Production Scheduling Problem of an Automobile Components Primer Painting Factory

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Abstract. In an automobile components primer painting factory, various automobile parts are attached to overhead hangers in a conveyor line and go through a series of coating processes. All the components go through the same processes and the conveyor line moves constantly so that the factory productivity depends on the hanger occupancy rate. An overhead hanger has a capacity limit and can hold a different number of components depending on the component type. If the hanger capacity is not fully filled, it causes productivity loss. To increase the productivity, the company mixes parts and attached them to hangers. A hanger sometimes carries two or more types of components and we call mixed hanger. Since mixed hangers require heavy workload, however, the company wants to reduce the number of mixed hangers. Thus, a good production schedule in the factory requires a minimum number of mixed hangers while it fills the hangers as much as possible. After the coating processes, components are wrapped at the packaging station. Since each component requires different packaging time and materials, the packaging process should be well balanced by a proper components sequence to prevent bottle neck effect. We develop a problem specific mathematical formulation. The mathematical model can provide meaningful lower bounds for real-world problems. Also, we propose a heuristic based solution approach for the problem.

Keywords: single machine scheduling, MIP, bin-packing, automobile components primer painting, heuristic algorithms

1. INTRODUCTION

In the automobile manufacturing process, various bodyworks need anti-corrosion coating. Electrodeposition coating is one of the anti-corrosion painting methods using a continuous hanger line. In this paper, we solve a single machine scheduling problem for an automobile components primer painting factory. In this factory, thousands of various automobile parts are delivered and each of them should be coated within a given due date. The series of coating processes is followed by a packaging step. All the components go through the same processes and the conveyor line moves at constant speed so that productivity of the line depends on the hanger occupancy rate.

Each hanger unit has the capacity limit which varies depending on the component type that it holds. If the hanger units are not fully filled, the total production cost of the company increases and completion time is delayed. If the number of ordered components is not exactly the multiples of the hanger capacity, some of the order components may be mixed with other types of components and be hung to an overhead hanger together.

At the packaging step each component is packed manually according to its specification. The packing specification can be categorized into three groups based on its workload. For some export components, packing workload is much higher than others. Due to the continuity of the hanger line, the workload of within a certain length of consecutive hangers for packaging workers should be well balanced among whole line. If not, the workers at the packaging step will have heavy fatigue.

Thus, there are three objective considerations in the scheduling problem: minimizing the productivity loss, minimizing the maximum workload of packing workers and minimizing the number of mixed hangers.

Considering the painting line as a machine, the scheduling problem is a single machine scheduling problem. Du and Leung (1990) showed that the single machine scheduling problem with an objective function of minimizing total tardiness is NP-hard by reducing the even-odd partitioning problem to the problem. In addition, they proved that the minimizing total tardiness and minimizing early work are equivalent under particular conditions. Using this result, Ben-Yehoshua and Mosheiov (2016) showed that a single machine scheduling problem minimizing early work is also NP-hard and proposed a dynamic programming algorithm for the problem.

On the other hand, various mixed integer programming (MIP) models for a single machine scheduling were thoroughly summarized by Wolsey (1997). He considered a general single machine for multi-item production. Production cost, setup cost, start-up cost, changeover cost and stock storage cost are considered. He also considered a sequence independent changeover model and a sequence dependent changeover model.

Nip et al. (2016) summarized various types of machine scheduling problems under linear constraints. When the number of machine is two or more and constraints are arbitrary, the problems are NP-hard. For the classical single machine case, he mentioned that it can be solved in polynomial time by the interior point method.

Although our problem belongs to the category of single machine scheduling problems, the consideration of hangers with mixing products and workload balancing at the packaging stage makes our problem unique. To the best of our knowledge, no previous studies addressed this kind of single machine scheduling problem.

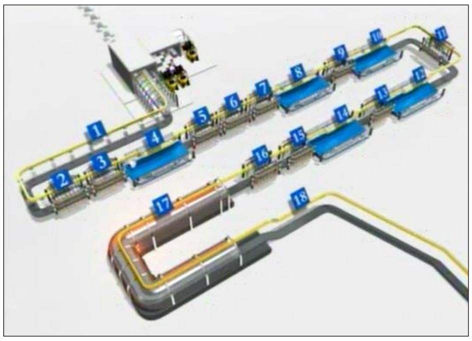

Figure 1: Aerial view of painting factory

This paper is organized as follows. The problem is described more formally in the next section. The mathematical model is presented in Section 3. A heuristic solution method is proposed and its computational results are presented in Section 4. Conclusions are followed in Sections 5.

2. PROBLEM DESCRIPTION

Figure 1 shows an aerial view of the automobile component painting factory. The numbers in the figure indicates the coating steps that component items pass through after they are attached to hangers. Finally, the workers at packaging station carry the items and pack them according to the product specification.

When it comes to the end of work in a day, the conveyor line should be stopped and the items already hung on the hanger will wait till the next day. When the line stops, therefore, the hangers at some of steps that involve the liquid painting process should be empty. In other words, those hangers should not have any items at the end of a day. In addition, the number of packing workers decreases so hard packing items should be avoided in the lunch time. With the consideration of above constraints, we put serial numbers to the hangers and set the eligibility of hangers for items. Note that a physically same hanger has multiple numbers because the hangers circulate the factory more than once in a day.

The input data form, objective functions, and output data form of the problem are explained below.

Table 1: Input data form

Input order	Delivery date	Ordering company	Type code	Type name	Group	Input amount	Hanger capacity	Packing specification
1	20160425 8:00	High-tech Inc.	770041R361	Door	DH	40	6	AF
2	20160425 8:34	East-metal Co.	66402W000	Hood	DH	20	5	BW
3	20160425 8:59	Auto-tech Inc.	760034F010	Door	EK	34	5	AF

Table 2: Scheduled output data form

Output order	Amount	Date	Start hanger	Finish hanger
1	20	2016-04-25 11:00	106	109
2	40	2016-04-25 11:05	110	116
3	34	2016-04-25 11:12	116	123

2.1 Input data form

Input data contains the list of information about types of items. For instance, input items can be grouped as domestic or export. More specifically there are 66 types of item types and each type has hanger capacity, ordering company, painting and packing specification. Table 1 shows a sample input data form.

2.2 Objectives

As mentioned above, the productivity loss is the first measure to minimize in scheduling. The hanger capacity means the amount of an item type that can be hung on a hanger: if the capacity is 5, a hanger can contain up to 5 of that item. If the hanger is not fully utilized, there is remaining capacity and it means productivity loss.

If the number of input amount of an item is not exactly the multiples of the hanger capacity of the item, some of the items should be mixed with other types of items to minimize the productivity loss. At the same time, however, we should consider the workload of packing workers that caused by mixed items. To minimize the workload, the same packing specification and same type items should be mixed. If that is not possible, the same packing specification items can be mixed and then the same types but not the same packing specification items can be mixed.

Finally, we want to balance the packing workload. Each packing specification is categorized by three levels according to its workload. If items that have heavy packing workload are assigned continuously, packing workload will greatly increase. In order to measure the packing workload for a period of time, we consider ten consecutive hangers together.

2.3 Output data form

Table 2 represents the desired solution output format. The scheduling result contains the information about when the item is hung, the amount of the item and its start and finish hanger number and times.

3. MATHEMATICAL FORMULATION

We present our MIP formulation of this problem. Let *i* be input order of item (1, …, *I*) and h be the hanger number (1, …, *H*). If the worktime is 9 hours, *H* is 600.

The parameters of the problem are as follows: a_i is the input amount of item *i;* c_i is the capacity proportion of an item *i* on a hanger (1/hanger capacity 0~1, real); p_i is the packing workload level of item *i* (1~3, integer); v_{ij} is the weight value considered if item *i* and *j* are mixed on a same hanger (1 if same type and same packing, 2 if just same packing, 3 if just same type and 100 if neither are the same); e_{ih} is 1 if item *i* can be hung on hanger *h*, otherwise 0; w_1, w_2, w_3 are the weight coefficients.

Decision variables are as follows:

$MaxP$: the maximum packing workload among 10 consecutive hangers.

m_{ij}: 1, if item i and j are mixed; 0, otherwise

x_{ih}: the amount of item *i* on hanger *h*

y_{ih}: 1, if item *i* is hung on hanger *h;* 0, otherwise

s_i: start hanger number of item *i*

f_i: last hanger number of item *i*

The MIP model is as follows:

$$\min w_1 * MaxP + w_2 \sum_i \sum_{j>i} m_{ij} * v_{ij} + w_3 \sum_h \left(1 - \sum_i x_{ih} * c_i\right) \quad (1)$$

s.t.

$$y_{ih} + y_{jh} - 1 \le m_{ij} \quad \forall\ j < i,\ \forall h \quad (2)$$

$$y_{ih} \le e_{ih} \quad \forall i, \forall h \tag{3}$$

$$\sum_i x_{ih} * c_i \le 1 \ \ \forall h \tag{4}$$

$$x_{ih} * c_i \le y_{ih} \ \ \forall i, \forall h \tag{5}$$

$$\sum_h x_{ih} = a_i \ \ \forall i \tag{6}$$

$$\sum_h y_{ih} \le c_i * a_i + 2 \ \ \forall i \tag{7}$$

$$MaxP \ge \ \sum_h^{h+10} \sum_i p_i * x_{ih} \ \ \forall i, h \le H - 10 \tag{8}$$

$$\sum_h y_{ih} \ge 1 \ \ \forall i \tag{9}$$

$$s_i \le H * (1 - y_{ih}) + h * y_{ih} \ \ \forall i, \forall h \tag{10}$$

$$f_i \ge h * y_{ih} \ \ \forall i, \forall h \tag{11}$$

$$f_i - s_i \le \sum_h y_{ih} - 1 \ \ \forall i \tag{12}$$

$$x_{ih} \in Z^+ \cup \{0\}, \ \ m_{ij} \in R^+ \cup \{0\}, \ \ y_{ih} \in \{0,1\}, \tag{13}$$

$$MaxP \in R^+, s_i \in R^+, f_i \in R^+ \ \ \forall j < i, \forall h$$

The objective function (1) minimizes the sum of maximum workload, mixing cost and productivity loss. We set weight coefficients of the objective components to 1, 10, and 100, respectively. On account of *MaxP* which is the maximum value of the sum of 10 consecutive hangers' workloads, the mix cost weight factor was chosen as 10 times larger. Since the productivity loss is calculated in hanger capacity ratio, w_3 is chosen to 100 to offset the scale factor. Constraints (2) ensure that m_{ijh} is 1 if item *i* and item *j* are hung on the same hanger *h*. Constraints (3) keep the eligibility condition of the hangers and item type. Constraints (4) indicate that the amount of item *i* cannot exceed the hanger capacity. Constraints (5) ensure that y_{ih} is 1 if item *i* is hung on hanger *h*. Constraints (6) indicate that the total amount of scheduled item cannot be more than the input amount. Constraints (7) limit the minimum needed hangers. Constraints (8) set *MaxP* to the maximum packing workload among 10 consecutive hangers. Constraints (9) reflect at least one hanger should be assigned for item *i*. Constraints (10) get the start hanger number of the item *i* and constraints (11) get the finish hanger number. Constraints (12) ensure that each item should be processed continuously from the start hanger to the finish hanger, i.e., each item should be processed in a batch.

4. HEURISTIC ALGORITHM AND COMPUTATIONAL RESULTS

First of all, to check the practicality of the proposed MIP model, we experimented with different data sizes. The result is on Figure 2. Since the problem has 600 hangers in a day, the MIP model as it is cannot be used for the real-world cases. Thus, we developed heuristic algorithms.

To minimize the mix cost, the list of input items is sorted by remaining amount ratio which is calculated by

$$r_i = (a_i \ mod \ (1/c_i)) * c_i \tag{14}$$

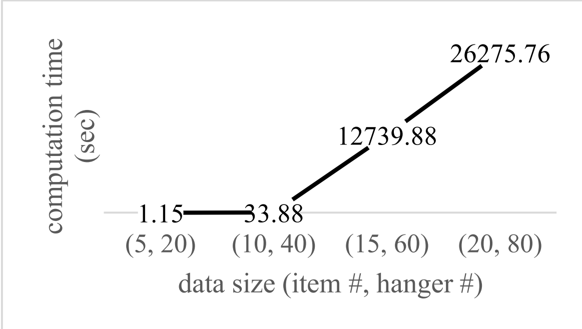

Figure 2: Computation time for different data sizes

```
Best Fit Decreasing (BFD) algorithm
sorted list of items I and empty hanger list H
  for i in I
     for h in H
        if (r_i < k_h) then
           if (f(i, h) < f̂) then
              f̂ := f(i, h)
           end if
        end if
     end for
end for
```

Figure 3 BFD pseudo code

```
2-Opt algorithm
sorted list of items I and empty hanger list H
  for i in I
     for j from i+1 to H
        v` := swap(i, j) in v
        if (f(v`) < f̂) then
           v := v`
        end if
     end for
  end for
```

Figure 4: 2-Opt pseudo code

Here we define k_h as the remaining capacity of hanger h. We can apply *best-fit decreasing* (BFD) algorithm of Figure 3. The input list is sorted by (14) in decreasing order and then each item is assigned to the hanger that has the minimum remaining capacity ratio among those whose remaining capacity is large enough to accommodate the item.

Note that $f(.)$ is the cost function of the solution calculating capacity loss and packaging cost within a hanger according to function (1). Then $\hat{f}$ is the current best cost value. After searching the minimum cost fit for whole components, it generates sort of job sequence. According to this information, we can make a schedule.

Secondly, we suggest 2-opt algorithm as shown in Figure 4. Here we denote v as input item sequence and $v`$ as job sequence after swap the orders of two items. If the cost of swapped input sequence is lesser than that of original one, the input item sequence will be saved as swapped.

In order to test the MIP model and the heuristic approach, we made a small data set shown in Table 3. The data set requires mixed loading, i.e. input amount is not multiple of capacity so that at least two items ought to be mixed.

Table 3: Small input data

Input order	Type code	Amount	Capacity	Package level
1	770031R361	17	6	1
2	657102H010	35	8	2
3	76004B8020	19	5	1
4	657103A100	38	10	3
5	664002W000	26	6	1

The MIP model was solved using CPLEX (12.6) and its result is shown in Table 4. The heuristic algorithms were implemented in C++ language and their results are shown in Tables 5 and 6.

Table 7 compares the experimental results. The weight factors are introduced beforehand and the values of this table is calculated by the objective function (1).

Table 4: MIP result for small data

Input Order \ Hanger number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1								1	6	6	4									
2																3	8	8	8	8
3					5	5	5	4												
4	10	10	10	8																
5											2	6	6	6	6					

Table 5: BFD heuristic result for small data

Input Order \ Hanger number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	6	6	5																	
2											3	8	8	8	8					
3			1	10	10	10	7													
4							1	5	5	5	3									
5																6	6	6	6	2

Table 6: 2-Opt heuristic result for small data

Input Order \ Hanger number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	6	6	5																	
2											3	8	8	8	8					
3							2	10	10	10	6									
4				5	5	5	4													
5																6	6	6	6	2

Table 7 compares the experimental results. The weight

factors are introduced beforehand and the values of this table is calculated by the objective function (1).

Table 7: Cost comparison

Cost	MIP	Heuristic	
		BFD	2-Opt
MaxP	146	91	99
Mix cost	20	90	60
Capacity loss	85.8	107	107
Total	251.8	288	266

The total cost of the MIP is smaller than that of the heuristic algorithm. As shown in Tables 4 and 5, the MIP solution has two mixed hangers (hanger number 8 and 11) whereas BFD solution has three (hanger numbers 3, 7 and 11) and 2-Opt solution has two (hanger numbers 7 and 11) with larger cost than MIP. 2-Opt outperforms the BFD.

To apply the heuristic algorithm to practical problem size, we tested it with an input data of 86 item types and total number of items is 7419. For the practice size problem, the hanger eligibility for breaks and end of day consideration was additionally considered as cost. The cost coefficients for three components are maintained the same with previous test and the eligibility cost coefficient was chosen as 10. For practical size problems, the eligibility constraints (3) can make the solution infeasible. Due to that reason, we consider the eligibility constraints as a part of objective function as (15)

$$\min w_4 * y_{ih} * (1 - e_{ih}) \qquad (15)$$

The cost coefficient w_4 is chosen as 10. The summary of result is shown in Table 8.

Table 8: Result of 2-Opt implementation

Minimum hanger	Mix not allowed	522	Cost	MaxP	2000
	Mix allowed	491		Mix cost	560
Used hanger		495		Capacity loss	436
				Eligibility	310

5. CONCLUSION

In this research, we developed a MIP model and two heuristic algorithms for a single line automobile components basic painting factory. While minimizing the productivity loss is similar to other problems, the consideration of hangers with mixing products and workload balancing at the packaging stage makes our problem unique.

The MIP model cannot be used for real-size problems due to long computation so we developed two heuristic algorithms, BFD and 2-Opt. The latter outperforms the former.

This research demands further studies. We think the problem is NP-hard but it is not proved formally. More sophisticated algorithms using metaheuristic approaches can be developed.

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