Cost Analysis of Two Dimensional Warranties Considering Lemon Laws

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Abstract. In this paper, we consider a repairable product sold with a two-dimensional free replacement warranty (FRW) and protected by lemon laws during the warranty period. The product is presumed a lemon if either (i) the car has been returned to the dealer four times to have the same problem fixed, but the dealer was unable to repair the problem satisfactorily, or (ii) the car has been out of service more than 30 days due to one or more defects. Two cases are considered studied – (i) refund and (ii) replacement cases. We gives numerical examples to illustrate the expected warranty servicing cost.

Keywords: lemon laws, two dimensional warranty, refund, replacement, expected warranty cost

1. INTRODUCTION

In automotive industry, all new motor vehicles are sold with a two dimensional warranty. For instance, a new car is warranted for three years or 100,000 km, whichever comes first. This warranty only requires manufacturers to repair a failed vehicle that occurs during the warranty period. The manufacturers have no obligation to replace a failed vehicle or refund the purchase price of a failed vehicle. If a consumer has an unreliable vehicle that fails several times during the warranty, then the vehicle is out of service repeatedly for repairs, and this in turn result in a significant inconvenience to the consumer.

With lemon laws, consumers are protected against a defective vehicle that does not conform to standards of quality and performance. The lemon laws provide the consumers the right to have a new product or a full refund when the vehicle is declared to be a lemon. The lemon laws have been enacted since 1982 in state of Connecticut US and in the next five year time, all states and the District of Columbia had enacted the lemon laws protecting new-car buyers from defective automobiles. Since then the consumers are allowed to return the defective vehicle (which is presumed a lemon) to get refund or replacement with a new one. Before 1982 the consumers of a new vehicle had bad experiences with frustration, delays, expense, and uncertainty to get the failed car fixed (Kegley and Hiller (1986)). Nowadays, the adoption of lemon laws spread out to outside US – such as Canada, Europe, Australia, Singapore to name a few.

"An automobile turns to be a lemon if either (i) the car has been returned to the dealer four times to have the same problem fixed, but the dealer was unable to repair the problem satisfactorily, or (ii) the car has been out of service more than 30 days due to one or more defects." However, not all component defects are classified as a major defect and result in a lemon. Only failures of components or sub-systems (such as gearbox, transmission, steering or braking systems, etc) that cause a safety-related problem are considered.

The lemon laws, in one side, provide protection to the consumers against repeat failures occurring during the warranty period. But, in the other side, the lemon laws require the car manufacturer to replace a defective car or refund its purchase price should the car is declared a lemon. As a results, cars protected by the lemon laws may result in a significant additional cost to the manufacturer for rectifying a defective car, and this in turn affects the manufacturer's profits.

Study of lemon law warranties has received less attention in literature. Smithson and Thomas (1988) studied automobile lemon laws to estimate the value of lemon protection to consumers. Centner and Wetzstein (1995) examined tractor lemon laws and make comparison between automobile and tractor lemon laws. They used a principal agent to model the economic efficiency of lemon laws. In this paper, we study automobile lemon laws from the manufacture's view point and obtain the expected warranty servicing cost. From the manufacturer point of view, to obtain an accurate estimate of the servicing warranty cost for a car sold with warranty and protected by lemon laws, is an issue of great interest to manufacturers. As lemon laws give more burden (additional cost) to the manufacturer for servicing the lemon-law warranty. Iskandar and Husniah (2016) studied cost analysis of lemon law warranties for one dimensional warranty case. In this paper we extend it to the case of two dimensional warranties.

The paper is organized as follows. In Section 2 we define a lemon-law warranties in a two dimensional case and give the details of the model formulation. Refund and replacement cases of a lemon-law warranty have been considered. Section 3 presents a numerical example for illustrating the estimate of the servicing warranty cost for the two lemon-law warranty cases. Finally, we conclude with a brief discussion of topics for future research in Section 4.

2. MODEL FORMULATION

2.1 Warranty Policy

We consider a repairable product (e.g. Automobiles, Trucks) sold with a two-dimensional free replacement warranty (FRW). The warranty has two limits (i.e. age and usage limits) which form a region Ω in a two-dimensional plane. There are several different warranty regions considered (See Murthy and Wilson [8]). In this paper we confine to a rectangle shape and hence the warranty expires when it reaches an age *W* or the total usage exceeds a level *U*. Under the FRW policy, should a failure occur with age at failure less than *W* and usage at failure less than *U*, the manufacturer rectifies the failure at no cost to the buyer. Since the product is repairable so the rectification of a failed item can be achieved through either repair or replacement.

2.2 Lemon Laws

We consider that a product is protected by lemon laws which are enforceable *during the warranty.* The product turns to be a lemon if either of two conditions are met: (i) the car has been returned to the dealer *k* times to have the same problem fixed or (ii) the car has been out of service more than τ unit time (e.g., 30 days) due to one or more defects. Two cases are considered – namely Cases 1 and 2.

- Case 1: Look at the case where the lemon law only deals with number of failures and not the time out of action
- Case 2: Take into account the number of failures as well as the downtime.

We assume that (i) the returned "lemon" is scrapped so there is no resale value to the manufacturer, and (ii) repair times are negligible.

2.3 Failure modelling

2.3.1 Approaches to modelling failures

For the cost analysis of two-dimensional warranty policies, we consider item failures as random points occurring over the warranty region. Three approaches can be used to modelling such failures.

Approach 1:

In this approach, the time to first failure is modelled by a bivariate distribution function $F(t,u)$. Murthy et al [14] have used this approach for the cost analysis of two-dimensional warranty policies.

Approach 2:

This approach assumes that the two measurement scales (age and usage) are combined to provide a single composite scale Z ($z = at + bu$ is an illustrative example) and failures are modelled as a counting process using this composite scale (see Kordonsky and Gertsbakh (1993)).

Approach 3:

This approach assumes that the usage rate *R* (e.g. distance travelled per unit time for an automobile) varies from customer to customer but is constant for a given customer. *R* is a random variable that can be modelled using a density function $g(r), 0 \le r < \infty$. Conditional on $R = r$, the total usage u at age x is given by $u = rx$. For a given usage rate r the conditional hazard (failure rate) function for the time to first failure is given by $h(x|r) \ge 0$ which is a non-decreasing function of the item age x and r. Then, failures over time are modelled by a one dimensional counting process. If failed items are replaced by new ones, then this counting process is a renewal process associated with the conditional distribution $F(x|r)$ which can be derived from $h(x|r)$ (See Murthy and Wilson (1991), Iskandar et al (2005)). In this paper, we use the third approach.

2.3.2 Modelling first failure

Let $X_{1|r}$ denote the time to first failure conditional on $R = r$. The distribution function of $X_{1|r}$ is given by

$$F\left(x|r\right) = 1 - \exp\left\{-\int_0^x \lambda\left(t|r\right)dt\right\} \qquad (1)$$

Let $N(t|r)$ denote the number of failures over $[0,t)$ conditional on $R = r$. If every failure is restored by minimal repair then the counting process is characterised by a conditional intensity function $\lambda(x|r)$ which is a non-decreasing function of x and

r, then $\lambda(x|r) = h(x|r)$. We assume a relationship of the form (as in Iskandar et al [4]).

$$\lambda(t|r) = \theta_0 + \theta_1 r + \theta_2 t + \theta_3 u = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)t \qquad (2)$$

Where the total usage u at age t is $u = rt$ and $\theta_i \geq 0, i = 0,...,3$.

2.4 Warranty servicing cost

Case 1: **Lemon Law Warranty**

Conditional on $R = r$, the item is declared a 'lemon' if it fails K times in $[0,\Psi)$ where Ψ is time instant of the warranty expiry (see Figure1) and it is given by $\Psi = Min\{W,\Gamma\}$, where $\Gamma = U/r$ If the product is a "lemon", then the manufacturer has to refund the sale price to the customer or replace the failed item with a new item together with a new warranty policy at the time of the k^{th} failure. As a results, for Case 1, we have two rectification actions - i.e. refund or replace the failed item.

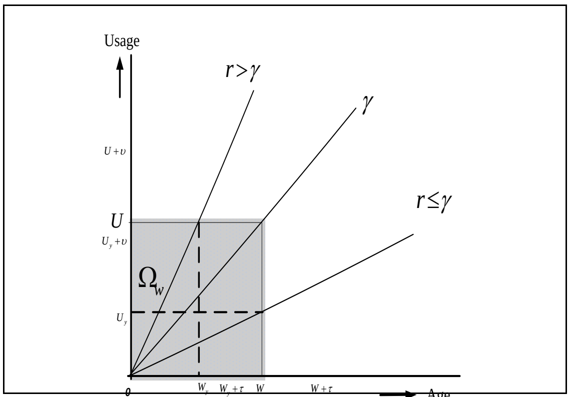

Figure 1: Warranty region.

Notations:

W	:Warranty period
C_p	:Item sale price
C_f	:Item manufacturing cost
C_m	:Average repair cost
C_c	:Collateral charges (incurred by the manufacturer if the item is declared a lemon under warranty)
$C(W;k\|r)$	:The cost to the manufacturer to service the warranty
$X_{1\|r}$	:The time to failure of the new item
$F(x\|r)$	:Distribution function and density function
$f(x\|r)$	for $X_{1\|r}$
$\lambda(x\|r)$, $\Lambda(x\|r)$	:Hazard function and cumulative hazard function associated with $F(x\|r)$
$X_{j\|r}$	:The operating time to the next item failure after $(j-1)$ repairs have been performed, $j \geq 1$.
$S_n = \sum_{j=1}^{n} X_{j\|r}$	:The time of the n^{th} failure, $n \geq 1$, has distribution function $F_n(x\|r)$, density function $f_n(x\|r)$ and survivor function $\bar{F}_n(x\|r)$
$N(t\|r)$	:The number of failures occuring in the interval $(0,t]$
$\Phi(z)$	:The standard normal distribution function

Case 1(i): [Refund]

Customer starts to use the item at time $t = 0$, time instant of sale. If the i-th failure ($i<k$) under warranty is minimally repaired by the manufacturer at an average cost C_m. In this, the item is declared a lemon under warranty if the k^{th} failure occurs before Ψ. If the item is a "lemon", then the manufacturer has to refund the total sale price to the customer should a critical component fail k times under warranty. Let $X_{1|r}$ be the time to failure of the new item for a given usage rate $R = r$. $X_{1|r}$ has distribution function $F_n(x|r)$, , density function $f(x|r)$, hazard function $\lambda(x|r)$ and cumulative hazard function $\Lambda(x|r)$. Let $X_{1|r}$ be the operating time to the next item failure after $(j-1)$ repairs have been performed,

$j \geq 1$. Define, $S_n = \sum_{j=1}^{n} X_{j|r}$, the time of the n^{th} failure, $n \geq 1$, has distribution function $F_n(x|r)$, density function $f_n(x|r)$ and survivor function $\bar{F}_n(x|r)$, . If $N(t|r)$ is the number of failures that occur in the interval $(0,t]$ for a given $R=r$, and each failure is fixed by a minimal repair, then $N(t|r)$ is a Nonhomogeneous Poisson process with intensity function $\lambda(t|r)$. Conditional on $R=r$, the probability of *n* successive minimal repairs in $(0,t]$ is given by $\Pr\{N(t|r)=n\} = F_n(t|r) - F_{n+1}(t|r)$ where $P(S_{n|r} \leq t|r) = F_n(t|r) = 1 - \sum_{i=0}^{n-1} \frac{e^{-\Lambda(t|r)}\Lambda(t|r)^i}{i!}$ (See Iskandar, et.al. (2012)). A lemon is declared if the k^{th} failure occurs during the warranty or if (i) $S_{k|r} \leq W,\ r \leq r_1$ or (ii) $S_{k|r} \leq \Gamma,\ r > r_1$.

Warranty servicing cost:

As the time instant of the warranty expiry dependent on the usage rate of a vehicle, then we need to look at two different cases – Case (a) $r \leq r_1$ and Case (b) $r > r_1$.

Case (a): $r \leq r_1$

Here the warranty ends at *W*. Let $C_1(W;k|r)$ be the cost to the manufacturer to service the warranty for $R=r$, for the case of refund. The warranty servicing cost $C_1(W;k|r)$ is $C_m N(W|r)$ if $S_{k|r} > W\ (\Leftrightarrow N(W|r) < k)$ and $(k-1)C_m + C_p + C_c$ if $S_{k|r} \leq W.$, conditional on $R=r$, the expected warranty servicing cost is

$$\begin{aligned} E\left[C_1(W;k|r)\right] &= \sum_{n=0}^{k-1} nC_m P\{N(W|r)=n\} + [(k-1)C_m + C_p + C_c]P\{S_{k|r} \leq W\} \\ &= C_m \sum_{n=1}^{k-1} n\left[F_n(W|r) - F_{n+1}(W|r)\right] + [(k-1)C_m + C_p + C_c]F_k(W|r) \\ &= C_m \sum_{n=1}^{k-1} F_n(W|r) + (C_p + C_c)F_k(W|r) \end{aligned} \tag{3}$$

The second moment of the warranty servicing cost is

$$\begin{aligned} E\left[C_1^2(W;k|r)\right] &= \sum_{n=1}^{k-1} (nC_m)^2 \left[F_n(W|r) - F_{n+1}(W|r)\right] \\ &\quad + [(k-1)C_m + C_p + C_c]^2 F_k(W|r) \\ &= C_m^2 \sum_{n=1}^{k-1} (2n-1) F_n(W|r) + (C_p + C_c)\left[C_p + C_c + 2(k-1)C_m\right] F_k(W|r) \end{aligned} \tag{4}$$

The variance of the warranty servicing cost is

$$Var\left[C_1(W;k|r)\right] = E\left[C_1(W;k|r)^2\right] - \left(E\left[C_1(W;k|r)\right]\right)^2 \tag{5}$$

Let C_L be the warranty servicing cost limit. If $C_1(W;k|r)$ is normally distributed, then the probability that $C_1(W;k|r)$ will exceed some defined limit, C_L is given by

$$\Pr\{C_1(W;k) > C_L|r\} = 1 - \Phi\left(\frac{C_L - E\left[C_1(W;k|r)\right]}{\sqrt{Var\left[C_1(W;k|r)\right]}}\right), \tag{6}$$

where $\Phi(z)$ is the standard normal distribution function.

Case (b): $r > r_1$

In this, the warranty ceases at Γ. $E\left[C_1(\Gamma;k|r)\right]$, $E\left[C_1^2(\Gamma;k|r)\right]$, $Var\left[C_1(\Gamma;k|r)\right]$ and $\Pr\{C_1(\Gamma;k) > C_L|r\}$ are given in (3), (4), (5) and (6), respectively, replacing W with Γ.

Finally, the removing the conditioning of $R=r$, we obtain the first and second moments, and variance of warranty servicing cost for the whole population of usage rates are given by

$$E\left[C_1(\Omega;k)\right]=\int_0^{r_1}E\left[C_1(W;k|r)\right]dG(r)+\int_{r_1}^{\infty}E\left[C_1(\Gamma;k|r)\right]dG(r) \tag{7}$$

$$E\left[C_1^2(\Omega;k)\right]=\int_0^{r_1}E\left[C_1^2(W;k|r)\right]dG(r)+\int_{r_1}^{\infty}E\left[C_1^2(\Gamma;k|r)\right]dG(r) \tag{8}$$

$$Var\left[C_1(\Omega;k|r)\right]=\int_0^{r_1}Var\left[C_1(W;k|r)\right]dG(r)+\int_{r_1}^{\infty}Var\left[C_1(\Gamma;k|r)\right]dG(r) \tag{9}$$

Hence,

$$\Pr\left\{C_1(\Omega)>C_L|r\right\}=\int_0^{r_1}\Pr\left\{C_1(W;k)>C_L|r\right\}dG(r)+\int_{r_1}^{\infty}\Pr\left\{C_1(\Gamma;k)>C_L|r\right\}dG(r). \tag{10}$$

Case 1(ii):[Replacement with a new warranty]

If the item turns to be a lemon under warranty (i.e. $S_{k|r}\le W,\ r\le r_1$ or (ii) $S_{k|r}\le\Gamma,\ r>r_1$), then the manufacturer has to replace the failed item with a new item with a new warranty, and hence have a renewing warranty.

As in the case of refund, here, we need to consider two cases – Case (a) $r\le r_1$ and Case (b) $r>r_1$.

<u>Case (a):</u> $r\le r_1$

Let $C_2(W;k|r)$ be the cost to the manufacturer to service the warranty for the case of replacement, conditional on $R=r$. We obtain $E\left[C_2(W;k|r)\right]$ by conditioning on $S_{k|r}$, the time of the k^{th} failure.

$$E\left[C_2(W;k)\middle|R=r,S_{k|r}=s\right]=\begin{cases}E\left[N(W|r)\middle|N(W|r)<k\right]C_m & if\ s>W\\(k-1)C_m+C_f+C_c+E\left[C_2(W;k)\right] & if\ s\le W\end{cases} \tag{11}$$

Removing the conditioning of $S_{k|r}=s$, we have

$$\begin{aligned}E\left[C_2(W;k|r)\right]&=\sum_{n=0}^{k-1}nC_mP\{N(W|r)=n\}\\&+\left[(k-1)C_m+C_f+C_c+E\left[C_2(W;k|r)\right]\right]F_k(W|r)\\&=C_r\sum_{n=1}^{k-1}F_n(W|r)+(C_f+C_c)F_k(W|r)\\&+E\left[C_2(W;k|r)\right]F_k(W|r),\end{aligned} \tag{12}$$

then

$$E\left[C_2(W;k|r)\right]=\frac{C_m\sum_{n=1}^{k-1}F_n(W|r)+(C_f+C_c)F_k(W|r)}{\bar{F}_k(W|r)}. \tag{13}$$

Note: $E\left[C_2(W;1|r)\right]=(C_f+C_c)\left(\frac{F(W|r)}{\bar{F}(W|r)}\right)$ is the expected servicing cost to the manufacturer of a renewing warranty when the item fails during the warranty period and it is replaced by a new one.

Using a similar conditioning argument,

$$E\left[C_2^2(W;k)|R=r,S_k=s\right]=\begin{cases}E\left[(N(W|r)|N(W|r)<k)^2\right]C_m^2 & if\ s>W\\E\left[\left((k-1)C_m+C_f+C_c+C_2(W;k|r)\right)^2\right] & if\ s\le W\end{cases} \tag{14}$$

Removing the conditioning of $S_{k|r}=s$, we have

$$E\left[C_2^2\left(W;k|r\right)\right]=$$

$$=\Omega\frac{1}{\bar{F}_k\left(W\right)^2}\left\{\begin{array}{l}C_m^2\bar{F}_k\left(W|r\right)\sum_{n=1}^{k-1}(2n-1)F_n\left(W|r\right)\\+2C_m\begin{bmatrix}(k-1)C_m\\+C_f+C_c\end{bmatrix}F_k\left(W|r\right)\sum_{n=1}^{k-1}F_n\left(W|r\right)\\+\left(C_f+C_c\right)\begin{bmatrix}C_f\\+C_c+2(k-1)C_m\end{bmatrix}F_k\left(W|r\right)\\+\left(C_f+C_c\right)^2F_k\left(W|r\right)^2\end{array}\right\}$$

(15)

Variance of $C_2\left(W;k|r\right)$ is given by

$$Var\left[C_2\left(W;k|r\right)\right]=E\left[C_2\left(W;k|r\right)^2\right]-\left(E\left[C_2\left(W;k|r\right)\right]\right)^2$$

(14)

As a result, the probability that $C_2\left(W;k|r\right)$ is greater than a pre specified limit, C_L, $\Pr\left\{C_2\left(W;k\right)>C_L|r\right\}$ is given by (6) with $E\left[C_2\left(W;k|r\right)\right]$ and $Var\left[C_2\left(W;k|r\right)\right]$ given in (9) and (12).

<u>Case (b):</u> $r>r_1$

In this, the warranty ceases at Γ, and $E\left[C_2(\Gamma;k|r)\right]$, $E\left[C_2^2(\Gamma;k|r)\right]$, $Var\left[C_2\left(\Gamma;k|r\right)\right]$ and $\Pr\left\{C_2\left(\Gamma;k\right)>C_L|r\right\}$ are given in (11), (12), (14), and (6) respectively, replacing W with Γ.

As a result, using a similar approach as in case (a), for the whole population, we have $E\left[C_2(\Omega;k)\right]$, $E\left[C_2^2(\Omega;k)\right]$, $E\left[C_2^2(\Omega;k)\right]$ and $\Pr\left\{C_2\left(\Omega\right)>C_L|r\right\}$ given by (5), (6), (7), and (8) respectively, replacing W with Γ.

Case 2: Lemon Law Warranty

An item is declared a 'lemon' if it fails k times during the warranty period or if the total time taken to repair the item (total downtime) under warranty exceeds τ. If the lemon law is invoked by one of these two events, the manufacturer refunds the sale price to the customer [Case2-(i)] or replaces the failed item with a new item together with a new warranty policy [Case2-(ii)].

Let $S_{k|r}$ be the time when the k^{th} failure occurs and the total downtime up to this point does not exceed τ [$S_{k|r}$ occurs at a time instance of failure], conditional on $R=r$. If for a given $R=r$, $L_{\tau|r}$ is the time when the total downtime first exceeds τ and less than k failures have occurred up to this point [$L_{\tau|r}$ occurs during a repair period], then $L_r=\min\left(S_{k|r},L_{\tau|r}\right)$. As a result, the item is declared a lemon under warranty if and only if $L_r\le W, r\le r_1$ or $L_r\le\Gamma, r>r_1$.

Case 2: [Refund]

An item is a "lemon", if an item fail k times under warranty or the total repair time of the item (total downtime) under warranty exceeds τ, whichever comes first. Here, when the item is declared a lemon then the manufacturer has to refund the total sale price to the customer.

Case 2: [Replacement with a new warranty]

Here, if the item is declared a lemon under warranty, then the failed item is replaced with a new item with a new warranty, and hence have a renewing warranty. (Note: as the mathematical formulations for Case 2 are more involved and hence we leave them as future works)

4. NUMERICAL EXAMPLE

We consider the conditional failure function given in (1) with $\theta_0 = \theta_1 = 0$ for simplicity. Hence we have $\lambda(t|r) = (\theta_2 + \theta_3 r)t$ and $F(t|r) = 1 - e^{-0.5(\theta_2+\theta_3 r)t^2}$ or in form of Weibull distribution given by $F(t|r) = 1 - \exp\{-(t/\alpha)^2\}$ with $\alpha = \sqrt{2/(\theta_2 + \theta_3 r)}$ and $\beta = 2$.

The following parameter values are used: C_p = 100, C_c = 0.05C_p, C_m = 0.7C_p, C_r = 0.05C_p, C_L = 0.3C_p , W(U)=1 year(1x20000km) and k = 4 . Table 1 shows $E[C(W,k)]/C_p$ and $P\{C(W,k) > C_L\}$ for $\theta_2 = 0.075$ and θ_3 = 0.15 to 0.20.For $r \le r_1$, the warranty expires at age W and for $r > r_1$ the warranty expires due to the usage at age W_r , $W_r < W$. In general, the refund case gives a smaller cost than that of the replacement case, and this is as expected. For the refund case, the expected warranty servicing cost ranges from 0.1-27% and from 0.1-31% for the replacement. The expected warranty servicing cost is the highest for $r = 1.0$ as the customer has the maximum coverage for age and usage. For $r = 2.0$ as warranty ends at $W_r = 0.5$ then the expected warranty cost is not much influenced by the effect of higher usage, but the effect would be significant when W is greater than 1 year. The expected warranty cost and the risk increases as θ_3 increase. This is as expected as greater θ_3 means higher failure rate. The risk of the warranty servicing cost exceeds C_L ranges from 2.5-3.62% for refund case, and the risk ranges from 2.52-3.37% for replacement case. As in the case of expected warranty cost, the risk is maximum when $r = 1.0$.

Table 1. Results for refund and replacement cases with $\theta_2 = 0.075$

r	θ_3	$E[T\|r]$	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
			$E[C(W,k)\|r]/C_p$	$P\{[C(W,k)\|r] > C_L\}$	$E[C(W,k)\|r]/C_p$	$P\{[C(W,k)\|r] > C_L\}$
0.80	0.150	2.8381	0.0009	0.0230	0.0009	0.0230
	0.175	2.7029	0.0011	0.0231	0.0011	0.0231
	0.200	2.5853	0.0013	0.0232	0.0013	0.0232
1.00	0.150	1.1323	0.0447	0.0730	0.0448	0.0573
	0.175	1.1209	0.0473	0.0793	0.0474	0.0615
	0.200	0.8700	0.2741	0.4763	0.3113	0.5111
1.50	0.150	2.2882	0.0010	0.0231	0.0010	0.0231
	0.175	2.1573	0.0012	0.0232	0.0012	0.0232
	0.200	0.7147	0.2621	0.4647	0.2948	0.4947
2	0.150	2.0466	0.0008	0.0230	0.0008	0.0230
	0.175	1.9224	0.0011	0.0231	0.0011	0.0231
	0.200	0.6208	0.2562	0.4588	0.2868	0.4864

5. CONCLUSIONS

In this paper, we have studied warranty servicing cost for a product sold with warranty and protected with lemon laws enforceable during the warranty period. Two cases (refund and replacement cases) are considered when an item is declared a lemon that is if and only if it fails *k* times during the warranty period. In general, the lemon law is invoked by one of these two events (i) it fails k times during the warranty period or (ii) the total time taken to repair the item (total downtime) under warranty exceeds τ . This has been indicated earlier as one further research topic. One can model failure over the warranty region by a bivariate distribution function (using Approach 1) and hence we have a new model formulation for the expected warranty servicing cost. These two further research topics are under investigation.

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