Network DEA: A Modified Non-radial Approach

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Abstract. DEA is a non-parametric and linear programming based technique that attempts to maximize a decision making unit's (DMUs) relative efficiency, expressed as a ratio of outputs to inputs, by comparing a particular unit's efficiency with the performance of a group of similar DMUs that are delivering the same service. The traditional DEA models treat DMUs as black boxes whose internal structure is ignored. Recently, network DEA models have been introduced that treats the internal structure of a DMU as a network system. The increased interest in network DEA also produced different type of model formulations including the slacks-based measure network DEA (SBM-NDEA). SBM is a non-radial approach suitable for measuring efficiencies when inputs and outputs may change non-proportionally, which is a sharp contrast as compared to traditional DEA models that measures input and output changes proportionally. However, just like other DEA models, SBM-NDEA has its assumptions that limit its applicability and discriminative power in efficiency measurement. The proposed model differs from existing SBM-NDEA approaches in that it considers the exogenous inputs and outputs at the system level instead of at the process level and takes into account the presence of intermediate products in the model's objective function.

Keywords: Data envelopment analysis; efficiency; non-radial; slacks-based measure

1. INTRODUCTION

DEA is an optimization methodology that measures the relative efficiency of a set of DMUs with multiple inputs and outputs to identify best practices (or efficient) frontier (Charnes et al., 1978). This indicates that DEA provides not only efficiency scores for DMUs, but also identify frontier projections for inefficient units onto an efficient frontier. Originally, the DEA methodology considers a DMU as a black box, that is, as a single process that consumes inputs and produces outputs. In other words, the internal structure of a DMU is ignored in the efficiency assessment (see, for example, Beasley, 1990; Luo et al., 2012; Premachandra et al., 2009; Sarkis, 2000; Sherman and Gold, 1985).

A major drawback of treating DMUs as black boxes is that it prohibits the identification of inefficiencies borne out of the sub processes that make up the internal structure of the DMU, and thus limiting the amount of information that can be gained to improve the overall system efficiency (Chen et al., 2016; Färe and Grosskopf, 2000). Disregarding the presence of sub processes may obtain misleading results, for instance, an overall system may be regarded as efficient even though all of its sub processes are not (Kao and Hwang, 2008). More significantly, there are cases in which all sub processes of a DMU have performance that are worse than those of another DMU, and yet the former still has the better system performance (Kao and Hwang, 2010). These findings suggest that the internal structure of DMUs, treating it as a network system, is required to produce accurate results when measuring efficiencies.

Most production systems have network structure in its operations, where the end-to-end process of a DMU is divided into sub processes, which are then linked together by intermediate products (Fukuyama and Mirdehghan, 2012; Lewis and Sexton, 2004). Considering this, Färe and Grosskopf (2000) conceptualized the network DEA, which gained interest among researchers producing numerous research studies thereafter. Some created models under specified conditions to measure efficiencies (Fukuyama and Weber, 2010; Kao, 2014a; Tone and Tsutsui, 2009, 2010), some examine properties possessed by certain models (Avkiran and McCrystal, 2012; Chen et al., 2009; Chen et al., 2010), and others apply existing models to solve real world problems (Avkiran, 2009, 2015; Chen and Yan, 2011; Lozano et al., 2013; Matthews, 2013; Yu, 2010). Kao (2014b) conducted a comprehensive review about network DEA and found out that most of the existing studies concerning network DEA are based from radial DEA models.

A radial model, in which the standard DEA models fall into, measures the relative efficiency of DMUs under the assumption of how much it can proportionally increase (decrease) all of its outputs (inputs) given its inputs (outputs) under technological constraint (Charnes et al., 1978; Farrell, 1957). However, radial models and so the standard DEA models were recognized to have two primary limitations. First, there is a possibility that a DMU may be measured against and referenced into a weakly efficient point in the production possibility set. These weakly efficient points have positive slacks with respect to strongly Pareto-efficient points on the efficient frontier. Second, a significant proportion of DMUs can be regarded as efficient (Dyson et al., 2001). If a total ordering among the efficient DMUs is desired, one may impose an exogenously determined preference structure for inputs or outputs. For example, restrictions on the dual multipliers or preferential weights (see the review of Angulo-Meza and Lins, 2002). Otherwise, one may also use the super-efficiency model (Anderson and Petersen, 1993), which does not require a prior weight assignment in the DEA models with weight restrictions.

Tone (2001) developed the SBM model to address the issue of referencing non-Pareto-efficient targets by eliminating slacks-related biases in the efficiency measurement. Tone's SBM model computes efficiency scores as a function of input slacks and output shortfalls as compared to the radial efficiency measures of standard DEA models, in which efficiency is determined based on either equi-proportional input-contraction or output-expansion. One noticeable advantage of SBM is that it is guaranteed to identify Pareto-efficient reference point for the evaluated DMU (Tone, 2001; Tone and Tsutsui, 2009). Therefore SBM resolves the "slacks" issue found to exist in the radial DEA models.

The significance of analyzing the internal structure of DMUs in different network structures and the attractiveness of the SBM model both contributed to a new research front in DEA, which Tone and Tsutsui (2009) coined as SBM-NDEA approach. The SBM-NDEA approach refers to a non-radial DEA model using slacks-based measure where the presence of intermediate products in a network system is considered formally. Several studies have already used the approach for real world applications such as in banking and finance (Lozano, 2016; Lu et al., 2014), transportation (Yu, 2010; Zhao et al., 2011), telecommunications (Moreno et al., 2013), and government (Amatatsu et al., 2012). However, the efficiency assessment of all of these studies is based on efficiency scores computed as a ratio of the average input reduction and output increases of the different processes in a DMU. Such formulation indicates that the only way to increase system efficiency is through making each process efficient as possible without increasing (decreasing) the inputs (outputs) of each of the processes.

Alternatively, one can consider a system perspective instead of a process perspective in taking into account DMU's exogenous inputs and outputs. Input slacks and output shortfalls measured at the system level provides the opportunity to the different processes to increase some inputs or decrease some outputs if it is deemed beneficial to the entire system. Such opportunity does not exist when the slacks and shortfalls are measured at the process level. Lozano (2015) emphasized that what is more important is to maintain the total input consumption and output production, irrespective of their allocation to the different processes. Therefore, taking the system perspective may uncover more sources inefficiencies and provide more ambitious targets through effective allocation of resources within the DMU than a process perspective.

2. EXISTING SBM-NDEA APPROACH

The following SBM-NDEA approach is from Tone and Tsutsui (2009). Suppose there exist n structurally homogenous DMUs (j = 1…n), i.e. all of them consists the same number of processes P (p =1…P) and for all the DMUs the inputs and outputs of each process are the same. Let m_p and k_p be the set of exogenous inputs consumed and of outputs produced by process p, respectively. The non-oriented weighted network SBM model under free link activities is as follows:

$$\varepsilon_0 = \min \frac{\sum_p \omega_p \left(1 - \frac{1}{m_p} \sum_{i \in m_p} \frac{s_i^{p-}}{x_{i0}^p}\right)}{\sum_p \omega_p \left(1 + \frac{1}{k_p} \sum_{i \in k_p} \frac{s_r^{p+}}{y_{r0}^p}\right)}$$

$$\text{s.t.} \sum_j \lambda_j^p x_{ij}^p = x_{i0}^p - s_i^{p-}, \qquad i = 1, \dots, m_p$$

$$\sum_j \lambda_j^p y_{rj}^p = y_{r0}^p + s_r^{p+}, \qquad r = 1, \dots, k_p$$

$\sum_j \lambda_j^p z_{dj}^{(p,t)} = \sum_j \lambda_j^t z_{dj}^{(p,t)}, \qquad d = 1, \dots, D$

$\lambda_j^p \geq 0 \quad \forall j \forall p,\ s_i^{p-} \geq 0 \quad \forall i \forall p,\ s_r^{p+} \geq 0 \quad \forall r \forall p \qquad (1)$

where ε_0 is the overall efficiency of DMU 0; x_{ij}^p is the observed amount of input i consumed by process p of DMU j; y_{rj}^p is the observed amount of output r produced by process p of DMU j; $z_{dj}^{(p,t)}$ is the observed amount of intermediate product d generated from process p to process t. D is the total number of linking intermediate products. In addition, s_i^{p-} and s_r^{p+} are input and output slacks of process p, respectively while λ_j^p is the intensity variable of process p of DMU j.

It is assumed here that an intermediate product d cannot be consumed and produced simultaneously by a process. That is, if $P^{in}(d)$ is the set of processes that generate the intermediate product d while $P^{out}(d)$ is the set of processes that consumed the intermediate product d, then $P^{in}(d) \cap P^{out}(d) = \emptyset$ for all d. Also, without a loss of generality, we assume that the intermediate products are completely generated and consumed within the own DMU.

The objective function denotes the fraction of the weighted average input reduction of the different processes to the weighted average output expansion of the different processes. The numerator is always less than or equal to one while the denominator is always greater than or equal to one. The first constraint measures the potential input reduction for each process. The non-negativity of the slack variables s_i^{p-} means that no increase is considered in any of the processes. The second constraint computes the potential output expansion for each process. The non-negativity of the shortfall variables s_r^{p+} forbids that any process reduces any output. The third constraint corresponds to the global balance equations for the intermediate products, i.e. the amount consumed by the different process must be equal the amount produced. This represents to the free links case introduced by Tone and Tsutsui (2009) where linking activities are freely determined (discretionary) while keeping continuity between input and output. The fixed links case where linking activities are kept unchanged (non-discretionary) would substitute the third constraint in model (1) by

$\sum_j \lambda_j^p z_{dj}^{(p,t)} = z_{dj}^{(p,t)}, \qquad d = 1, \dots, D$

$\sum_j \lambda_j^t z_{dj}^{(p,t)} = z_{dj}^{(p,t)}, \qquad d = 1, \dots, D \qquad (2)$

Note that model (1) corresponds to the CRS case. Adding $\sum_j \lambda_j^p = 1$ in the constraints will make the model assume VRS. As mentioned, model (1) follows non-oriented case but it can be easily transformed into input-oriented and output-oriented cases by the changing the objective function to

$$\varepsilon_0 = \min \sum_p \omega_p \left(1 - \frac{1}{m_p} \sum_{i \in m_p} \frac{s_i^{p-}}{x_{i0}^p}\right) \qquad (3)$$

and

$$(\xi_0)^{-1} = \min \sum_p \omega_p \left(1 + \frac{1}{k_p} \sum_{i \in k_p} \frac{s_r^{p+}}{y_{r0}^p}\right), \qquad (4)$$

respectively.

3. PROPOSED NON-RADIAL NETOWRK DEA APPROACH

The full list of the notations used in the proposed non-radial network DEA approach can be found in the Appendix. Our approach is inspired in network SBM approach of Lozano (2015) and in the relational network DEA approach of Kao and Hwang (2008) and Kao (2009). The proposed non-radial DEA approach can be formulated with an objective function as

$$\xi_0 = \min \frac{1 - \frac{1}{|\mathbb{I}^p + \mathbb{M}^p|}\left[\omega_i * \left(\sum_{i \in \mathbb{I}^p} \frac{s_i^-}{x_{i0}^p} + \sum_{i \in \mathbb{M}^p} \frac{t_f^-}{z_{f0}^p}\right)\right]}{1 + \frac{1}{|\mathbb{O}^p + \mathbb{N}^p|}\left[\mu_k * \left(\sum_{i \in \mathbb{O}^p} \frac{s_k^+}{y_{r0}^p} + \sum_{i \in \mathbb{N}^p} \frac{t_g^+}{z_{g0}^p}\right)\right]} \qquad (5)$$

subject to

$$\sum_p \sum_j \lambda_j^p x_{ij}^p = x_{i0} - s_i^- \qquad \forall i \in \mathbb{I}^p \qquad (6)$$

$$\sum_p \sum_j \lambda_j^p y_{rj}^p = y_{r0} + s_r^+ \qquad \forall r \in \mathbb{O}^p \qquad (7)$$

$$\sum_{p \in \mathbb{P}^{out}(g)} \sum_j \lambda_j^p z_{fj}^p \geq \sum_{p \in \mathbb{P}^{in}(f)} \sum_j \lambda_j^p z_{gj}^p \qquad \forall f, g \qquad (8)$$

$$\lambda_j^p \geq 0 \qquad \forall j \forall p \qquad (9)$$

$$s_i^-, s_k^+, t_f^-, t_g^+ \geq 0 \qquad \forall i, k, f, g \qquad (10)$$

The objective function (5) represents the fraction of the average total input reduction to the average total output expansion. The numerator is always less than or equal to one while the denominator is greater than or equal to one. Constraints (6) calculate the potential reduction that can be achieved for each input. Likewise, constraints (7) compute the potential expansion in the total amount produced of each output. Constraints (8) are the relaxed version of the corresponding free links SBM-NDEA constraints in model (1). The equality sign in model (1) corresponds to letting the shadow prices of the intermediate products free whereas the inequality character in constraints (8) follows the assumption of non-negative shadow prices. Relaxing the constraint s has the advantag e of enlarging the feasibility

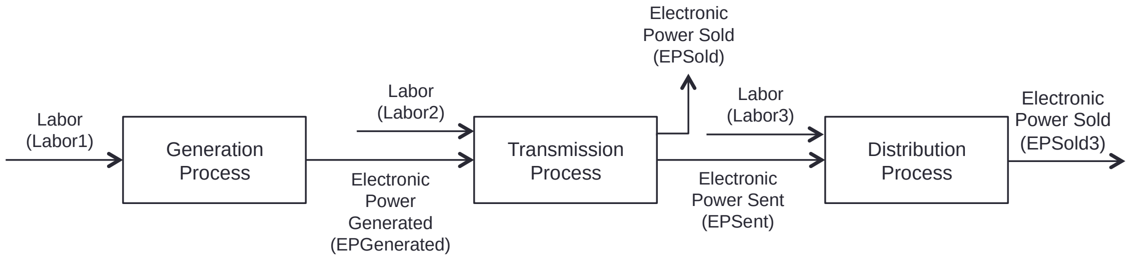

Figure 1: Tone and Tsutsui (2009) electric utility companies.

region of the optimization model, thus increasing the discriminating power of the approach. Further, in contrast to the SBM-NDEA model (1), the slacks in model (5-10) are computed at the system level, that is, with respect to the total consumption of inputs and outputs. Such view indicates that what is more vital is the total consumption and output production, irrespective of their allocation to the different processes.

2.1 Identifying Characteristics of the Non-radial DEA Approach

The proposed approach differs from existing SBM-NDEA approaches particularly with that of Tone and Tsutsui (2009) in terms of the following reasons. First, Tone and Tsutsui's SBM-NDEA approach impose non-negative slacks in each process while the proposed non-radial network approach compute the slacks in the whole system and that global slack is the one that should be non-negative. If an input or an output is consumed or produced by just one process then there is no difference. However, if an input/output is consumed/produced in more than one process then the proposed non-radial network approach take a system view that considers that what matters is the total input/output consumption/production. With the relaxation of the intermediate products balance constraints, the proposed approach involved a larger feasibility region and thus has more discriminating power than Tone and Tsutsui's SBM-NDEA approach.

Second, Tone and Tsutsui's objective function is a ratio in which the numerator is the weighted average input reduction of the different processes and the denominator is analogously a weighted average of the output increases of the different processes. Whereas, the proposed approach's objective function is a ratio in which the numerator is the (average) global input reduction and the denominator the (average) global output reduction. It must be noted again that the adoption of this objective function is in parallel with the SBM logic only that at the global (i.e. system) level.

Lastly, Tone and Tsutsui's objective function does not consider slacks of intermediate products in the determination of the overall efficiency and process efficiency because the model employ the equality constraints $\sum_j \lambda_j^p z_{dj}^{(p,t)} = \sum_j \lambda_j^t z_{dj}^{(p,t)}, \forall d$ for intermediate products in model (1). However, as Fukuyama and Mirdehghan (2012) asserted ignoring slacks in intermediate products is inappropriate when one is after the efficiency status of DMUs and their processes. Therefore, the proposed approach included slacks variables t_f^-, t_g^+ associated with the intermediate products in the objective function.

4. NUMERICAL ILLUSTRATION

The dataset from Tone and Tsutsui (2009) will be used to illustrate the application of the proposed non-radial network DEA approach. It consists of 10 DMUs corresponding to electric utility companies (see Figure 1). Each electric utility company consists of three processes connected in series: generation, transmission, and distribution. Each process has an exogenous input referred to as Labor that corresponds to the number of employees. There are two outputs that correspond to the Electric Power Sold to large customers (exogenous output of Transmission process) and to small customers (exogenous output of Distribution process). Lastly, there are intermediate products: Electric Power Generated (which is produced by Generation process and consumed by Transmission process) and Electric Power Sent (which is produced by Transmission process and consumed by Distribution process). Table 1 presents the data for the inputs, outputs, and intermediate products of the 10 DMUs.

The proposed non-radial network DEA model was built and run for analysis using the optimization modeling software called LINGO 15.0. It must be noted that the DEA models built for this specific analysis alone follows the input-oriented case where the linear programming model is configured so as to determine how much the input use of a DMU could reduce if used efficiently in order to achieve the same output level. More so, a variable returns-to-scale was assumed. These assumptions were made in to order to be able to compare the results of the proposed model with the results of both the black box and SBM-NDEA models published in Tone and Tsutsui's (2009) work.

Table 1: Tone and Tsutsui's (2009) sample data set for electric utility companies.

DMU	Labor1	Labor2	Labor3	EPSold2	EPSold3	EPGenerated	EPSent
A	0.838	0.277	0.962	0.879	0.337	0.894	0.362
B	1.233	0.132	0.443	0.538	0.18	0.678	0.188
C	0.321	0.045	0.482	0.911	0.198	0.836	0.207
D	1.483	0.111	0.467	0.570	0.491	0.869	0.516
E	1.592	0.208	1.073	1.086	0.372	0.693	0.407
F	0.790	0.139	0.545	0.722	0.253	0.966	0.269
G	0.451	0.075	0.366	0.509	0.241	0.647	0.257
H	0.408	0.074	0.229	0.619	0.097	0.756	0.103
I	1.864	0.061	0.691	1.023	0.38	1.191	0.402
J	1.222	0.149	0.337	0.769	0.178	0.792	0.187
Average	1.020	0.127	0.560	0.763	0.273	0.832	0.290

Three DEA models (i.e., black box model, SBM-NDEA model, and proposed model) were compared and the resulting efficiency scores are shown in Table 2. It can be seen that the efficiency scores of the black box model tend to be higher than those of the SBM-NDEA and proposed models. Eight out of the 10 DMUs were considered efficient when using the black box model whereas no DMUs were considered as overall efficient when using the SBM-NDEA and proposed models. Such results are not surprising as black box models do not consider the internal structure of DMUs unlike the network DEA models which provides the latter that advantage of determining more sources of inefficiencies in the sub processes. Thus, it is apparent that when one uses a network DEA model it would hardly provide an efficient DMU, which the proposed model adhered to. This observation also indicates that the black box model is inferior in terms of the discriminate power to that of the SBM-NDEA and the proposed models.

Figure 2 clearly shows that the discriminate power of the black box model is inferior to that of the SBM-NDEA and proposed models. In addition, the figure shows that the ranks of the scores of the three models are not always corresponding. For example, DMU B is scored worse in the black box model, while better in both the SBM-NDEA and proposed models. However, when comparing SBM-NDEA and proposed models it can be observed that ranks of scores of the former is always higher than the latter, indicating higher discriminate power of the latter. These observations in the rankings between the SBM-NDEA and proposed models can be attributed to the proposed model's inclusion

Table 2: Efficiency scores for the three DEA models.

DMU	Black Box	SBM-NDEA	Proposed
A	1.000	0.478	0.475
B	0.531	0.739	0.612
C	1.000	0.968	0.968
D	1.000	0.719	0.719
E	1.000	0.456	0.428
F	0.681	0.719	0.708
G	1.000	0.947	0.830
H	1.000	0.969	0.871
I	1.000	0.832	0.832
J	1.000	0.590	0.577

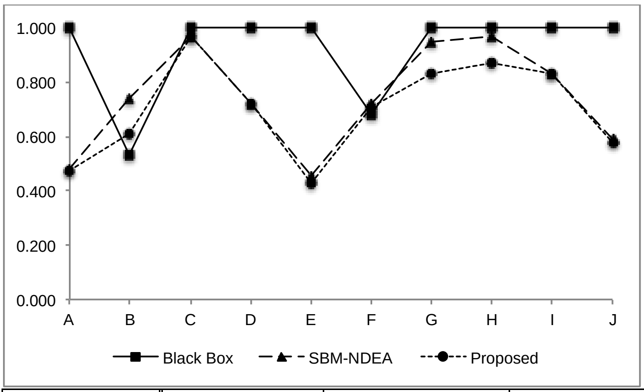

Figure 2: Comparisons of scores among the three DEA models.

of intermediate slacks in the objective function and computing the slacks in the system level. The consistencies in the rankings between the SBM-NDEA and the proposed model may suggest that both models work in the same way except that the latter has more discriminate power than the former. More so, from these results one can safely conclude that the black-box model is inferior in providing accurate efficiency measurement as opposed to the SBM-NDEA and proposed models.

Table 3: Process efficiencies of SBM-NDEA and proposed models.

DMU	SBM-NDEA Model			Proposed Model		
	Process 1	Process 2	Process 3	Process 1	Process 2	Process 3
A	0.633	0.339	0.393	0.633	0.321	0.378
B	0.349	1.000	1.000	0.261	0.539	1.000
C	1.000	1.000	0.919	1.000	1.000	0.919
D	0.297	1.000	1.000	0.297	1.000	1.000
E	0.263	1.000	0.377	0.202	1.000	0.368
F	1.000	0.403	0.596	1.000	0.362	0.588
G	1.000	1.000	0.868	0.712	1.000	0.863
H	0.922	1.000	1.000	0.787	0.780	1.000
I	1.000	1.000	0.581	1.000	1.000	0.579
J	0.288	0.377	1.000	0.263	0.359	1.000

Table 3 presents the process efficiencies derived from both the SBM-NDEA and the proposed models. It must be noted that the overall efficiency scores reported in Table 2 could be derived using the process efficiencies indicated in Table 3 with the corresponding input weights for each process, $\omega_1 = \omega_3 = 0.4$ and $\omega_2 = 0.2$. Given that the intermediate products constraints of the proposed model are the relaxed version of the SBM-NDEA, then it is logical that former's process efficiencies are lower than or equal to those reported using the latter model. The differences are, however, small with the largest difference found in process 1 of DMU G amounting to 0.288 (difference between 1.000 and 0.712). More so, it can be observed from the resulting process efficiencies why none of the DMUs is overall efficient. That is because the overall efficiency is a weighted average of the three processes, which based from Table 3 no DMU had all three processes fully efficient. The identifying difference between the results of SBM-NDEA and the proposed model is that the latter provides lower process efficiencies as compared to the former. Such results are attributable to the combination of relaxing the intermediate products constraints, considering intermediate product slacks in the objective function, and computing these slacks at the system level in the proposed model.

5. CONCLUSION

A modified non-radial approach is proposed for network DEA. Particularly, the proposed approach differs from Tone and Tsutsui's (2009) existing SBM-NDEA approach for the following reasons. The input slacks and output shortfalls of the proposed approach are measured at the system level thus giving freedom to the different processes to increase some inputs or decrease some outputs if that is deemed beneficial to the entire system. That is something not allowed when the input slacks and output shortfalls are measured at the process level just as in the case of Tone and Tsutsui's SBM-NDEA model. It has been observed from the existing SBM-NDEA models that all adhered to a process perspective in computing for the efficiency of DMUs, that is, the input slacks and output shortfalls are computed in each process. The adoption of process perspective restricts the different processes to increase/decrease some inputs/outputs and thus limits the overall improvement that a DMU can achieve. With that in mind, it may be beneficial if a system perspective will be adopted in the efficiency measurement. That is, it gives freedom to the different processes to increase/decrease some inputs/outputs if that is advantageous to the whole system. Also, by taking the system perspective it may uncover more sources inefficiencies and provide more ambitious targets through effective allocation of resources within the DMU than a process perspective.

Furthermore, aligned with the adoption of the system perspective in efficiency measurement, the objective function of the proposed approach considers the ratio of the average global input reduction to the average global output reduction. This is in contrast with Tone and Tsutsui's (2009) SBM-NDEA approach where the objective function is a ratio of the weighted average input reduction of the different processes to the weighted average of the output increases of the different processes. Lastly, the proposed approach considers the slacks of intermediate products in determining the overall and process efficiency scores that are not consider previously. The presence of intermediate products is an essential part in considering the internal structure of DMUs as it provides evidence on how processes are interconnected to each other, which then justifies the need for it to be accounted directly in the efficiency measurement.

Future research can further relax the assumption that all sub processes utilize exactly the same set of inputs to generate the same set of outputs. In many real-world settings, the homogeneity of input and output factors may not hold. Consider, for instance, the case of different production lines in a manufacturing factory, or differently functioning business units operating within an organization. One may opt to extend previous studies into a more general scenario considering intermediate measures and no matter what type of input and output measures are consumed and produced, respectively. The general scenarios that can considered with respect to input and output measures are: (a) all sub processes within any DMU have disjoint output sets, (b) some output measures are shared among different sub processes, (c) all inputs can be split up in terms of proportions across sub processes, and (d) some inputs cannot be readily split up and distributed to various sub processes.

Appendix A. Notations

The following are the notations used in the proposed non-radial network DEA approach:

n	Number of DMUs
P	Number of processes
$\mathbb{I}^p$	Set of exogenous inputs of process *p*
x_{ij}^p	Observed amount of exogenous input *i* consumed by process *p* of DMU *j*
$\mathbb{O}^p$	Set of exogenous outputs of process *p*
y_{rj}^p	Observed amount of exogenous output *r* produced by process *p* of DMU *j*
$\mathbb{M}^p$	Set of endogenous inputs of process *p*
z_{fj}^p	Observed amount of endogenous input *f* produced by other processes *p*
$\mathbb{N}^p$	Set of endogenous outputs of process *p*
z_{gj}^p	Observed amount of endogenous output *g* utilized by other processes *p*
$\mathbb{p}^{out}(g)$	Set of processes that generate the endogenous output *g*
$\mathbb{p}^{in}(f)$	Set of processes that consume the endogenous input *f*
s_i^-	Slack of exogenous input *i*
s_i^+	Shortfall of exogenous output *r*
t_f^-	Slack of endogenous input *f*
t_g^+	Shortfall of endogenous output *g*
λ_j^p	Intensity variable used for process *p* of DMU *j* when computing linear combinations of the observed DMUs
0	DMU being evaluated/projected

REFERENCES

Amatatsu, H., Ueda, T., and Amatatsu, Y. (2012). Efficiency and returns-to-scale of local governments. *Journal of Operational Research Society*, **63**, 299-305.

Andersen, P. and Petersen, N.C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, **39**, 1261-1264.

Angulo-Meza, L., and Lins, M.P.E. (2002). Review of methods for increasing discrimination in data envelopment analysis. *Annals of Operations Research*, **116**, 225-242.

Avkiran, N.K. (2009). Opening the black box of efficiency analysis: An illustration with UAE banks. *Omega*, **37**, 930-941.

Avkiran, N.K. and McCrystal, A. (2012). Sensitivity analysis of network DEA: NSBM versus NRAM. *Applied Mathematics and Computation*, **218**, 11226-11239.

Beasley, J.E. (1990). Comparing university departments. *Omega*, **18**, 171-183.

Charnes, A., Cooper, W.W., and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, **2**, 429-444.

Chen, Y., Cook, W.D., Li, N., and Zhu, J. (2009). Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research*, **196**, 1170-1176.

Chen, Y., Cook, W.D., and Zhu, J. (2010). Deriving the DEA frontier for two-stage processes. *European Journal of Operational Research*, **202**, 138-142.

Chen, Y., Li, Y., Liang, L., Salo, A., and Wu, H. (2016). Frontier projection and efficiency decomposition in two-stage processes with slacks-based measures. *European Journal of Operational Research*, **250**, 543-554.

Chen, C. and Yan, H. (2011). Network DEA model for supply chain performance evaluation. *European Journal of Operational Research*, **213**, 147-155.

Cook, W.D., Zhu, J., Bi, G.B., and Yang, F. (2010). Network DEA: Additive efficiency decomposition. *European Journal of Operational Research*, **207**, 1122-1129.

Dyson, R.G., Allen, R., Camanho, A.S., Podinovski, V.V., Sarrico, C.S., and Shale, E.A. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research*, **132**, 245-259.

Färe, R. and Grosskopf, S. (2000). Network DEA. *Socio-Economic Planning Sciences*, **34**, 35-49.

Farrell, M.J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society*, **120**, 253-290.

Fukuyama, H. and Mirdehghan, S.M. (2012). Identifying the efficiency status in network DEA. *European Journal of Operational Research*, **220**, 85-92.

Fukuyama, H. and Weber, W.L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega*, **38**, 398-409.

Hsieh, L.F. and Lin, L.H. (2010). A performance evaluation model for international tourist hotels in Taiwan: An application of the relational network DEA. *International Journal of Hospitality Management*, **29**, 14-24.

Kao, C. (2009). Efficiency decomposition in network data envelopment analysis: A relational model. *European Journal of Operational Research*, **192**, 949-962.

Kao, C. (2014a). Efficiency decomposition in network data envelopment analysis with slacks-based measures. *Omega*, **45**, 1-6.

Kao, C. (2014b). Network data envelopment analysis: A review. *European Journal of Operational Research*, **239**, 1-16.

Kao, C. and Hwang, S.N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operations Research*, **185**, 418-429.

Kao, C. and Hwang, S.N. (2010). Efficiency measurement for network systems: IT impact on firm performance. *Decision Support Systems*, **48**, 437-446.

Lewis, H. F. and Sexton, T.R. (2004). Network DEA: Efficiency analysis of organizations with complex internal structure. *Computers & Operations Research*, **31**, 1365-1410.

Lin, T.Y. and Chiu, S.H. (2013). Using independent component analysis and network DEA to improve bank performance evaluation. *Economic Modelling*, **6**, 4187-4192.

Lozano, S. (2015). Alternative SBM Model for Network DEA. *Computers and Industrial Engineering*, **82**, 33-40.

Lozano, S. (2016). Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector. *Omega*, **60**, 73-84.

Lozano, S., Gutiérrez, E., and Moreno, P. (2013). Network DEA approach to airports performance assessment considering undesirable outputs. *Applied Mathematical Modelling*, **37**, 1665-1676.

Luo, Y., Bi, G., and Liang, L. (2012). Input/output indicator selection for DEA efficiency evaluation: An empirical study of Chinese commercial banks. *Expert Systems with Applications*, **39**, 1118-1123.

Lu, W.M., Wang, W.K., and Kweh, Q.L. (2014). Intellectual capital and performance in the Chinese life insurance industry. *Omega*, **42**, 65-74.

Matthews, K. (2013). Risk management and managerial efficiency in Chinese banks: A network DEA framework. *Omega*, **41**, 207-215.

Moreno, P., Lozano, S., and Gutiérrez, E. (2013). Dynamic performance analysis of U.S. wireline telecommunication companies. *Telecommunications Policy*, **37**, 469-482.

Premachandra, I.M., Bhabra, G.S., and Sueyoshi, T. (2009). DEA as a tool for bankruptcy assessment: A comparative study with logistic regression technique. *European Journal of Operational Research*, **193**, 412-424.

Sarkis, J. (2000). Analysis of the operational efficiency of major airports in the United States. *Journal of Operations Management*, **18**, 335-351.

Sherman, H.D. and Gold, F. (1985). Bank branch operating efficiency: Evaluation with data envelopment analysis. *Journal of Banking and Finance*, **9**, 297-315.

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, **130**, 498-509.

Tone, K. and Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research*, **197**, 243-252.

Tone, K. and Tsutsui, M. (2010). Dynamic DEA: A slacks-based measure approach. *Omega*, **38**, 145-156.

Yu, M.M. (2010). Assessment of airport performance using the SBM-NDEA model. *Omega*, **38**, 440-452.

Zhao, Y., Triantis, K., Murray-Tuite, P., and Edara, P. (2011). Performance measurement of a transportation network with a downtown space reservation system: A network-DEA approach. *Transportation Research Part E: Logistics and Transportation Review*, **47**, 1140-1159.