An Optimal Switching Model Considered the Risks of Production, Quality and Due Data for Limited-Cycle with Multiple Periods

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Abstract. - This paper aims to derive an optimal switch model considered the risks of production, due date and quality for limited-cycle with multiple periods. In the gobble supply chain environment, optimal operation management for horizontal integration of production network has been paid to attention recently. Due to the customer needs of reducing cost and delivery date shorting, prompt change in the production plan became more important. In the multi period system (For instance, production line.) where target processing time exists, production, idle and delay risks occur repeatedly for multiple periods. In such situations, delay of one process may influence the delivery date of an entire process. In this paper, we discuss minimum expected cost including production, due date and quality in a production process, where the risk depends on the previous situation and occurs repeatedly throughout multiple periods. Also, the policy of optimal switching for parallel production system will be analyzed.

Keywords: operational research, supply chain management, production management.

1. INTRODUCTION

This paper aims to derive an optimal switch model considered production, due date and quality for production system. In the gobble marketing environment, the horizontal integration of cooperation by information systems of enterprises or factories that are related to make a product is an important problem in Supply Chain Management.

On the other hand, variable production networks is not only one of the most important issues in manufacture management and operation research but also a significant factor affecting supply chain (SC). The supply collaborate concept requires formation and optimization of production or supply networks, characterized by intensive communication between the distributed entities. The goal is to allocate among the collaborating partners the production demand. This capability provides the entire network with the required flexibility to respond quickly to demand disruption for the products and services (Y. Nof et al., 2015). This paper analyzes an optimal switching model for parallel production system with multiple periods in SCS.

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In any social system or production process with multi periods and predetermined target time, system idleness and process delay risks exists throughout the multiple periods. In such situations, delay of one process sometimes affect the delivery date of the entire process. This kind of problem is called "a limited-cycle problem with multiple periods", and is seen in production lines, time-bucket balancing, production seat systems and so on (Yamamoto H. et al., 2006; Matsui M., 2008) In this paper, we discuss minimum expected risk (cost) in a parallel production process, where the risk depends on the previous situation and occurs repeatedly throughout multiple periods in smart supply chain environment.

As limited-cycle problems of production line, Verzijl (1976) analyzed the element and construction of the production system. Enns (2001) presented a framework for the analysis of delays within the production system. Benders (2002) gave a review for the origin and solution of period batch control system. Xia and Wu (2005) presented an easily implemented hybrid algorithm for the multi-objective flexible job-shop problem. Recently, Wu and Zhou (2008) concerned with the problem in scheduling a set of jobs associated with random leadtime on a single machine so as to minimize the expected maximum lateness in stochastic environment.

As limited-cycle problems of operation management, Safaei and Tavakkoli-Moghaddam (2009) proposed an integrated mathematical model of the multi-period cell formation and production planning in a dynamic cellular manufacturing system to minimize costs through a mixed integer programming technique. Porkka et al. (2003) proposed a mixed integer linear programming based capacitated lot sizing models that included carryovers incorporating set-up times with associated costs. Moreno et al. El Hafsi and Bai (1998) employed an optimal multi-period production plan for a single product over a finite planning horizon to minimize the total inventory and backlog costs by solving a nonlinear programming problem. Li et al. (2010) described an optimal solution structure by the dynamic programming approach for a joint manufacturing and remanufacturing system in a multi-period horizon.

Under uncertain conditions, the result and efficiency of a certain production cycle period and a certain process are influenced not only by the risks that exist in the current period but also by the risks that existed in the foregoing periods. Therefore, we discuss the minimum expected risk of the case mentioned above, in which the risk depends on the foregoing situation and occurs repeatedly for multiple periods. Whether the process (period or site) satisfies the time limit (restriction) usually depends on the state of the foregoing process, as seen in Verzijl (1976), Benders (2002), and Wright (1974).

In this paper, first, the optimal switching problem is systematically classified. Next, the mathematic formulation of the total expectation considered production, due date and quality for the production system is proposed. Finally, the optimal switching point is investigated by numerical experiments.

2. OPTIMAL SWITCHING PROBLEM FOR PRODUCTION SYSTEM CONSITERED PRODUCTION, DUE DATE AND QUALITY

This paper considers cases in which the above two risks not only occur in the single period, but also in multiple periods repeatedly. The problem of minimizing the expected risk in such a situation is a limited-cycle problem with multiple periods. The multi period problem could be classified according to whether the periods are independent or not.

For this problem, one result is the general form of production rate and waiting time by a station-centered approach as discussed in Matsui el al. (1997) (2008). The explicit form is obvious and consists of the product form in the period-independent case, such as a single line, but it is untraceable in the period-dependent case such as a mixed or tandem line. The mixed line has a absorbing barrier, but the tandem line has a reflective barrier at the end. This paper presents a cost approach for the latter.

The purpose of this paper is discussing the switching problem for parallel production system shown in Figure 2.

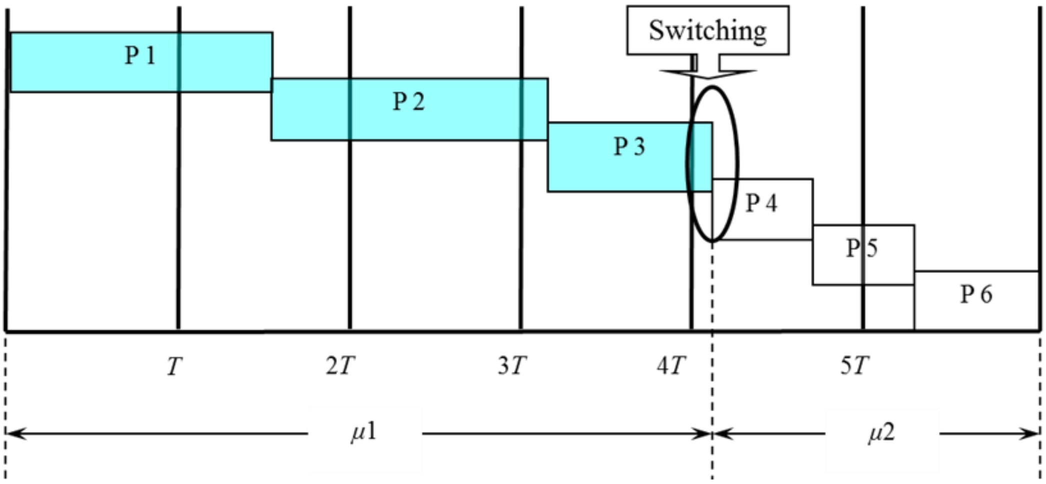

Figure 1: Optimal Switching Problem for Production and Due date Risks of Multiple Periods

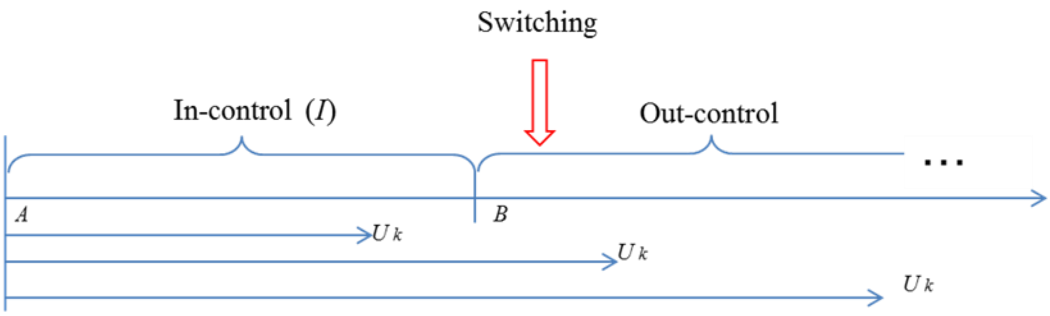

Figure 2: Optimal Switching Problem for Quality problem of Multiple Periods

3. THE OPTIMAL SWITCHING MODEL

3.1 The assumption and notation

The Optimal switching model for the parallel production system with multiple periods is considered based on the following assumptions:

(1) One product is made by a process with n processes.

(2) For i=1, 2, …, n; j=1, 2, …, m, the production time of process i of line j is denoted by T_{ji} which is assumed to be statistically independent, respectively. The usual processing rate is μ_{j1}, and the emergency processing rate is μ_{j2}.

(3) For i=1, 2, …, n , the target production time of process i is denoted by iT, and the due time of the entire process (n periods) is nT.

(4) k is switching point.

(5) The cost per unit time ($C_s^{(h)}$) occurs when a process is executed before the target production time of the process. (h=1 means before switching and h=2 means after switching)

(6) The cost per unit time ($C_p^{(h)}$) occurs when a process is executed after the target production time of the process (h=1 means before switching and h=2 means after switching).

(7) When $X_i = \sum_{l=1}^{i} T_l >$ due time (U_n) of n periods, the delay cost C_p occurs.

(8) When $X_i = \sum_{l=1}^{i} T_l <$ due time (U_n) of n periods, the idle cost C_s occurs.

Some notations are also defined.

For i=1, 2, …, n,

$C(T_1, T_2, \ldots, T_n)$: the total cost of the production process.

$C(i)$: the production cost of period *i*.
T_i : the production time of period i.
X_i : the production time of i periods ($X_i = \sum_{l=1}^{i} T_l$).
$\Pr(X_n > U_n)$: the probability of delay.
$\Pr(X_n \le U_n)$: the probability of idle.
β : Probability of quality problem occurring in production process ($\Pr(\sum_{i=1}^{n} T_i > I)$)

3.2 The assumption and notation

The objective of proposed model is shown as following:

$$E[Z_j] = \min_k (E \begin{bmatrix} C_1(k;T_1,T_2,...T_n)+ \\ C_2(k;T_1,T_2,...T_n)+ \\ C_3(k;T_1,T_2,...T_n) \end{bmatrix}) \quad (1)$$

Z_j is the total expected cost of line *j*,
$C_1(k;T_1,T_2,...T_n)$: The production cost,
$C_2(k;T_1,T_2,...T_n)$: The due date penalty cost,
$C_3(k;T_1,T_2,...T_n)$: The quality penalty cost.

(i) Production Cost
From assumptions (1)-(7) mentioned in Section3.1, we can easily see,

$$E[C_1\ (k;T_1,T_2,...,T_n)] = \sum_{i=1}^{n} E[C(i)] \quad (2)$$

Where, $E[C(i)]$ is the expected cost of period *i*.
In this research, the production time *T*i is assumed to be exponential distributed and statistically independent, respectively.
Then, for, $i = 1,2,...,k$

$$\begin{aligned} E[C(i)] &= E[(C_p^{(1)} - C_s^{(1)}) \cdot S_i + C_s^{(1)} \cdot T_i] \\ &= (C_p^{(1)} - C_s^{(1)})E[S_i] + C_s^{(1)} E[T_i] \\ &= (C_p^{(1)} - C_s^{(1)})E[S_i] + \frac{C_s^{(1)}}{\mu_1} \end{aligned} \quad (3)$$

and for, $i = k+1,\ k+2,\ldots,\ n$

$$\begin{aligned} E[C(i)] &= (C_p^{(1)} - C_s^{(1)})E[S_i \mid \sum_{i=1}^{k} T_i \le U_k] \cdot \Pr\{\sum_{i=1}^{k} T_i \le U_k\} \\ &+ (C_p^{(2)} - C_s^{(2)})E[S_i \mid \sum_{i=1}^{k} T_i > U_k] \cdot \Pr\{\sum_{i=1}^{k} T_i > U_k\} \\ &+ \frac{C_s^{(1)}}{\mu_1} \cdot \Pr\{\sum_{i=1}^{k} T_i \le U_k\} + \frac{C_s^{(2)}}{\mu_2} \cdot \Pr\{\sum_{i=1}^{k} T_i > U_k\} \end{aligned} \quad (4)$$

(ii) Due Date Penalty Cost

$$E[C_2\ (k;T_1,T_2,...,T_n)] = C_p \Pr(X_n > U_n) + C_s \Pr(X_n \le U_n) \quad (5)$$

where, $C_p \Pr(X_n > U_n)$ is the delayed expected cost, $C_s \Pr(X_n \le U_n)$ is the idle expected cost.

$$\Pr\{\sum_{i=1}^{k} T_i > U_k\} = \sum_{l=0}^{k-1} \frac{(\mu_1 U_k)^l}{l!} e^{-\mu_1 U_k} \quad (6)$$

$$\begin{aligned} \Pr\{\sum_{i=1}^{k} T_i \le U_k\} &= 1 - \Pr\{\sum_{i=1}^{k} T_i > U_k\} = 1 - \sum_{l=0}^{k-1} \frac{(\mu_1 U_k)^l}{l!} e^{-\mu_1 U_k} \\ &= \sum_{l=k}^{\infty} \frac{(\mu_1 U_k)^l}{l!} e^{-\mu_1 U_k} \end{aligned} \quad (7)$$

(ii) Quality Penalty Cost
For, $i = 1,2,..., k$

$$E[C_3\ (k;T_1,T_2,...,T_n)] = C_{01}(E[\max(U_k - I,\ 0)])\Pr\{\sum_{i=1}^{n} T_i > U_k\} \quad (8)$$

and for $i = k+1,\ k+2,\ldots,\ n$,

$$E[C_3\ (k;T_1,T_2,...,T_n)] = C_{02}\ (E[\max(\sum_{i=1}^{n} T_i - I,\ 0)])\Pr\{\sum_{i=1}^{n} T_i \le U_k\} \quad (9)$$

where, C_{01} and C_{02} are the quality cost per unit time.

In this research, the in-control time of a production process (I) is assumed to be exponential distributed which the mean value is 1/λ.

4. EXPERIMENTAL CONSIDERATION

In this section, we consider the optimal switching time to minimize the total expected cost by numerical experiments, where $C_s^{(1)}=1$, $C_p^{(1)}=2$, $C_s^{(2)}=3$, $C_p^{(2)}=6$, $C_p=60$, $C_s=20$, $C_{01}=C_{02}=100$, $\beta=0.5$, $\lambda=0.3$, $T=5$, $m=2$ and $n=10$.

Table 1 Production penalty cost by change of the usual processing rate

	µ1=0.2	µ1=0.3	µ1=0.4	µ1=0.5
k=1	69.50	47.45	34.27	25.89
k=2	70.88	45.29	30.92	22.74
k=3	71.42	43.35	28.88	21.38
k=4	71.68	41.81	27.67	20.78
k=5	71.82	40.61	26.95	20.52
k=6	71.87	39.68	26.53	20.41
k=7	71.83	38.96	26.29	20.36
k=8	71.72	38.42	26.15	20.34
k=9	71.62	38.02	26.07	20.33
k=10	71.66	37.78	26.04	20.33

Table 2 Quality penalty cost by change of the usual processing rate

	µ1=0.2	µ1=0.3	µ1=0.4	µ1=0.5
k=1	88.7	53.8	32.7	20.0
k=2	92.6	35.6	21.2	31.0
k=3	60.8	38.4	93.6	186.8
k=4	49.3	129.7	307.4	451.8
k=5	88.4	336.3	587.3	665.5
k=6	201.9	619.1	820.7	775.1
k=7	397.9	904.3	964.8	816.6
k=8	662.1	1136.4	1036.4	829.2
k=9	963.0	1297.1	1066.7	832.4
k=10	1264.9	1395.2	1077.9	833.1

Figure 3 show the behavior of the optimal switching time by change of the usual processing rate when emergency processing rates of system is 0.6. From Figure 3, it can be noted that when usual processing rates of line 1 and line 2 are 0.2, 0.3, 0.4 and 0.5, the optimal switching times are 4T, 2T, 2T and 1T, respectively.

From Figure 3, it also can be noted that when usual processing rates of line 1 is constant, the optimal switching time parameter k decreases with the increase of the usual processing speed of line 2. This is because the quality penalty cost is large in this case, so the the behavior of total expected cost follows the behavior of quality penalty cost.

Table 3 Due date penalty cost by change of the usual processing rate

	µ1=0.2	µ1=0.3	µ1=0.4	µ1=0.5
k=1	29.7	21.6	20.118	20.0073
k=2	28.3	21.4	20.107	20.0052
k=3	27.9	21.32	20.102	20.0049
k=4	28.0	21.30	20.101	20.0053
k=5	28.9	21.4	20.105	20.0056
k=6	30.3	21.5	20.114	20.0061
k=7	32.2	21.7	20.129	20.0066
k=8	34.5	22.0	20.150	20.0074
k=9	36.8	22.5	20.177	20.0082
k=10	38.3	22.8	20.200	20.0089

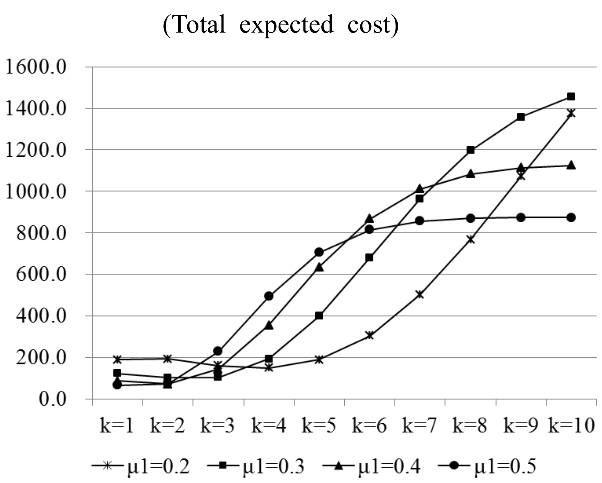

Figure 3: Behavior of the optimal switching time by change of the usual processing rate

5. CONCLUSIONS

In this paper, we considered the optimal switching point (time) to minimize the total expected cost considered production, due date and quality in production system with multiple periods. First, we systematically explained the multi period problem and optimal switching problem. Next, we proposed an optimal switching time model and showed the corresponding mathematical formulations. Finally, by investigating behaviors of the optimal switching time, the optimal switching point could be found.

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