A Commodity Trading Policy to Maximize the Survival Probability

Wen-Chih Chen †
Department of Industrial Engineering and Management
National Chiao Tung University, Hsinchu, Taiwan
Tel: (+886) 3- 573-1857, Email: wenchih@faculty.nctu.edu.tw

Cheng-Hung Wu
Institute of Industrial Engineering
National Taiwan University, Taipei, Taiwan
Tel: (+886) 2-3366-9505, Email: wuchn@ntu.edu.tw

Ping-Chung Chi
Department of Industrial Engineering and Management
National Chiao Tung University, Hsinchu, Taiwan
Tel: (+886) 3-571-2121 ext. 57348, Email: slalom2009@gmail.com

Abstract. This paper determines an optimal trading strategy for a decision maker that needs to purchase a commodity from the upstream in a fixed quantity and price at each period, and resell to the downstream at a high market price. This trading problem is typically found in the non-profit organizations to support the poors in the industry upstream, and thus the objective is to maximize the sustainability, not profits or growth. We model this problem as a stochastic dynamic programming problem to maximize the survival probability; optimal strategy is determined and managerial insights are also discussed.

Keywords: Commodity trading, stochastic dynamic programming, non-profit organization

1. INTRODUCTION

Motivated by a real case, we investigate a 3-tier supply chain, in which a wholesale need to purchase storable commodities from the upstream suppliers at a fixed quantity and a fixed price. The wholesale stores the commodities and resells to the downstream market when the spot market price is favorable. This business model can be found in some non-profit organizations to support the poors in the industry upstream. Thus the objective of the organization is to maximize the sustainability, not profits or growth.

The objective of the paper is to determine the optimal trading (selling) decision, i.e., at each period, what is the quantity level to sell when the spot price is observed. We model this problem as a stochastic dynamic programming problem to maximize the survival probability.

Our research question is related to the inventory problems. Inventory problem is an important and classic research area that determines when to order and/or by how many to fulfil the demand. Considering different cost factors, e.g. setup cost, holding cost, shortage cost, etc., the decision intends to maximize the expected profits for one or multiple periods (see e.g., Adeyemi and Salami, 2012; Federgruen and Heching, 1999; Zipkin, 2000). Some works consider the external constraints in practices, for example, the lead time and capacity of the suppliers (e.g. Federgruen and Zipkin(a), 1986; Federgruen and Zipkin(b), 1986; Ciarallo et al., 1994; Wang and Gerchak, 1996). In the last decade, the internal capability constraints, such as finance condition, have drawn attention (e.g. Xu and Birge, 2005; Chao et al., 2008). In addition to the profitability, some works, such as Archibald et al. (2002) and Thomas and Archibald (2003), consider the sustainability especially for the startup companies.

Back to 1948, Cahn studied how to optimally trade (buy/produce and sell/hold) for profitability when observe the market prices. He considered deterministic price variability and storage capacity. Dreyfus (1957) showed that although in a multi-period environment, only one of the three actions is optimal at each period: doing nothing, selling all inventory on hand, and buying as many as possible.

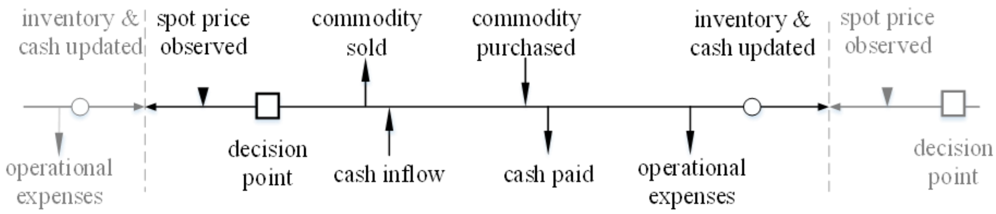

Figure 1: Decision sequence for a period

Trading problem can apply to storable commodity, such as oil (Devalkar et al., 2011), fuel (Martínez-de-Albéniz and Vendrell, 2008), natural gas (Secomandi, 2010) and electricity (Cruise et al., 2014; Cruise and Zachary, 2015). And methodology-wise, they are typically formulated as linear programs (e.g., Charnes and Cooper,1955) or dynamic programs (e.g. Bellman, 1956; Secomandi, 2010). Stochastic price, an important characteristic in reality, is studied since Charnes et al. (1966), and there is a stream of research also consider the transactions capacity, e.g. Rempala (1994), Martínez-de-Albéniz and Vendrell (2008), Secomandi (2010) and Cruise et al. (2014). However, to our best knowledge, the literature has only focused on the profit maximization, but not the sustainability that is the most important feature in this work.

The paper is unfolded as follows. Section 2 presents the optimization model and solution approach. Section 3 presents a simulated case study to show the correctness and effectiveness of the proposed model, and Section 4 concludes.

2. MODEL and SOLUTION APPROACH

In this section, we formally develop a dynamic programming model to reflect the situation we studied and to provide a decision analysis aid for the wholesaler under study. Hereafter we call the wholesaler as the decision maker to generalize the model.

Consider the decision maker needs to purchase a storable commodity from the upstream in a fixed quantity (d) and fixed price (β) at each period, and resell to the downstream later at a suitable market price. There is a fixed operating and administration cost H for each period, including rent, personnel, depreciation, etc.

We define period t as that there are t periods left. The interpretation of t is different from the conventional multiple period decision model to reflect our special concern on the objective of the business. A stochastic process $\{\tilde{s}_t, t = 1,2,3,\ldots\}$ represents the spot price at different time periods. We denote the corresponding realized value as s_t, and the density function as $f_{\tilde{s}_t}$. This can be considered as a backward stochastic process due to the definition of the time index.

Figure. 1 shows the decision sequence at each period. In the beginning of the period, the decision maker observes the spot price, she determines the selling quantity, and sells the stocks to the downstream. She then purchases the commodities from the upstream and stores in the storage. We assume that all trading transactions are paid in cash.

Denote the a_t be the level of trading activity, i.e., the quantity selling to the downstream, at period t. Let m_t and i_t be the cash and inventory level at the beginning of period t, respectively. m_t and i_t also represent the cash and inventory level at the end of period $t+1$ due to law of material convention and the definition of the time index. We have the following relationships:

$$m_{t-1} = m_t + a_t s_t - d\beta - H; \qquad (1)$$

$$i_{t-1} = i_t - a_t + d. \qquad (2)$$

A trading decision a_t is determined by not only the observed spot price s_t, but also constrained by many factors. a_t is bounded by the trading transition capacity $\bar{A}$, i.e., cannot exceed the maximum level of selling due to manpower and space, and the inventory i_t, i.e., must having something to sell. This requirement is stated as follows:

$$a_t \le min(\bar{A}, i_t). \qquad (3)$$

a_t is also constrained by the storage capacity C, i.e., selling is needed to clean out the space for incoming stock d due to the required purchasing at each period even when the spot price is not favorable. It is represented as follows:

$$a_t \ge \max\{0, i_t + d - C\}, \qquad (4)$$

where $i_t + d - C$ is the inventory space needs to sold for the new purchasing d. Further, (1) indicates a_t should be determined to ensure $m_{t-1} \ge 0$ due to the underlying objective of the firm. In summary, a_t depends on the status of i_t and m_t. Further, determining a_t should consider the current cash level and the future cash flows as the function (eq. (1)) of future spot prices, which are uncertain.

```
                    The Backward Induction Algorithm
[INPUT:] ℙ, T,  m_T,  i_T,  p_T
[OUTPUT:] Q(T,m_T,i_T, p_T)
one period left:
  𝕊_1 = listAllPossibleStates(1, m_T, i_T, p_T, ℙ)
  For s in 𝕊, Do:
      ℚ_1(s) = findOptSurvivalProbsForPeriodOne(s, ℙ)
  End for.
t periods left:
  For t = 2,…,T, Do:
  𝕊_t = listAllPossibleStates(t, m_T, i_T, p_T, ℙ )
      For s in 𝕊, Do:
          ℚ_t(s) = findOptSurvivalProbs(s, ℚ_{t-1}, ℙ)
      End for.
  End for.
```

Figure 2: Pseudocode of the backward induction

Survival is defined by the cash level. When the cash level is below 0, the firm files the bankruptcy, otherwise it survives. Denote the maximum probability to survive next t periods given the observed spot price and current inventory and cash levels as $Q(t, m_t, i_t, s_t)$. $Q(t, m_t, i_t, s_t)$ can be expressed in the following recursive form:

$$Q(t, m_t, i_t, s_t) = \max_{a_t \in \mathbb{K}} \sum_{s_{t-1}} f_{\tilde{s}_{t-1}}(s_{t-1}) Q(t-1, m_t + a_t s_t - d\beta - H, i_t - a_t + d, s_{t-1}) \quad t > 1 \tag{5}$$

$$Q(1, m_1, i_1, s_1) = \begin{cases} 0 & m_1 + a_1 s_1 - d\beta - H < 0 \\ 1 & otherwise \end{cases} \quad for \quad a_1 \in \mathbb{K} \tag{6}$$

$$Q(t, m_t, i_t, p_t) = 0 \quad if \quad m_t < 0 \quad t \geq 1 \tag{7}$$

$\mathbb{K}$ is the feasible region for a_t $t \geq 1$, which is defined by (3) and (4). Eq. (5) specifies the relation between t periods left to $t-1$ periods left. In the last period to operate, there are only two possible outcomes: survival or bankruptcy, i.e. survival probability can be either one or zero. Eq. (6) expresses this consideration. Eq. (7) reflects the fact that once the firm faces the bankruptcy, it cannot survive afterward.

We use backward induction to solve the problem. Figure. 2 shows the pseudocode of the backward induction. The first part associates with Eq. (6) reflecting the situation with one period left. The second part implements the recursive structure of Eq. (5). In both cases, we simply enumerates all possible states of the inventory and cash levels to search for an optimal solution, and Eq. (7) should be followed.

3. NUMERICAL RESULTS

This section illustrates the effectiveness of the proposed model and the optimal solutions provided. We implement the proposed dynamic program model to obtain the optimal survival probability conditioning on the status of observed spot price, inventory and cash levels as well as the target time frame to operate. Of course, the corresponding optimal trading decisions are also provided

We set the operational storage and transition capacities as $C = 250$ tons and $\bar{A} = 190$ tons/week. Each week, it is required to purchase $d = 20$ tons at unit price $\beta = 3$ NTD/kg, and the weekly operation cost is $H = 50{,}000$ NTD. The parameters are based on the data sources for the waste paper recycle industry in Taiwan so that it can properly reflect the reality. We collect the weekly waste paper wholesale price information from July 2005 to March 2016. We round the prices to integer, which ranges 2 to 7 NTD/kg, and summarize the frequency for the corresponding values as the empirical probability function of the spot market price. Table 1 summarizes the probabilities. We assume the spot prices distribute independently and identically following the probabilities.

We assume the selling quantities are in the unit of 10 tones, i.e., we can only sell 10, 20, 30 tones etc. This assumption narrows down the solution space to be searched, which is not necessary, but is also reasonable in real practice since it is the capacity of a medium-size truck. We present our results based on the current inventory is 120 tones, ~50% of the storage capacity, and the cash level is 500,000 NTD, which is roughly

Table 1: Empirical distribution of the spot price

Price (NTD/kg)	2	3	4	5	6	7
Probability	.0445	.4152	.2055	.1724	.0963	.0661

Table 2: Survival probabilities obtained by the policy

spot price (NTD/kg)	optimal survival probability	simulated proportion of survival	p-value*
2	.930	.934	.62
3	.930	.94	.22
4	.930	.939	.27
5	.941	.939	.79
6	.973	.978	.33
7	.989	.987	.54

*H_0: real survival probability equals the optimal probability computed.

prepared for operating one month and commonly observed in practice.

Table 2 shows the survival probability for operating 200 weeks (~4 years) based on 120 tones inventory and 500,000 cash on hand associated with 6 different price observed. Column 2 lists the maximum probabilities are obtained by the proposed model, and Column 3 is the probabilities based on simulation study.

We first validate the correctness of our model, the optimal survival probability is correct, by simulation. We test the null hypothesis that the true survival probability based on the trading policy equals to the value listed in Column 2. In simulation, we generate 200 spot prices, representing the prices for the next 200 weeks, independently and identically based on the empirical probability function shown in Table 2. At each week, when the price, generated in the beginning, is observed, we follow the trading policy suggested by the proposed model. After 200 weeks, the information of whether or not to survive is collected. We repeat the experiment 1000 times, and collect the proportion of survival as the simulated survival probability (Column 3). We conduct a statistical inference for the hypothesis using the simulated survival probability as the estimate, and the p-values are also listed in Table 2. The high p-values do not provide sufficient evidences to reject the null hypothesis, and we conclude that when following the policy proposed, the survival probability equals the optimal survival probability provided by the program.

In the simulation with the same parameter setting, we further apply three trading rules to see if the policy proposed in this paper is superior. The first policy is a single-period cash flow balance rule: at any week, the cash inflow from selling commodity should equal all cost expenses including purchasing and operating. This policy does not consider the possible variability of the spot price.

The first policy ignores the fact that it is reasonable to selling more when the price is very high. The second policy reflects this observation by setting the selling quantity linear to the spot price, i.e. the selling quantity is linear to the gap between the breakeven unit cost considering purchasing and operation costs and unit spot price observed. For example, suppose the breakeven unit cost is 4 NTD/kg and the maximum spot expected is 7 NTD/kg, we sell 1/3 of the inventory when 5 NTD/kg is observed or 2/3 of the inventory when 6 NTD/kg is observed.

The third policy is modified from the one proposed by Dreyfus (1957): sell all inventory when the market price is higher than the purchasing cost; otherwise buy as much as you can. We note that Dreyfus (1957) tries to maximize the expected profits, not the survival probability, for free purchasing i.e. a decision maker can decide to purchase any quantity (including nothing) from the upstream supply to benefit herself. To fit into our context, we define the threshold as the breakeven nit costs used in the second policy. And the decision maker sells all inventory when the market price is higher than the threshold; otherwise buy as much as she can.

Table 3 shows the comparison of four different policies. The values are the proportion of the survival in 1,000 simulation trials based on the corresponding trading rules. It is found that cash flow balance gives the worst performance (no survival among 1,000 trials), and our policy outperforms all other three with a significant gap. The comparison provides evidence to support the effectiveness of our optimal trading policy.

Table 3: Simulation results for different polices.

spot price (NTD/kg)	Simulated survival probability			
	Our policy	cash flow balance	linear	Dreyfus
2	.934	.000	.285	.405
3	.940	.000	.273	.396
4	.939	.000	.258	.310
5	.939	.000	.294	.472
6	.978	.000	.374	.578
7	.987	.000	.499	.677

4. CONCLUDING REMARKS

Motivated by a real case, we develop an optimal trading policy for a special 3-tier supply chain scenario. In the scenario, a wholesale need to purchase storable commodities from the upstream suppliers at a fixed quantity and a fixed price. The wholesale stores the commodities and resells to the downstream market when the spot market price is favorable. Different from the literature maximizing the expected profits, we focus on the sustainability of the operations, i.e. maximizing the survival probability. We model the problem as a stochastic dynamic program. The simulated case studies show the model can provide a policy with superior decision quality.

ACKNOWLEDGMENTS

The first and third authors are supported by the research grant from the Ministry of Science and Technology, Taiwan (MOST104-2628-E-009-010-MY3).

REFERENCES

Archibald, T.W., Thomas, L.C., Betts, J.M., Johnston, R.B. (2002). Should start-up companies be cautious? Inventory policies which maximize survival probabilities. *Management Science*, 48(9): 1161-1174A

Adeyemi, S. L., & Salami, A. O. (2010). Inventory management: A tool of optimizing resources in a manufacturing industry a case study of Coca-Cola bottling company. *Ilorin Plant. Journal of Social Sciences*, 23(2), 135-142.

Bellman, R. (1956). On the theory of dynamic programming: A warehouse problem. *Management Science*, 272-275.

Chao, X., Chen, J., & Wang, S. (2008). Dynamic inventory management with cash flow constraints. *Naval Research Logistics*, 55(8), 758-768.

Charnes, A., & Cooper, W. W. (1955). Generalizations of the Warehousing Model. *Journal of the Operational Research Society*, 6(4), 131-172.

Charnes, A., Dreze, J., Miller, M. (1966). Decision and horizon rules for stochastic planning problems: A linear example. *Econometrica*, 34:307-330.

Ciarallo, F. W., Akella, R., & Morton, T. E. (1994). A periodic review, production planning model with uncertain capacity and uncertain demand—optimality of extended myopic policies. *Management Science*, 40(3), 320-332.

Cruise, J., Flatley, L., Gibbens, R., & Zachary, S. (2014). Optimal control of storage incorporating market impact and with energy applications. *arXiv preprint arXiv*:1406.3653.

Cruise, J., & Zachary, S. (2015). The optimal control of storage for arbitrage and buffering, with energy applications. *arXiv preprint arXiv*:1509.05788.

Devalkar, S. K., Anupindi, R., & Sinha, A. (2011). Integrated optimization of procurement, processing, and trade of commodities. *Operations Research*, 59(6), 1369-1381.

Dreyfus, S.E. (1957). An analytic solution of the warehouse problem. *Management Science*, 4: 99-104.

Federgruen, A., & Heching, A. (1999). Combined pricing and inventory control under uncertainty. *Operations Research*, 47(3), 454-475.

Federgruen, A., & Zipkin, P. (1986a). An inventory model with limited production capacity and uncertain demands I. The average-cost criterion. *Mathematics of Operations Research*, 11(2), 193-207.

Federgruen, A., & Zipkin, P. (1986b). An inventory model with limited production capacity and uncertain demands II. The discounted-cost criterion. *Mathematics of Operations Research*, 11(2), 208-215.

Martinez-de-Albeniz, V., & Vendrell, J. M. (2008). A Capacitated Commodity Trading Model with Market Power. *SSRN Working Paper Series*.

Rempala, R., (1994). Optimal strategy in a trading problem with stochastic prices. *Springer Berlin Heidelberg*, Chapter System Modelling and Optimization, 560-566.

Secomandi, N., (2010). Optimal commodity trading with a capacitated storage asset. *Management Science*, 56(3): 449-467.

Thomas, L. C., Possani, E., & Archibald, T. W. (2003). How useful is commonality? Inventory and production decisions to maximize survival probability in start-ups. *IMA Journal of Management Mathematics*, 14(4), 305-32

Wang, Y., & Gerchak, Y. (1996). Periodic review production models with variable capacity, random yield, and uncertain demand. *Management Science*, 42(1), 130-137.

Xu, X., & Birge, J. R. (2005). Joint Production and Financing Decisions: Modeling and Analysis. *SSRN Working Paper Series*.

Zipkin, P. H. (2000). *Foundations of Inventory Management*, McGrawHill, Boston