Using Latin Hypercube Sampling to Improve Solution for a Supply Chain Network Design Problem

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Abstract. In this paper, we focus on a supply chain network design problem in determining the number and location of facilities, as well as the flows between them. Due to globalization and complexity in supply chain environment, recent supply chain network design studies have incorporated uncertainty and disruption risk into the design decisions in order to make supply chain network more robust and resilience. To deal with the unexpected disruptions, a Latin Hypercube Sampling (LHS) method is used to generate disruptive scenarios for model evaluation. We compare the solutions with those from the Monte Carlo Sampling (MCS) technique. The results show that LHS provides smaller standard deviation than the MCS technique as it yields a good approximation of the sample space. This further indicates that LHS use less sample size leading to more efficient computational time in obtaining solutions.

Keywords: Supply chain network design, Facility disruption, Two-stage stochastic program, Simulated annealing, Latin hypercube sampling

1. INTRODUCTION

Supply chain network design is a crucial strategic decision as it effects will last for many years under dynamic business environment (Klibi et al., 2010). In addition, uncertainties and risks are intensified in the supply chain network where demand and cost vary and disruptions are unpredictable. Therefore, the design of supply chain network should be resilience to uncertainty and possible disruptions in the network. In many supply chain network design studies, Monte Carlo Sampling technique has been applied in order to generate scenarios for model evaluation.

In this paper, we focus on the use of a sampling technique called Latin Hypercube Sampling for supply chain network design under facility disruptions.

In this study we use a two-stage stochastic programming model from Rienkhemaniyom et al. (2016) that formulated for a supply chain network design problem considering possibility of facility disruption. The model is used for a single-product supply chain network which includes suppliers, manufacturing plants, warehouses, and retailers where the facilities are located in different places around the world. The objective function is to maximize the expected profit under facility disruptions. In the first stage, suppliers and warehouses are selected. In the second stage, the quantity of product purchased, produced and transported between facilities are determined under disruptive scenarios so that the expected profit is maximized. A simulated annealing meta-heuristic algorithm (SA) is used to find near optimal solutions. In terms of sampling technique, the Monte Carlo Sampling (MCS) technique is used in constructing the sample average problem in order to get the estimate of the objective function value.

Beside MCS, there are several sampling techniques available to reduce the variance of the estimate and to improve the quality of solutions, for example Latin Hypercube Sampling (LHS), and Antithetic Variates Sampling (AVS).

In this paper, we apply Latin Hypercube Sampling (LHS) technique to generate disruptive scenarios. Several studies have discussed the advantages of LHS, for instance, Fattahi et al. (2014) and Shi et al. (2013) stated that LHS covers more of domain of the random variables.

Hence, the objective of this study is to improve solution of a supply chain network design problem by using LHS technique to sampling disruptive scenarios.

2. LITERATURE REVIEW

For a stochastic linear program, it can be solved approximately by generating a subset of all possible random scenarios. The value of the optimal solution to the sampled problem provides an estimate of the true objective function value. However, the estimator is known to be optimistically biased (Freimer et al., 2012). The bias and variance of the solution are also related to which sampling method has been used. Freimer et al. (2012) studied the impact of sampling methods: antithetic variates and Latin Hypercube sampling, on the bias and variance reduction in a two-stage stochastic linear program for a newsvendor problem. The first-stage solution is to choose the order quantity, while the second-stage solution decides on how much of the available stock after demand has been realized. The results show that both sampling method are effective at reducing variance, however LHS sampling approach outperforms the antithetic variates method in both bias and variance reduction.

K.R.M dos Santos et al. (2015) presented a benchmark studies that comparing MCS with four modern sampling techniques, including Importance Sampling Monte Carlo, Asymptotic Sampling, Enhanced Sampling, and Subset Simulation. The sampling techniques are combined with three schemes for generating the samples, which consists of Simple Sampling, LHS, and Antithetic Variates Sampling. The results from this paper show that Importance Sampling is extremely efficient for evaluating small failure probabilities but it can be an issue for some problems. Subset Simulation yields very good performance for all problems. Enhanced Sampling has performance better than Asymptotic Sampling. LHS outperforms Simple Sampling and Antithetic Variates Sampling.

The Latin Hypercube Sampling (LHS) is a stratified-random procedure where samples are obtained from their distributions. The cumulative distribution for each variable is divided into N equiprobable intervals. Assuming that the variables are independent, a value is selected randomly from each interval. (Yushumito et al., 2012).

LHS has been used in various applications, including supply chain network design problem as it becomes a very large and complex problem when disruption is considered (Garcia-Herreros and Grossmann, 2013). Keyvanshokooh (2015) has applied LHS to generate scenarios for transportation cost. Govindan and Fattahi (2015) used LHS to generate a set of demand scenarios. Yushumito et al., (2012). Deleris et al. (2005) proposed a Monte Carlo based technique to evaluate supply chain network under risks.

From the review, we observe that Monte Carlo Sampling (MCS) is a common and widely used technique for scenario generations, including disruption, in supply chain network design problem. For LHS, it has also been used to generate demand and cost scenarios. However, very few studies have used Latin Hypercube Sampling for generating disruptive scenarios. In this paper, we use LHS technique to generate disruptive scenario for a supply chain network design proposed by Rienkhemaniyom et al. (2016). Then, we compare the solution quality, in terms of variance, and computational time with the original work.

3. SUPPLY CHAIN NETWORK DESIGN PROBLEM

3.1 Problem description

The original study of a supply chain network design problem proposed in Rienkhemaniyom et al. (2016) involves a two-stage stochastic program for a supply chain network design under facility disruption for a four-stage supply chain which consists of suppliers, manufacturing plants, warehouses, and retailers (as show in Figure 1) under single-product supply chain network environment. The facilities are located around the world. Locations of manufacturing plants and retailers are known, while suppliers and location of warehouses are decisions to be made. The objective function is to maximize the expected profit under facility disruptions. Simulated annealing meta-heuristic algorithm is used to find a good solution, and MCS technique is used for generating disruptive scenarios. In this study, a disruption may occur at any region, which leads to disruption of suppliers, manufacturing plants, or warehouses that are located in the disrupted region. The decision variables are to select the suppliers and the location of warehouses so that the expected profit of the supply chain is maximized.

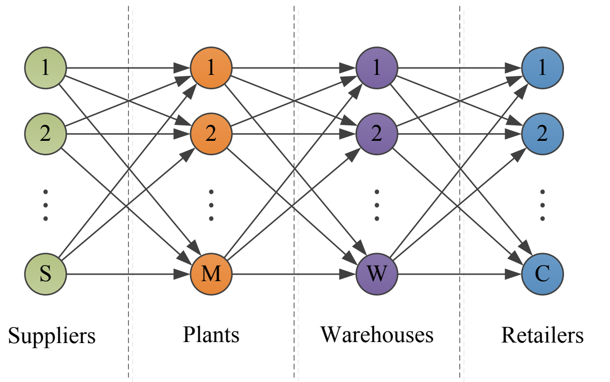

Figure 1. Supply chain network

3.2 Mathematical model

The two-stage stochastic programming model of a supply chain network design problem that proposed in Rienkhemaniyom et al. (2016) can be summarized as follows.

Notation:

Sets

S	Set of suppliers
M	Set of manufacturing plants
W	Set of warehouses
C	Set of retailers
L	Set of warehouse capacities
K	Set of disruptive scenarios

Indices

s	Index of suppliers $s \in S$
m	Index of manufacturing plants $m \in M$
w	Index of warehouses $w \in W$
c	Index of retailer $c \in C$
l	Index of warehouse capacities $l \in L$
k	Index of disruptive scenarios $k \in K$

Parameter

cap_m	Production capacity at manufacturing plant m
cap_s	Capacity at supplier s
cap^l_w	Capacity at warehouse w of size l
d_c	Demand for products at retailer c
msm	Minimum transportation quantity form supplier s to manufacturer m
f^l_w	Fixed cost of opening a warehouse w of capacity l
pm_{sm}	Purchasing cost of material from supplier s by plant m
tr_{sm}	Transportation cost per unit from plant m to warehouse w
tr_{wc}	Transportation cost per unit from warehouse w to retailer c
pc_m	Production cost for a product at plant m
np	Price of a product
ls_c	Lost sales cost at retailer c

Random Parameters

$$\alpha_{sk} = \begin{cases} 1 & \text{if supplier } s \text{ is operated in scenario } k \\ 0 & \text{if a disruption occurs at supplier } s \text{ in scenario } k \end{cases}$$

$$\delta_{wk} = \begin{cases} 1 & \text{if warehouse } w \text{ is operated in scenario } k \\ 0 & \text{if a disruption occurs at warehouse } w \text{ in scenario } k \end{cases}$$

$$\beta_{mk} = \begin{cases} 1 & \text{if plant } m \text{ is operated in scenario } k \\ 0 & \text{if a disruption occurs at plant } m \text{ in scenario } k \end{cases}$$

p_k The probability of disruption occurs in scenario k

First-stage Decision Variables

$$x^l_w = \begin{cases} 1 & \text{if warehouse } w \text{ is operated with size } l \\ 0 & \text{otherwise} \end{cases}$$

$$y_s = \begin{cases} 1 & \text{if supplier } s \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

Second-stage Decision Variables

QSM_{smk}	Quantity of raw material purchased from supplier s by plant m in scenario k
QMW_{mwk}	Quantity of products shipped from plant m to warehouse w in scenario k
QWC_{wck}	Quantity of products shipped from warehouse w to retailer c in scenario k
LD_{ck}	Quantity of sales lost at retailer c in scenario k

3.2.1 Objective Function

The objective is to maximize the expected supply chain profit (Z) which is the difference between the total cost and the expected revenue. The first-stage cost is the fixed cost of opening warehouse and the second-stage cost is the difference of the expected revenue and operation costs.

$$\begin{aligned} \text{Maximize} \quad Z = & -\sum_{w \in W} \sum_{l \in L} f^l_w x^l_w + \sum_{k \in K} p_k \Bigg[np \left(\sum_{w \in W} \sum_{c \in C} QWC_{wck} \right) \\ & - \left(\sum_{s \in S} \sum_{m \in M} pm_{sm} QSM_{smk} \right) \\ & - \left(\sum_{m \in M} \sum_{w \in W} tr_{mw} QMW_{mwk} + \sum_{w \in W} \sum_{c \in C} tr_{wc} QWC_{wck} \right) \\ & - \sum_{m \in M} pc_m \left(\sum_{w \in W} QMW_{mwk} \right) - \sum_{c \in C} ls_c LD_{ck} \Bigg] \end{aligned} \tag{1}$$

3.2.2 Constraints

Supplier capacity

$$\sum_{m\in M} QSM_{smk} \le cap_s \alpha_{sk} y_s, \quad \forall s \in S, k \in K \tag{2}$$

Inter-stage flow

$$msm \cdot \alpha_{sk} y_s \le QSM_{smk} \le cap_s \alpha_{sk} y_s, \quad \forall s \in S, k \in K \tag{3}$$

Production capacity

$$\sum_{w\in W} QMW_{mwk} \le cap_m \beta_{mk}, \quad \forall m \in M, k \in K \tag{4}$$

Material flow between suppliers and plants

$$\sum_{s\in S} QSM_{smk} = \sum_{w\in W} QMW_{mwk}, \quad \forall m \in M, k \in K \tag{5}$$

Warehouse capacity

$$\sum_{m\in M} QMW_{mwk} \le \sum_{l\in L} cap_w \delta_{wk} x_w^l, \quad \forall w \in W, k \in K \tag{6}$$

$$\sum_{l\in L} x_w^l \le 1, \quad \forall w \in W \tag{7}$$

Product flow between warehouse and retailers

$$\sum_{m\in M} QMW_{mwk} = \sum_{C\in C} QWC_{wck}, \quad \forall w \in W, k \in K \tag{8}$$

Demand requirement

$$\sum_{w\in W} QWC_{wck} + LD_{ck} = d_c, \quad \forall c \in C, k \in K \tag{9}$$

Non-negativity

$$QSM_{smk}, QMW_{mwk}, QWC_{wck}, LD_{ck} \ge 0 \tag{10}$$

For disruptive scenarios, the authors use the number of disasters from the Centre for Research on the Epidemiology of Disasters – CRED (http://www.emdat.be/). The number of disasters occurred in each region is used to estimate the occurrence or probability of disruption (as shown in Table 1). Hence, this probability will be used for generating disruptive scenarios by LHS.

Table 1. Number of disasters reported during 1966-2015.

Region	Number of disasters	Probability of occurrence
Africa	3,954	0.2
Asia	8,560	0.42
Australia	628	0.03
Europe	2,576	0.13
North America	2,575	0.13
South America	1,923	0.09

3.3 Solution Methodology

A simulated annealing meta-heuristic algorithm (SA) is applied in Rienkhemaniyom et al. (2016) for finding the solution. The first-stage solution is obtained outside the model, and the second-stage solution is obtained by re-optimizing the associated second-stage problem from a given first-stage solution. In this study LHS technique is used to construct the sample average problem instead of solving problem with all possible scenarios. This technique is known as the sample average approximation (SAA) technique in A. Shapiro (2003). The solver CPLEX is used to solve the second-stage sample average problem in order to estimate the objective function value. The general framework of the algorithm is showed in Figure 2

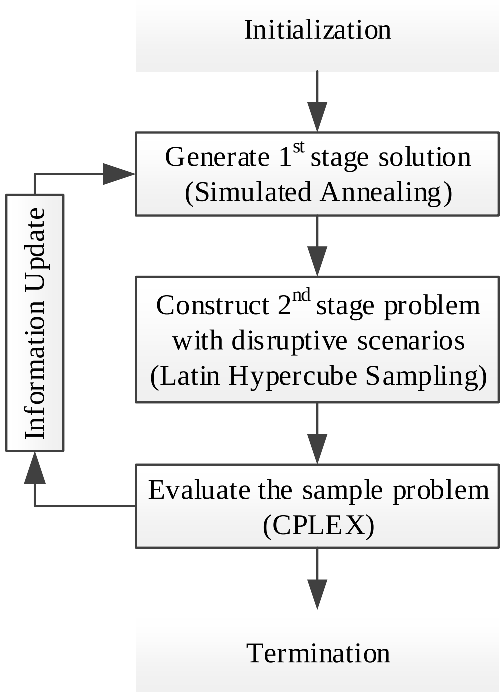

Figure 2. The general framework of the proposed methodology

3.3.1 Simulated Annealing

Simulated annealing (SA) is a probabilistic technique for solving global optimization problem in Aarts et al. (2005). The algorithm has inspiration come from annealing in metallurgy, a thermal process involving heating and controlled cooling of a solid for the particles to arrange themselves in the ground state.

Parameter

r	Index of iterations
c_r	Control parameter at iteration
c_0	Initial value of the control parameter
c_{min}	Minimum value of the control parameter
a, b	First-stage solution vectors
T	Number of solution evaluated in each iteration
N	Sample size (the number of sampled scenarios)
$\hat{Z}_N$	Estimate of the objective function value by using sample size N
$\mathcal{P}$	The probability vector of the first-stage variables (x and y)
U	Random number in [0, 1)
ρ	Coefficient of the control parameter update
λ	Coefficient of the probabilities update

$$exp\left(\frac{\hat{Z}_N(b)-\hat{Z}_N(a)}{c_r}\right) \tag{11}$$

The parameters ρ, λ, c_0, T, N and the probability vector $\mathcal{P}$ are initialize parameters. The algorithm generates a feasible first-stage solution vector from the probability vector $\mathcal{P}$. In each iteration, the algorithm will search for a better solution. The objective values are compared between current solution a and new solution b. The new solution b will be accepted if its objective value is better than the current objective values or if the probability of acceptance in Equation (11) is greater than a randomly generated number U. The control parameter c_r plays the role of temperature control in the cooling progress, which controls the probability of accepting solution in the algorithm. The value of c_r is decreased by a factor of ρ in each iteration, which reducing the chance of accepting worse solutions. At the end of each iteration, the probability vector is updated from the current solution vector. The algorithm terminates when c_r is less than a given value c_{min}.

4. LATIN HYPERCUBE SAMPLING

LHS technique was introduced by McKat et al. (1979). It is one of the sampling techniques that has been used in computer experiment for finding simulation solution. In this study, we use LHS to generate disruptive scenarios to ensure that all disruptive scenarios will be occurred in each input variables. Due to the advantages of LHS technique, we expect to obtain solutions that have a smaller standard deviation of the sample mean compared to those from the MCS technique in the original paper.

4.1 Latin Hypercube Design

A Latin hypercube design (LHD) with N samples and S input variables, denoted by LHD(N, S), is an $N \times S$ matrix (i.e. a matrix with N rows and S columns). Hence, a LHS can be generated by the following steps.

Step 1. The matrix $P(N,S)$ was generated, which each column of matrix P consists of a random permutation of the integer 1 to N.

$$P(N,S)=\begin{bmatrix} p_{1,1} & \cdots & p_{1,S} \\ \vdots & & \vdots \\ p_{N,1} & \cdots & p_{N,S} \end{bmatrix}$$

Step 2. The random numbers in Monte Carlo methods (Simply random sampling) $u_{k,j} \sim U(0, 1)$, $k = 1, \ldots, N$, $j = 1, \ldots, S$ which are mutually independent are generated to form the matrix $U(N,S)$.

$$U(N,S)=\begin{bmatrix} u_{1,1} & \cdots & u_{1,S} \\ \vdots & & \vdots \\ u_{N,1} & \cdots & u_{N,S} \end{bmatrix}$$

Step 3. The matrix $L(N,S)$ was generated from Equation (12), and this matrix is used for generating the disruptive scenarios.

$$L(N,S)=\frac{1}{N}\left[P(N,S)-U(N,S)\right] \tag{12}$$

Figure 3 shows a scatter plot of MCS for sample size N = 40 with two input variables. There is no pattern of points in each row and each column. The portions of its range are not represented in each of the input variables.

Figure 4 shows a scatter plot of LHS for sample size N = 40. We can see that each row and each column contain eight points, and the points are uniformly distributed in the corresponding row and column. It shows that each input variables has all portions of its range covered. For sample size N=40, when dividing the sample space of LHS into ten regions, reach region will contain exactly four points.

In this study, LHS technique is used to draw N samples from the set of original scenarios K. The objective function value Z in Equation (1) is estimated by substituting the first-stage solution obtained from SA, and then the sample average second-stage problem is constructed by using LHS technique. The random vector in the model is replaced by the sampled scenarios, and the probability p_k is changed to $1/N$ in the objective function. The resulting sample average problem is then solved by using CPLEX.

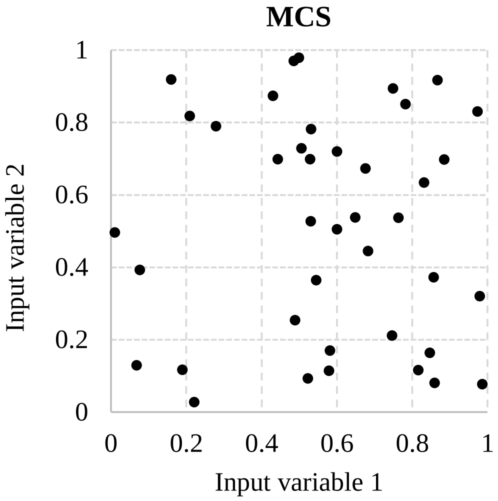

Figure 3. Scatter plot of samples generated by MCS

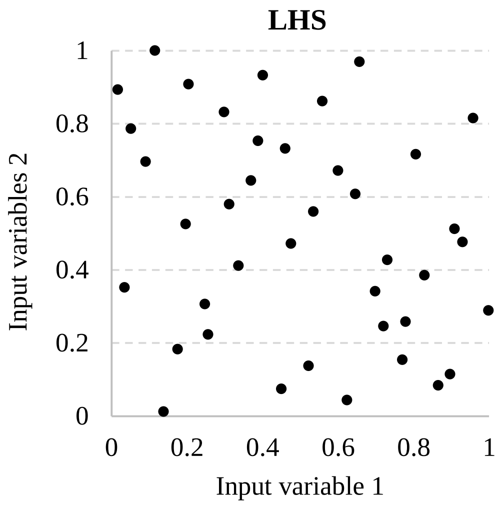

Figure 4. Scatter plot of samples generated by LHS

5. RESULTS AND DISCUSSIONS

Table 2 presents a summary of comparison of the supply chain profit values under disruptive scenarios between the MCS and our proposed LHS sampling from running 10 experiment trials in different cases. In most cases, the LHS provides better objective values and smaller standard deviations compared to the MCS. For example, at N = 50, the LHS sampling method yields a standard deviation of profit of $281,123.69, while it requires N = 100 for the MCS method to achieve the similar result (the standard deviation of profit under MCS sampling method is $293,121.62). When considering disruption with sample size 100, the MCS technique has an average profit value $9,833,097.05, and the standard deviation of profit value is $293,121.62. On the other hand, the supply chain network solutions from the LHS has an average profit value $10,023,344.56, and the standard deviation of profit value is $239,643.36. The results show that the standard deviation of solution when using LHS with sample size 100 can be reduced by 18.24 percent, and the average profit value is also improved, but it is not significantly different.

6. CONCLUSION

This paper presents an application of LHS technique in a supply chain network design problem. LHS is used to generate disruptive scenarios. The results show that the standard deviations of solutions are reduced as compared to the results from the MCS technique when using the same sample size as in Rienkhemaniyom et al. (2016). In solving large stochastic programming model for a supply chain network problem, LHS is an effective sampling technique that could provide good solutions while using small sample sizes. In future study, we will consider other new sampling techniques to reduce sample sizes and standard such as Optimal Latin Hypercube Sampling technique to generate disruptive scenarios.

Table 2: Comparison of objective values between two sampling techniques with different sample sizes

Sampling technique	Objective values	N = 20	N = 50	N = 100
Monte Carlo	Average	11,723,298.14	10,370,222.87	9,833,097.05
	Standard deviation	472,645.30	382,156.18	293,121.62
Latin Hypercube	Average	11,897,192.75	10,366,350.96	10,023,344.56
	Standard deviation	400,506.95	281,123.69	239,643.36
	Average Difference	1.48%	-0.04%	1.93%
	Standard deviation Difference	-15.26%	-26.44%	-18.24%

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