Option Contract with Put and Call Option: A Case of One Buyer and Two Suppliers

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Abstract. This research studies a case of one retailer and two suppliers under option contract with put and call option. Each of suppliers offers the contract with the related prices. Then, the retailer decides how much to order and places initial order and option quantity. When there is updated demand information, the retailer will adjust initial order by exercising option from the supplier that is more profitable in option exercise price. To analyze the contract, mathematical models are firstly developed. Then, numerical experiments are conducted to examine the effects of contract parameters of the model in numerical experiments. Through numerical experiments, this research finds that the retailer can have more profit from using bidirectional option contracts with two suppliers. However, the supplier with higher put exercise price faces negative effect from this contract.

Keywords: Supply chain management, Option contract, One buyer and two suppliers, Single-period newsvendor problem

1. INTRODUCTION

Nowadays, most of the retailers face a big problem of uncertainty in demand of consumer. Reducing lead time, i.e., time between order release and order receipt, can be a solution for this problem. Another solution is to purchase products in advance based on forecast and keep them in inventory. In principle, there is no way to match demand and supply perfectly. In order to increase the efficiency of supply chain, supply contract is a widely-used solution. By sharing the risk between the retailer and the supplier, profits of both members can be increased. Option contract is a commonly used supply contract nowadays. It gives the retailer the right to buy, called call option, and the right to return, called put option, by paying for option reservation. Option contract plays an important role in dealing with uncertainty in demand in supply chain. By using option contract, the retailer places the initial order based on the forecast in the first stage and has the right to adjust the initial order later. The put option will be employed to adjust the order downward while the call option will be used to adjust the order upward just before selling season. However, using only put option or call option can lead the retailer to shortage or over stock in an extreme scenario. Hence, the option contract with both put and call options is better in dealing with the supply risk.

Option contract between one supplier and one retailer is a commonly used contract. However, the use of a single supplier in option contract cannot mitigate the supply risk. Therefore, our research presented here aims to reduce this kind of risk by investigating the use of two suppliers for the same item. We believe that this contract type can reduce the supply risk while can still excel in dealing with demand uncertainty. The problem addressed here is to help the retailer to split the order between the two suppliers to whom he has signed the option contract. The possibility of the option contract to allocate profit among the members of the supply chain will also be examined and compared with the single supplier contract type. The rest of this paper is organized as follows. We present literature review in Section 2. In Section 3, we construct the mathematical models and analyze for optimal solution. Section 4 presents the numerical experiments in order to investigate coordination and conduct sensitivity analysis. Lastly, section 5 provides a brief conclusion and recommendations for future research.

2. LITERATURE REVIEW

This section presents a review on the literature related to supply contracts with the focus on option contract, mixed contract, and mixed contract with spot market. It is noted that many research works in supply contract used the single newsvendor problem as the standard problem such as Gomez-Padilla and Mishina (2009), Zhao et al. (2013), Jörnsten et al. (2013), and Cai et al. (2015). Various types of supply contract have been examined in many research works for their flexibility and efficiency. Cai et al. (2015) investigated a supply contract for a simple supply chain with one supplier and one retailer who sells goods in a pre-sale season and a sales season. Unsold items can be sold as salvage and there will be holding cost for inventory carried from the pre-sale season to the sales season. In the contract the supplier will offer an expedited-delivery option for additional purchase to satisfy the customer of retailer on time. A two-period newsvendor model for this problem has been derived and it was found that there exists a range of the expedited-delivery capacity of the supplier that provides benefits for both supplier and retailer. This range depends on the cost of delivery and the minimum order quantity. Jörnsten et al. (2013) conducted the comparison between a mixed contract and a real option contract in which the buyer has the right to buy one unit of goods at a fixed price. From the result of this research, it can be concluded that the mixed contract is better than the real option contract only when the manufacturer is risk-averse. Zhao et al. (2013) also studied a two-echelon supply chain with one manufacturer and one retailer under bidirectional option contract. The authors developed initial order strategy and option purchasing strategy and explored how contract strategy would help achieve supply chain coordination. They also derived a closed-form expression for retailer's optimal order strategy. However, the results of this research did not tell clearly in which scenarios we should adopt high or low initial order.

Protection against supplier reliability risk has become an objective in many recent research works about supply contract. The method to reduce the supply risk is studied in many ways such as using multiple suppliers, spot purchase or mixed strategy. The use of multiple suppliers has been examined by Gomez-Padilla and Mishina (2009). In this research, the authors simulated the two-supplier case for a supply contract. They analyzed the supply contract which contains options and also derived a model to help determine the option premium. The results confirmed that employing the proposed supply contract is better than using no-contract. Fu (2015) also studied the effect of supply options. In this research, a single period procurement problem with a set of contingent options and the use of spot market is examined. In the same direction with the research of Fu (2015), Merzifonluoglu et al. (2015) also considered a portfolio contract which consists of forward contract, option contract and spot purchase in a two-stage decision framework in both risk-neutral in risk-averse environments.

Kim et al. (2014) emphasized the important of using multiple suppliers. Their research focused on a quantity flexibility contract with heterogeneous suppliers. A linear programming model was developed from retailer's perspective, and a rolling-horizon strategy is derived and suggested for efficiency of the contract. The results showed that the proposed strategy is efficient, easy to use, reliable and can help to lower the cost.

When a retailer changes from one supplier to many suppliers, there are impacts that have to be considered. Chambolle and Villas-Boas (2015) argued that rival retailers may choose to differentiate their supplying producers, even at the expense of downgrading the quality of the product, to improve the buyer power. They showed that, through the differentiation of suppliers, a retailer may obtain a larger slice of a smaller bilateral joint profits. Thus, the impact of buyer power, i.e., retailer power, should be considered.

The mixed strategy of using spot purchase and a long-term contract was also studied by Li et al. (2009). In their paper, the authors considered a supply problem in which the buyer faces non-stationary stochastic price and demand. First, they compared two pure strategies: (i) periodically purchasing from the spot market; and (ii) a long-term contract with a single supplier. The results showed that the selection of suppliers can be complicated by many parameters affected by price uncertainty. They then develop a stochastic dynamic programming model to incorporate mixed strategies, purchasing commitments and contract cancellations. The results showed that increases in price uncertainty favor long-term suppliers and increases in demand uncertainty favor short-term suppliers. By examining the two-way interactions of contract factors which are price, demand, purchasing bounds, learning and technology effect, salvage values, they noticed that when the learning and technology improvement effect is high, the long-term supplier becomes favorable, but when the variability of both demand and price is high, the short-term suppliers are preferred. When cancellation of the contract is allowed, cancellation cost can play the leverage role in selection of suppliers.

Risk measure is also an interesting topic in supply contract. Wang et al. (2012) analyzed the risks of introducing options, and found that even if providing a higher expected profit at the beginning of a planning horizon, supply contracts with options may have risks associated with a worse performance later compared with the traditional newsvendor contract model. They derived two important parameters for the buyer to estimate the risks of introducing options. One is a risk indicator that can show whether using option-based contract model has risks or not, and the other is a ratio to measure the probability of such risks.

Contributing to the current research stream on the use of multiple suppliers in option contract, in this current research we will examine the use of option contract between a retailer and two suppliers. The aim is to help the retailer to decide on how to spit the order between the two suppliers so as to maximize his own profit. Also, the ability of the option contract to allocate profit among the members is also examined and compared with the single-supplier contract.

3. MATHEMATICAL MODEL

3.1. Model Assumptions

This research studies a supply chain in which:

- The supply chain has one retailer and two suppliers
- The retailer signs option contracts to both suppliers
- Demand information are known by all parties.

The retailer problem is a single period newsvendor problem with two stages. At the beginning of the first stage, i.e., the pre-selling season, the retailer will place initial orders along with option quantities to both suppliers. Then, at the beginning of the second stage, i.e., the selling season, the retailer will place firm orders to the suppliers considering the use of put/call options after realizing the demand.

The following notations are used in this study:

$f(.)$ = Probability density function of demand
$F(.)$ = Cumulative distribution function of demand
Q_i = initial order quantity placed to supplier i (i =1, 2)
Q = Total initial order quantity
p = product selling price
W_{epi} = put exercise price with supplier i (i = 1, 2)
W_{eci} = call exercise price with supplier i (i = 1, 2)
q_i = quantity of option purchased from supplier i (i = 1, 2)
v = unit salvage value
w_i = wholesale price of supplier i (i = 1, 2)
o_i = option price of supplier i (i = 1 ,2)
qe_i = quantity of option exercised from supplier i (i = 1, 2)
g = unit shortage cost
c_i = unit cost of production of by supplier i (i = 1, 2)
l = proportion of initial order from supplier 1 ($0 \le l \le 1$)

To avoid trivial cases, following assumptions are made

1. To ensure that the retailer will accept the use of option contract, we must have
$$0 \le W_{epi} \le w \le W_{eci}$$
2. The option price should be positive to prevent the transfer of all the risks to supplier.
$$o_i > 0$$
3. To ensure that the retailer still have profit when exercising call option
$$o_i + W_{eci} \le p$$
4. To persuade the retailer to exercise put option rather than selling excess inventory at salvage price
$$0 \le v \le W_{epi} - o_i$$
5. Normal conditions on input parameters
$$0 \le v \le c_i \le w_i$$
6. Constraints for variables
$$0 \le q_1 + q_2 \le Q$$

3.2. Retailer's Profit Function

In the pre-selling season, the retailer has to pay

- Cost of purchasing initial order = $w_1 Ql + w_2 Q(1-l)$
- Cost of purchasing option = = $o_1 q_1 + o_2 q_2$

In the selling season, the retailer will firstly observe the demand x of the product. There are 4 cases to be considered:

Case 1: $0 < x \le Q - q_1 - q_2$
Case 2: $Q - q_1 - q_2 < x \le Q$
Case 3: $Q < x \le Q + q_1 + q_2$
Case 4: $Q + q_1 + q_2 < x$

In each case, the profit function is as follows.

Case 1: $0 < x \le Q - q_1 - q_2$

In this case, the retailer will exercise the maximum amount of put option and receive refunds from the suppliers. Also, the leftover product after exercising put option will be sold as salvage by the retailer. Therefore,

Retailer's profit = Refund from put option + Salvage + Sales revenue
$$= W_{ep1} \cdot q_1 + W_{ep2} \cdot q_2 + px + v(Q - q_1 - q_2 - x)$$
It is noted in this case that the exercised put option quantities are $qe_1 = q_1$, $qe_2 = q_2$.

Case 2: $Q - q_1 - q_2 < x \le Q$

In this case, the retailer will exercise put option so as to exactly fulfill the demand. Therefore,

Retailer's profit = Refund from put option + Sales revenue
$$= W_{ep1} \cdot qe_1 + W_{ep2} \cdot qe_2 + px$$
It is noted that:

- If $W_{ep1} > W_{ep2}$ then put option from supplier 1 is preferable, and hence
$$qe_1 = \min(Q - x,\ q_1) \text{ and } qe_2 = (Q - x) - qe_1$$

So,

If $0 \le Q - x \le q_1$ or $Q - q_1 \le x \le Q$: $qe_1 = Q - x$, $qe_2 = 0$
If $q_1 \le Q - x \le q_1 + q_2$ or $Q - q_1 - q_2 \le x \le Q - q_1$: $qe_1 = q_1$, $qe_2 = Q - x - q_1$

- If $W_{ep2} > W_{ep1}$ then put option from supplier 2 is preferable, and hence
$$qe_1 = (Q - x) - qe_2 \text{ and } qe_2 = \min(Q - x,\ q_2)$$

So,

If $0 \le Q - x \le q_2$ or $Q - q_2 \le x \le Q$: qe_1= 0, $qe_2 = Q - x$
If $q_2 \le Q - x \le q_1 + q_2$ or $Q - q_1 - q_2 \le x \le Q - q_2$:
qe_1= $Q - x - q_2$, $qe_2 = q_2$

Case 3: $Q < x \le Q + q_1 + q_2$

In this case, the retailer will exercise call option so as to exactly fulfill the demand. Therefore,

Retailer's profit = Sales revenue – Cost from call option
$= px - W_{ec1} \cdot qe_1 - W_{ec2} \cdot qe_2$

It is noted that:

- If $W_{ec2} > W_{ec1}$ then call option from supplier 1 is preferable, and hence
  qe_1= $\min(x - Q,\ q_1)$ and qe_2 = $(x - Q) - qe_1$

So,
If $0 \le x - Q \le q_1$ or $Q \le x \le Q + q_1$: qe_1= x – Q, qe_2 = 0
If $q_1 \le x - Q \le q_1 + q_2$ or $Q + q_1 \le x \le Q + q_1 + q_2$:
qe_1= q_1, qe_2 = $x - Q - q_1$

- If $W_{ec1} > W_{ec2}$ then call option from supplier 2 is preferable, and hence
  qe_1= $(x - Q) - qe_2$ and qe_2 = $\min(x - Q,\ q_2)$

So,
If $0 \le x - Q \le q_2$ or $Q \le x \le Q + q_2$: qe_1= 0, qe_2 = x – Q
If $q_2 \le x - Q \le q_1 + q_2$ or $Q + q_2 \le x \le Q + q_1 + q_2$:
qe_1= $x - Q - q_2$, qe_2 = q_2

Case 4: $x > Q + q_1 + q_2$

In this case, the retailer will exercise the maximum call option, and pay shortage cost for unsatisfied demand. Therefore,

Retailer's profit = Sales revenue – Cost from call option – shortage cost
$= p(Q + q_1 + q_2) - W_{ec1}q_1 - W_{ec2}q_2 - g(x - Q - q_1 - q_2)$

It is noted in this case that the exercised call option quantities are qe_1= q_1, qe_2 = q_2.

The expected retailer's profit can then be determined as

$$E[\pi_r(Q, q_i, q_e^*)] = -w_1 Ql - w_2 Q(1 - l) - o_1 q_1 - o_2 q_2 + E[\pi_r^{t_1}(Q_i, q_i, q_e^*)]$$

in which $E[\pi_r^{t_1}(Q, q_i, q_e^*)]$ is expected profit of selling season. There are four scenarios to consider as follows:

Scenario 1: If $W_{ep1} > W_{ep2}$ and $W_{ec2} > W_{ec1}$

In this scenario, the retailer will exercise both put and call options from supplier 1 first. Hence,

$$E\left[\pi_r^{t_1}\left(Q, q_i, q_e^*\right)\right] =$$
$$\int_0^{Q-q_1-q_2} \left[W_{ep1}q_1 + W_{ep2}q_2 + px + v\left(Q - q_1 - q_2 - x\right)\right] f(x)\,dx$$
$$+ \int_{Q-q_1-q_2}^{Q-q_1} \left[W_{ep1}q_1 + W_{ep2}\left(Q - x - q_1\right) + px\right] f(x)\,dx$$
$$+ \int_{Q-q_1}^{Q} \left[W_{ep1}\left(Q - x\right) + px\right] f(x)\,dx + \int_{Q}^{Q+q_1} \left[px - W_{ec1}\left(x - Q\right)\right] f(x)\,dx$$
$$+ \int_{Q+q_1}^{Q+q_1+q_2} \left[px - W_{ec1}q_1 - W_{ec2}\left(x - Q - q_1\right)\right] f(x)\,dx$$
$$+ \int_{Q+q_1+q}^{\infty} \left[p\left(Q + q_1 + q_2\right) - \left(W_{ec1}q_1 + W_{ec2}q_2\right) - g\left(x - Q - q_1 - q_2\right)\right] f(x)\,dx$$

Scenarios 2: If $W_{ep1} > W_{ep2}$ and $W_{ec1} > W_{ec2}$

In this scenario, the retailer will exercise put option from supplier 1 and call option from supplier 2 first. Hence,

$$E\left[\pi_r^{t_1}\left(Q, q_i, q_e^*\right)\right] =$$
$$\int_0^{Q-q_1-q_2} \left[W_{ep1}q_1 + W_{ep2}q_2 + px + v\left(Q - q_1 - q_2 - x\right)\right] f(x)\,dx$$
$$+ \int_{Q-q_1-q_2}^{Q-q_1} \left[W_{ep1}q_1 + W_{ep2}\left(Q - x - q_1\right) + px\right] f(x)\,dx$$
$$+ \int_{Q-q_1}^{Q} \left[W_{ep1}\left(Q - x\right) + px\right] f(x)\,dx + \int_{Q}^{Q+q_2} \left[px - W_{ec2}\left(x - Q\right)\right] f(x)\,dx$$
$$+ \int_{Q+q_2}^{Q+q_1+q_2} \left[px - W_{ec1}\left(x - Q - q_2\right) - W_{ec2}q_2\right] f(x)\,dx$$
$$+ \int_{Q+q_1+q}^{\infty} \left[p\left(Q + q_1 + q_2\right) - \left(W_{ec1}q_1 + W_{ec2}q_2\right) - g\left(x - Q - q_1 - q_2\right)\right] f(x)\,dx$$

Scenario 3: If $W_{ep2} > W_{ep1}$ and $W_{ec2} > W_{ec1}$

In this scenario, the retailer will exercise put option from supplier 2 and call option from supplier 1 first. Hence,

$$E\left[\pi_r^{t_1}\left(Q, q_i, q_e^*\right)\right] =$$
$$\int_0^{Q-q_1-q_2} \left[W_{ep1}q_1 + W_{ep2}q_2 + px + v\left(Q - q_1 - q_2 - x\right)\right] f(x)\,dx$$
$$+ \int_{Q-q_1-q_2}^{Q-q_2} \left[W_{ep1}\left(Q - x - q_2\right) + W_{ep2}q_2 + px\right] f(x)\,dx$$
$$+ \int_{Q-q_2}^{Q} \left[W_{ep2}\left(Q - x\right) + px\right] f(x)\,dx + \int_{Q}^{Q+q_1} \left[px - W_{ec1}\left(x - Q\right)\right] f(x)\,dx$$
$$+ \int_{Q+q_1}^{Q+q_1+q_2} \left[px - W_{ec1}q_1 - W_{ec2}\left(x - Q - q_1\right)\right] f(x)\,dx$$
$$+ \int_{Q+q_1+q}^{\infty} \left[p\left(Q + q_1 + q_2\right) - \left(W_{ec1}q_1 + W_{ec2}q_2\right) - g\left(x - Q - q_1 - q_2\right)\right] f(x)\,dx$$

Scenario 4: If $W_{ep2} > W_{ep1}$ and $W_{ec1} > W_{ec2}$

In this scenario, the retailer will exercise both put and call options from supplier 2 first. Hence,

$$E\left[\pi_r^{t_1}\left(Q,q_i,q_e^*\right)\right]=$$

$$\int_0^{Q-q_1-q_2}\left[W_{ep1}q_1+W_{ep2}q_2+px+v\left(Q-q_1-q_2-x\right)\right]f\left(x\right)dx$$

$$+\int_{Q-q_1-q_2}^{Q-q_2}\left[W_{ep1}\left(Q-x-q_2\right)+W_{ep2}q_2+px\right]f\left(x\right)dx$$

$$+\int_{Q-q_2}^{Q}\left[W_{ep2}\left(Q-x\right)+px\right]f\left(x\right)dx+\int_{Q}^{Q+q_2}\left[px-W_{ec2}\left(x-Q\right)\right]f\left(x\right)dx$$

$$+\int_{Q+q_2}^{Q+q_1+q_2}\left[px-W_{ec1}\left(x-Q-q_2\right)-W_{ec2}q_2\right]f\left(x\right)dx$$

$$+\int_{Q+q_1+q}^{\infty}\left[p\left(Q+q_1+q_2\right)-\left(W_{ec1}q_1+W_{ec2}q_2\right)-g\left(x-Q-q_1-q_2\right)\right]f\left(x\right)dx$$

3.3. Supplier's Profit Function

In the pre-selling season, we have:
Supplier 1's profit = $w_1Ql+o_1q_1-c_1(Ql+q_1)$
Supplier 2's profit = $w_2Q(1-l)+o_2q_2-c_2(Q(1-l)+q_2)$

In the selling season, it is noted that the production quantities of supplier 1 and supplier 2 are $Ql+q_1$ and $Q(1-l)+q_2$. Hence, the profit functions of the two suppliers can be derived accordingly as follows:

Case 1: $0 < x \le Q-q_1-q_2$
The amount that the retailer buys from supplier 1 is $Ql-q_1$. Hence,
Supplier 1's profit = Salvage value – refund for retailer
= $[(Ql+q_1)-(Ql-q_1)]v-Wep_1\cdot q_1$
= $2q_1\cdot v-Wep_1\cdot q_1$
The amount that the retailer buys from supplier 2 is $Q(1-l)-q_2$. Hence,
Supplier 2's profit = Salvage value – refund for retailer
= $[(Q(1-l)+q_2)-(Q(1-l)-q_2)]v-Wep_2\cdot q_2$
= $2q_2\cdot v-Wep_2\cdot q_2$

Case 2: $Q-q_1-q_2 < x \le Q$
The amount that the retailer buys from supplier 1 is $Ql-qe_1$. Hence,
Supplier 1's profit = Salvage value – refund for retailer
= $[(Ql+q_1)-(Ql-qe_1)]v-Wep_1\cdot qe_1$
= $(q_1+qe_1)v-Wep_1\cdot qe_1$
The amount that the retailer buys from supplier 2 is $Q(1-l)-qe_1$. Hence,
Supplier 2's profit = Salvage value – refund for retailer
= $[(Q(1-l)+q_2)-(Q(1-l)-qe_2)]v-Wep_2\cdot qe_2$
= $(q_2+qe_2)v-Wep_2\cdot qe_2$

Case 3: $Q < x \le Q+q_1+q_2$
The amount that the retailer buys from supplier 1 is $Ql+qe_1$. Hence,
Supplier 1's profit = Salvage value + Sale from option
= $[(Ql+q_1)-(Ql+qe_1)]v+Wec_1\cdot qe_1$
= $(q_1-qe_1)v+Wec_1\cdot qe_1$
The amount that the retailer buys from supplier 2 is $Q(1-l)+qe_2$. Hence,
Supplier 2's profit = Salvage value + Sale from option
= $[(Q(1-l)+q_2)-(Q(1-l)+qe_2)]v+Wec_2\cdot qe_2$
= $(q_2-qe_2)v+Wec_2\cdot qe_2$

Case 4: $x > Q+q_1+q_2$
The amount that the retailer buys from supplier 1 is $Ql+q_1$. Hence,
Supplier 1's profit = Salvage value + Sale from option
= $[(Ql+q_1)-(Ql+q_1)]v+Wec_1\cdot q_1$
= $Wec_1\cdot q_1$
The amount that the retailer buys from supplier 2 is $Q(1-l)+q_2$. Hence,
Supplier 2's profit = Salvage value + Sale from option
= $[(Q(1-l)+q_2)-(Q(1-l)+q_2)]v+Wec_2\cdot q_2$
= $Wec_2\cdot q_2$
The profit functions of the two suppliers are then derived as follows:

<u>The supplier 1's profit</u>

$$\pi_{s1}\left(Q,q_i,q_e^*\right)=w_1Ql+o_1q_1+c_1\left(Ql+q_1\right)+E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]$$

in which $E[\pi_{s1}^{t1}(Q,q_i,q_e^*)]$ is the expected profit in the selling season of supplier 1. There are four scenarios to consider:

Scenario 1: If $W_{ep1} > W_{ep2}$ and $W_{ec2} > W_{ec1}$

In this scenario, the retailer will exercise both put and call options from supplier 1 first. Hence,

$$E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]=\int_0^{Q-q_1-q_2}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$

$$+\int_{Q-q_1-q_2}^{Q-q_1}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$

$$+\int_{Q-q_1}^{Q}\left[\left(q_1-\left(Q-x\right)\right)v+W_{ep1}\left(Q-x\right)\right]f\left(x\right)dx$$

$$+\int_{Q}^{Q+q_1}\left[\left(q_1-\left(x-Q\right)\right)v+W_{ec1}\left(x-Q\right)\right]f\left(x\right)dx$$

$$+\int_{Q+q_1}^{Q+q_1+q_2}W_{ec1}q_1f\left(x\right)dx+\int_{Q+q_1+q_2}^{\infty}W_{ec1}q_1f\left(x\right)dx$$

Scenario 2: If $W_{ep1} > W_{ep2}$ and $W_{ec1} > W_{ec2}$

In this scenario, the retailer will exercise put option from supplier 1 and call option from supplier 2 first. Hence,

$$E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]=\int_0^{Q-q_1-q_2}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$
$$+\int_{Q-q_1-q_2}^{Q-q_1}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$
$$+\int_{Q-q_1}^{Q}\left[\left(q_1+\left(Q-x\right)\right)v-W_{ep1}\left(Q-x\right)\right]f\left(x\right)dx+\int_{Q}^{Q+q_2}q_1vf\left(x\right)dx$$
$$+\int_{Q+q_2}^{Q+q_1+q_2}\left[\left(q_1-\left(x-Q-q_2\right)\right)v+W_{ec1}\left(x-Q-q_2\right)\right]f\left(x\right)dx$$
$$+\int_{Q+q_1+q_2}^{\infty}W_{ec1}q_1f\left(x\right)dx$$

Scenario 3: If $W_{ep2} > W_{ep1}$ and $W_{ec2} > W_{ec1}$

In this scenario, the retailer will exercise put option from supplier 2 and call option from supplier 1 first. Hence,

$$E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]=\int_0^{Q-q_1-q_2}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$
$$+\int_{Q-q_1-q_2}^{Q-q_2}\left[\left(q_1+\left(Q-x-q_2\right)\right)v-W_{ep1}\left(Q-x-q_2\right)\right]f\left(x\right)dx$$
$$+\int_{Q-q_2}^{Q}q_1vf\left(x\right)dx+\int_{Q}^{Q+q_1}\left[\left(q_1-\left(x-Q\right)\right)v+W_{ec1}\left(x-Q\right)\right]f\left(x\right)dx$$
$$+\int_{Q+q_1}^{Q+q_1+q_2}W_{ec1}q_1f\left(x\right)dx+\int_{Q+q_1+q_2}^{\infty}W_{ec1}q_1f\left(x\right)dx$$

Scenario 4: If $W_{ep2} > W_{ep1}$ and $W_{ec1} > W_{ec2}$

In this scenario, the retailer will exercise both put and call options from supplier 2 first. Hence,

$$E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]=\int_0^{Q-q_1-q_2}\left(2q_1v-W_{ep1}q_1\right)f\left(x\right)dx$$
$$+\int_{Q-q_1-q_2}^{Q-q_2}\left[\left(q_1+\left(Q-x-q_2\right)\right)v-W_{ep1}\left(Q-x-q_2\right)\right]f\left(x\right)dx$$
$$+\int_{Q-q_2}^{Q}q_1vf\left(x\right)dx+\int_{Q}^{Q+q_2}q_1vf\left(x\right)dx$$
$$+\int_{Q+q_2}^{Q+q_1+q_2}\left[\left(q_1-\left(x-Q-q_2\right)\right)v+W_{ec1}\left(x-Q-q_2\right)\right]f\left(x\right)dx$$
$$+\int_{Q+q_1+q_2}^{\infty}W_{ec1}q_1f\left(x\right)dx$$

<u>**The supplier 2's profit**</u>

$$\pi_{s2}\left(Q,q_i,q_e^*\right)=w_1Q\left(1-l\right)+o_1q_1+c_1\left(Q\left(1-l\right)+q_1\right)+E\left[\pi_{s1}^{t1}\left(Q,q_i,q_e^*\right)\right]$$

in which $E[\pi_{s2}^{t1}(Q,q_i,q_e^*)]$ is the expected profit in the selling season of supplier 2. Similar to the case of supplier 1, four scenarios should be taken into consideration, and the associated expressions of $E[\pi_{s2}^{t1}(Q,q_i,q_e^*)]$ can be developed. However, due to similarities, these expressions are not presented here in details.

4. NUMERICAL EXPERIMENTS

4.1. The base case

In this section, the demands in both periods are assumed to follow uniform distribution. From the results in section 3, there are four scenarios to consider but it is noted that scenario 1 is similar to scenario 4 and scenario 2 is similar to scenario 3. Therefore, only scenarios 1 & 2 will be considered here.

<u>Scenario 1</u>: The case of put & call from supplier 1

It is assumed that the demand is uniformly distributed over the range [a, b] = [5000, 10000]. The following input parameters are used: p =200; w_1=100; w_2=100; g =70; v =20; W_{ec1} =120; W_{ec2} =130; W_{ep1} =60; W_{ep2} =50; o_1 =9; o_2 =9; c_1 = 40; c_2 = 40; l =0.5.

The optimal solutions are as follows: $Q=7391$; $q_1=2338$; $q_2=0$, and the profits of retailer, supplier 1, and supplier 2 are 691373.59; 241199.40; and 221130.00, respectively. From the results, it can be seen that the retailer will use option contract with only supplier 1.

<u>Scenario 2</u>: The case of put from supplier 1 & call from supplier 2

It is still assumed that the demand is uniformly distributed over the range [a, b] = [5000, 10000]. The same input parameters are used except that W_{ec1} =140

The optimal solutions are as follows: $Q=7469$; $q_1=1130$; $q_2=1192$, and the profits of retailer, supplier 1, and supplier 2 are 681281.75; 215418.70; and 256387.50, respectively. From the results, it can be seen that option contract will be used with both suppliers and the option quantity has been split to both suppliers. For comparison purpose, the use of option contract with only one supplier will be analyzed below

- If the retailer signs a single option contract with supplier 1, the optimal solutions are:

Q	q	Retailer's profit	Supplier1's profit
7618	2173	678971.66	474116.32

- If the retailer signs a single option contract with supplier 2, the optimal solutions are:

Q	q	Retailer's	Supplier2's

		profit	profit
7383	2311	678935.03	474844.07

From the optimal solutions of both single option contracts, it is clear that the retailer can have higher profit from using option contracts with two suppliers.

4.2. Sensitivity analysis with respect to call exercise price

In this part, the exercise price of call option is varied to examine the effect to order quantity and profit of retailer along with profit of each supplier. Input parameters of the first scenario of the base case will be considered in which W_{ec1} will be changed from 90 to 220 with the step size of 10.

The detailed results are present in Table 1 below.

Table 1: Sensitivity analysis with respect to W_{ec1}

Q	q_1	q_2	Profit of Retailer	Profit of Sup. 1	Profit of Sup. 2
7074	2549	0	706534.41	235232.37	212220.00
7230	2437	0	698556.88	238430.89	216900.00
7371	2338	0	691373.59	241199.39	221130.00
7500	2250	0	684874.99	243562.50	225000.00
7469	1130	1192	681281.75	215418.72	256387.50
7448	681	1668	680140.96	214090.55	258798.26
7435	453	1904	679599.19	215574.59	257904.62
7425	311	2046	679295.15	217167.23	256709.51
7415	208	2143	679112.67	218566.22	255632.21
7405	126	2215	679004.16	219761.29	254701.64
7394	54	2272	678948.19	220800.26	253891.55
7383	0	2311	678935.00	221490.00	253354.07
7383	0	2311	678935.00	221490.00	253354.07

From the results, it can be seen that when W_{ec1} is lower than 130, which means supplier 1 dominates both put and call options over supplier 2, the retailer will decide to buy option from supplier 1 only. Furthermore, there exists a range of value of W_{ec1}, i.e., [140,200], in which the retailer decides to split the option quantity. The proportion to split will depend on the exercise price of call option in which the higher the call option price from supplier 1, the lower the option quantity allocated to supplier 1. This trend is reasonable. Also, when W_{ec1} is more than 210, it can be seen that the value of W_{ec1} will have no effect on the retailer's profit anymore. The reason behind this trend is that supplier 2 in this situation will dominate supplier 1 even though the put option from supplier 1 is more attractive and the retailer will decide to choose only supplier 2 for option contract. The trend of initial order is also an increasing trend with W_{ec1} =100 to W_{ec1} =130. This is because it is more profitable for the retailer to have more initial order when the price to call from supplier 1 increases. However, after W_{ec1} = 140, the initial order has a decreasing trend. This happens because when exercising call option from supplier 2 becomes attractive, the retailer will reduce initial order in order to exercise more call option from supplier 2.

From the retailer's profit showed in Table 1, it can also be seen that the profit of retailer will decrease sharply at first when the call option exercise price, W_{ec1}, increases. However, when $W_{ec1} \geq 140$, the retailer's profit will decrease slightly when W_{ec1} increases. This is due to the fact that the retailer splits the option quantity to both suppliers, and hence, the profit gained from exercising put option from supplier 2 will help to compensate the negative effect on profit when W_{ec1} increases. Furthermore, it is noted that when $W_{ec1} \geq 210$ the retailer profit will not be affected anymore because he will use only supplier 2 for option.

There are three ranges to consider for supplier1's profit. First, in the range between $W_{ec1} = 100$ to $W_{ec1} = 130$, the retailer will decide to buy option only from supplier 1. Increasing W_{ec1} will lower the option purchased from supplier 1 but supplier 1 will get more profit. In the second range, between $W_{ec1} = 130$ to $W_{ec1} = 200$, the profit of supplier 1 decreases and then increases. It is important to note that after the order is split the option from supplier 1 will have negative effect on the his own profit. This is due to the fact that the retailer will exercise call option from supplier 2 first and put option from supplier 1 first. From the retailer's strategy, supplier 1 has to refund for excess inventory first and gains benefit only after the retailer exercises all the call option from supplier 2. This makes supplier 1's additional profit cannot compensate for the costs of production and to give refund for put option. Furthermore, increasing more option quantity from supplier 2 will shorten the range in which supplier 1 makes profit from call option. The trend of decreasing supplier 1's option quantity, q_1, gives supplier 1 less cost which contradicts the trend of increasing q_2. It can be seen from the results that after the value $W_{ec1} = 160$, the effect of decreasing trend of q_1 wins over the increasing trend of option quantity from supplier 2, q_2.

Same as supplier 1, there are three ranges to consider for supplier 2's profit. First, in the range from $W_{ec1} = 100$ to $W_{ec1} = 130$, supplier 2's profit increases due to initial order only. After the value $W_{ec1} = 130$, the profit of supplier 2 increases sharply because the retailer will split the option quantity and will exercise call option from supplier 2 first.

However, in the range from $W_{ec1} = 150$ to $W_{ec1} = 210$, the profit of supplier 2 decreases due to the decreasing trend of q_1. By reducing q_1, the range that supplier 2 does not have to refund for put option (i.e., $Q - q_1 < x < Q$) will be shorten. Hence, supplier 2 now has to refund for put option more and gains less profit. After the value $W_{ec1} = 210$, there will be no effect of W_{ec1} on the profit of supplier 2 because the retailer will place all option quantity to supplier 2.

4.3. Sensitivity analysis with respect to put exercise price

In this part, the unit exercise price of put option is varied to examine the effect to the order quantity and profit of retailer along with profit of each supplier. To prevent the situation that one supplier dominates the other, input parameters from the second scenario of the base case will be considered with the value of W_{ep1} is from 60 to 90. The results are presented in Table 2. From the results, it can be observed that most of values of W_{ep1} will lead to splitting of option order except when $W_{ep1} > 80$ where supplier 1 dominates supplier 2. There is also a trend that retailer will buy more initial order when W_{ep1} is higher. This is because when W_{ep1} is low the retailer tries to avoid exercising put option, and hence, he will buy less initial order. Therefore, when W_{ep1} increases, it is reasonable for the retailer to buy more initial order because of higher profit from exercising put option. However, when the initial order increases, the retailer will exercise less call option which is the cause of the decreasing trend of option quantity from supplier 2. Related to profit, it can be seen that the retailer's profit increases when W_{ep1} increases. This trend is understandable.

Table 2: Sensitivity analysis with respect to W_{ep1}

Q	q_1	q_2	Profit of Retailer	Profit of Sup. 1	Profit of Sup. 2
7469	1130	1192	681281.75	215418.72	256387.50
7534	1453	846	683686.01	217558.45	251398.31
7606	1695	571	686495.62	219411.77	246243.01
7685	1893	334	689633.78	220525.62	241407.00
7771	2066	116	693074.48	220866.22	236944.08
7839	2156	0	696793.93	218135.44	235170.00
7881	2161	0	700638.46	212047.44	236430.00

Related to the profit of supplier 1, it can be seen from Table 2 that the increase in q_1 has negative effect on profit of supplier 1 because the retailer prefers to place put option from supplier 1. However, due to the increase of initial order, the profit of supplier 1 will firstly increase but later it will decrease when the profit from the increase of initial order cannot compensate the loss due to the increase of put option price. Related to the profit of supplier 2, it can be seen that as W_{ep1} increases q_2 decreases. The decrease in q_2 has negative effect on profit of supplier 2 because the profit comes from call option will be reduced. However, the profit of supplier 2 will increase later (when $W_{ep1} > 80$) because the retailer will not buy option from supplier 2, but the initial order increases.

5. CONCLUSIONS

This research studies a case of one buyer and two suppliers using bidirectional contract. From numerical experiments, this research finds that only in case one supplier dominates call exercise price and the other supplier dominates put exercise price the retailer will split the option order. By studying the splitting option order case, it can be concluded that the contract can help the retailer to gain more profit in comparison to single option contract. However, from supplier perspective, the option contract will lead to better profit for the supplier who dominates call exercise price, but the other supplier who dominates put exercise price might face negative effect. This leads to unfair profit allocation between the two suppliers and less profit for the supplier with higher put exercise price. This issue should be tackled in future research.

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