2 / 3, \mathrm{v} 2=4 / 3$ ): (a) $\alpha=0.5$ (b) 0.75 (c) $\alpha=1$


Figure 4: Throughput percentage difference of BB \& workers with maximum collaboration for fully cross-trained, slowestfastest sequence ( $\mathrm{v} 1=2 / 3, \mathrm{v} 2=4 / 3$ ): (a) $\alpha=0.5$ (b) 0.75 (c) $\alpha=1$

(b)


Figure 5: Throughput percentage difference of BB and worker collaboration for fully cross-trained, fastest-slowest sequence ( $\mathrm{v} 1=4 / 3$, $\mathrm{v} 2=2 / 3$ ) with $\alpha=1$ : (a) minimum (b) maximum


Figure 6: Throughput percentage difference of BB and worker collaboration at partially cross-trained workers $\alpha=1.0$ : (a) slowest-fastest sequence (b) fastest-slowest sequence


Figure 7: Plotted throughput of worker with minimum collaboration, at $\alpha=1$, fully cross-trained, slowest-fastest sequence with velocity configuration: (a) $\mathrm{v} 1=1 / 3, v 2=4 / 3$ (b) $\mathrm{v} 1=2 / 3, \mathrm{v} 2=4 / 3$ and (c) $\mathrm{v} 1=1, v 2=4 / 3$

## 4. CONCLUSIONS

This paper has shown the other possibility to increase the performance of BB by using worker collaboration. We have identified conditions when worker collaboration can attain the maximum throughput. Based on analysis at threestation and two-worker production line, at fully and partially cross-trained team, both options of collaborative work can effectively reduce almost all the impact of blocking that usually occur at BB. But minimum collaboration comes with a limitation that it is still not capable to eliminate repetitive blocking under several work content distributions.

This research only considers a 3-work station and 2worker production line. Future work must focus on worker collaboration at larger systems ( $\mathrm{m}>\mathrm{n} \geq 2$ ) and minimizing the impact of starvation using worker collaboration especially at partially cross-trained teams.

## REFERENCES

Andradóttir, S., Ayhan, H. Down, D. G. (2001). Server assignment policies for maximizing the steady-state throughput of finite queuing systems. Management Science 47(10): 1421-1439.
Andradóttir, S., Ayhan, H. (2005). Throughput maximization for tandem lines with two stations and flexible servers. Operations Research 53(3): 516-531.
Andradóttir, S., Ayhan, H. Down, D. G. (2007). Dynamic assignment of dedicated and flexible servers in tandem lines. Probability Engineering Information Sciences 21(4): 497-538.

Bartholdi, J. J. III, Eisenstein, D. D. (1996). A production line that balances itself. Operations Research 44(1): 2134.

Bartholdi, J. J. III, Bunimovich, L. A., Eisenstein, D. D. (1999). Dynamics of two- and three-worker "bucket brigade" production line. Operations Research 47(3): 488-491.
Bartholdi, J. J. III, Eisenstein, D. D., Foley, R. D. (2001). Performance of bucket brigades when work is stochastic. Operations Research 49(5): 710-719.
Bartholdi, J. J. III, Eisenstein, D. D. (2005). Using bucket brigades to migrate from craft manufacturing to assembly lines. Manufacturing \& Service Operations Management 7(2): 121-129.
Bartholdi, J. J. III, Eisenstein, D. D., Lim, Y. F. (2006). Bucket brigades on in-tree assembly networks. European Journal of Operational Research 168(3): 870879.

Hopp, W. J., Van Oyen, M. P. (2004). Agile workforce evaluation: A framework for cross-training and coordination. IIE Transactions 36(10): 919-940.
Hirotani, D., Myreshka, Morikawa, K., Takahasi, K. (2006). Analysis and design of self-balancing production line. Computers and Industrial Engineering 50: 488-502.
Lim, Y. F., Yang, K. K. (2009). Maximizing throughput of bucket brigades on discrete work stations. Production and Operations Management 18(1): 48-59.
Van Oyen, M. P., Gel, E. G. S., Hopp, W. J. (2001). Performance opportunity for workforce agility in collaborative and noncollaborative work systems. IIE Transactions 33(9): 761-777.

