

# Hospital Outpatient Service Optimization with Revenue Management Approach

**Hai Shen**<sup>1,2</sup>

<sup>1</sup>International Business School  
Shaanxi Normal University, Xian, China

<sup>2</sup>Business School  
Xi'an International Studies University, Xian, China  
Email: xisushenhai@163.com

**Kin Keung Lai**<sup>1,2</sup>

<sup>1</sup>International Business School, Shaanxi Normal University, Xian, China

<sup>2</sup>Department of Management Sciences, City University of Hong Kong, Kowloon, Hong Kong  
Email: mskklai@cityu.edu.hk

**Abstract.** This paper applies revenue management ways of aviation industry to the hospital outpatients appointments. Considering the uncertainty of actual demand of patients on the outpatients ability and the no-show situation of the patients, adopting the model of adjustable robust optimization can overcome the dependence of the traditional optimization methods on assumption and strict estimation. Robust optimization model can compensate the shortcomings of the traditional random optimization and the method of demand distribution estimation, which provides a reference method for the formulation of pre-sale strategy of the hospital outpatient appointment. At the same time, using the revenue management to help us to better meet the needs of different patients, and improve the actual income of hospital.

Keywords: revenue management; robust optimization; uncertainty demand; outpatients appointment

## 1. INTRODUCTION

Optimization of capacity for providing medical services is a difficult problem all over the world and a lot of complaints are heard among patients, medical workers and taxpayers. Outpatients appointment is the first segment when patients enter the hospital. It is an important factor which influences the patients satisfaction with efficiency of the service as the waiting time of the patients determines the efficient use of the hospital's resources. How to establish a scientific management system for outpatients appointment through optimization of the hospital service procedures is a significant issue since it can enhance the hospital's medical resources utilization rate and the doctor's work efficiency.

Medical services is a kind of special service industry and its resources and equipments have the characteristics of finiteness, timeliness and exclusive, which means once this service is provided to a patient, other patients can not enjoy this service at the same time. And if no patients use it in a certain time, this service can not have any value. Therefore, the medical industry has very broad prospects for application of revenue management. In mainland China,

most of the research on revenue management has focused on industries such as aviation, hotels and so on, but the research on the medical industry has been sparse.

Revenue management is the process of generating incremental revenues from existing inventory or capacity through a better administration of the sale of a good or service. RM is attributable to bringing new ideas and models that changed the paradigm about doing business. In one form or another, RM applications and their consequences are felt more and more. More specifically, the application part of the Medical Outpatient Service focuses upon improving the Outpatient Service and allocation for elective procedures, using revenue management techniques and practices. At present, almost every hospital outpatient appointment system has the following problems: firstly, the no-show of patients. Since hospitals around the world use different appointment systems, patients' feeling of responsibility is also different. The situation of patients' no-show is a common phenomenon; some regions' no-show rate of patients is as high as 40%. Patients' no-show seriously interferes with the normal work of outpatient appointments system and the problem of limited and stretched medical resources becomes even more serious. Doctors scheduled work cannot be implemented effectively

and the work plan is seriously disturbed. Defife et al. (2010) report a 21% no-show rate in psychotherapy appointments and Moore et al. (2001) estimate the cost of no-shows and cancellations at 3-14% of revenue. When period of the appointment is long, the treatment will when period of the appointment is long, the treatment willingness of patients gradually declines and the time interval from appointment to acceptance of treatment is longer, the patients' no-show rate increases. Secondly, patient's demand. Patients' outpatient service demand is uncertain in each diagnosis cycle. Capital management in hospital has to solve optimization questions in the context of high uncertainty, both from hospital supply side (e.g. machine breakdown, surgeons called away for emergency (Van berkel et al., 2007)) and patients demand side (e.g. random patients arrivals, inherent variability in the time surgery takes and patients' actual length of stay (Adan et al., 2009)). If the date of an outpatient's appointment for diagnosis is a holiday patient demand declines. And if the appointment is during a specific season, demand may rise sharply. Allocation of outpatient appointments ability in a reasonable way and reducing the impact of the uncertainty is very important. Under the premise of full consideration of no-show of patients, meeting the demand of diagnosis patients can ensure optimal utilisation of medical resources. Therefore, how to determine the amount of overbooking and to make a balance between the profits and losses becomes one of the important points for revenue management.

Many foreign hospitals resort to overbooking in outpatient appointments to ensure the optimal number of admissions, to reduce the loss caused by patients' no-show. This strategy has advantages of reducing the loss of patients' no-show. Even though it can have both positive and negative impacts on revenues (Yoshinori 2002) and its negative side effects cannot be completely eliminated, overbooking is becoming a necessity. Overbooking received a great deal of attention from practitioners and academia and it has been researched for longer time than any other RM related problem (McGill and van Ryzin 1999). The more recent literature on overbooking is overwhelming in the number of issues and assumptions taken into account in the developed models and proposed optimal or heuristic solutions (Chatwin 1993; Chatwin 1999; Chatwin 1999; Karaesmen and Van Ryzin 2004), just to mention a few.

However, uncertainty of demand affects the hospital reservation strategy. Uncertainty demand as embodied in the fluctuation of outpatients' demand to medical service, because it is hard for the hospital to predict the number of patients who need the outpatient appointment service, and at the same time, it is also difficult to arrange appropriate amount of medical resources to meet the needs of patients. Due to the existence of the patient's no-show cases in the process of outpatient appointment, the number of patients

who actually come to outpatient clinic is often smaller than its own capacity, which wastes plenty of medical resources. Therefore, the allowed number of visits to the outpatient section is often greater than the number of outpatient section can accept. Adopting overbooking strategy of the revenue management, the outpatient section of hospital can obtain more outpatient resources. Even if the patients unsubscribe or no-show, the full-outpatient can also be ensured. This can provide medicare service for more patients and improve the hospital's revenue. However, if the hospital has an inaccurate prediction for the unsubscribe and no-show cases, it will cause the outpatients waiting for too long or even unable to have treatments. The hospital's reputation will be influenced. Therefore, under the premise of fully considering uncertainty of demand, adopting the corresponding reservation strategy in accordance with patients' different demands, so as to improve the patients' satisfaction with hospital outpatient service.

Patients have a certain uncertainty demand for the outpatient appointment. That is to say, during a single period, the hospital cannot predict the number of patients booking in the outpatient appointment system. Therefore, it cannot accurately arrange the number of doctors and medical resources. Therefore, the gap between patients' needs and medical resources is caused, resulting a decline of patient satisfaction. This uncertainty can be reflected by the fluctuations of patients demand for outpatient appointment, and it is generally difficult to use a specific distribution function to describe it. At the same time, it is very difficult to use chance to describe the specific factors that affect the demand. In the process of actual outpatient appointment, there are a lot of uncertainties which should be fully considered in the formulation of the overbooking strategy, so as to ensure the feasibility and optimality of the strategy. Robust optimization is a process that seeks to strike a balance between the robust solution and the model robustness by introducing the error term. As an uncertain information processing method, a robust plan can solve the influence caused by inaccurate data. and the best robust solution will be obtained. Therefore, the author uses robust optimization method to study the overbooking problem of outpatient appointment under uncertain demand.

## **2. Basic model of overbooking outpatient appointments**

### **2.1 Description of the problem**

According to the current characteristics of the hospital outpatient appointments practices, this paper defines the hospital outpatient appointment as an appointment a patient books through the network, telephone or message. Because of reservation through the network, patients may not have so much sense of responsibility. After making the

appointment, patients often ask for registration withdrawal or do not show up, resulting in actual number of patients that arrive being less than the number that a hospital can serve. This is not only a waste of medical resources, but also brings a certain loss to the hospital. Therefore, in order to reduce the loss of hospital, they often send out appointment numbers beyond the schedule, and the appointment numbers are larger than the capacity. If taking the overbooking strategy, the hospital can provide more resources for diagnosis and treatment for patients. Even if patients ask for registration withdrawal or do not show up, hospitals still can make full use of the limited medical resources and enhance the hospital income. However, all these need to be based on accurate prediction of refund and breaking of appointments (no-show case). Otherwise, there will be patients who have normal appointments but cannot be entertained by the hospital. In this way, the hospital not only has to pay high compensation, but it also damages the hospital's reputation. This requires the use of accurate prediction of demand to determine the optimal number of outpatient appointments, that is, outpatient appointment quota. However, a hospital can provide the best service for patients and also maximize its revenue. Hospital can limit the daily number of appointments in the reservation system and if the number of appointments exceeds the limit, the hospital will close the appointment system. If the number of appointments is less than the quota, the rest will be shifted to the next day to become available.

## 2.2 Variables and parameters of model

In this paper, Letter “ $R$ ” denotes the hospital outpatient capacity. Letter “ $T$ ” denotes overall days of outpatient appointments. Letter “ $t$ ” represents the day that patients can make an appointment after the outpatient appointment system opens  $0 < t \leq T$ . Letter “ $F$ ” represents the average compensation loss for breach of contract per patient. “ $Y_t$ ”, which changes with “ $t$ ”, represents the net income from a successful outpatient appointment. “ $D_t$ ” represents the demand for outpatient appointments on the “ $t$ ” day. In accordance with the number of bookings of patients, letter “ $q$ ” represents the proportion of patients failing to keep the appointment  $0 \leq q \leq 1$ . Letter “ $k$ ” denotes the ratio of the maximum number of appointments to hospital outpatient capacity, which generally ranges from 5% to 10%. Letter “ $X_t$ ” denotes the number of outpatient appointment on the “ $t$ ” day. Letter “ $V_t$ ” denotes the quota of the hospital outpatient appointment on the “ $t$ ” day. Based on these, a maximum revenue model of hospital outpatient appointment can be established as follows:

$$\max f(V, X, D) = \sum\{Y \min\{V, X\}\} - F * \max\{(1-q)\sum\{\min\{V, X\}\} - R, 0\} \quad (1)$$

$$s.t. \quad \sum X \leq \sum V \quad (2)$$

$$\sum\{\min\{V, X\}\} \leq kR \quad (3)$$

$$V_t \leq D_t ; X \geq 0; V \geq 0 \quad (4)$$

Eq. (1) is the objective function, which represents the maximum revenue.

$\sum\{Y \min\{V, X\}\}$ , denotes the net income from hospital outpatient appointments.

$F * \max\{(1-q)\sum\{\min\{V, X\}\} - R, 0\}$ , denotes the loss from overbooking, which contains two decision variables. The quota of the hospital outpatient appointment “ $V_t$ ” is a principal decision variable, and “ $X_t$ ” is an assistant decision variable. For constraints, Eq. (2) ensures that if outpatient appointments don't reach the quota on the “ $t$ ” day, the rest of the appointments are added onto the “ $t+1$ ” day. Eq. (3) denotes the maximum outpatient capacity. What's more, we should try to meet all the appointment demands.

## 3. The overbooking model of outpatient's appointment

The basic model can be easily solved if the demand distribution is known. However, demand “ $D_t$ ” for outpatient appointments is uncertain in actual situations, and the requirements of appointments also cannot be described by an accurate distribution model. Therefore, this paper adopts the method of adjustable robust optimization to optimize and solve the overbooking model of outpatient appointments. Robust optimization considers the influences of overbooking caused by uncertainty of demand. According to historical statistics of hospital outpatient appointments, it also obtains the demand fluctuation range rather than demand distribution and probability. So, this uncertainty problem will effectively transform into a certain problem. In order to avoid the problem of NP-hard, the robust problem will transform into an approximation problem. If  $X_t = \pi_t D_t$ , and  $\pi_t$  is new and not adjustable

decision variable in it. Therefore:

$$\min f(V, X, D) = [n(1-q)F - Y_t] \cdot \sum_t m_t V_t - nFR + \gamma \quad (5)$$

s.t.

$$n(1-q)F \sum_t (1-m_t) \pi_t D_t - \sum_t (1-m_t) Y_t \pi_t D_t \leq \gamma; \quad (6)$$

$$\sum_t \{m_t V_t + (1-m_t) \pi_t D_t\} \leq kR; \pi_t \in [0,1] \quad m_t = 0,1; \quad (7)$$

$$\left\{ R - (1-q) \sum_t \{m_t V_t + (1-m_t) \pi_t D_t\} \right\} \geq 0; \quad V_t \leq D_t; \quad (8)$$

$$X_t \geq 0; V_t \geq 0$$

In Eq. (5), “ $V$ ”, “ $X$ ”, “ $D$ ” are all in the equal position as decision variables. When the outpatient appointments system opens, they are likely to show some extreme values of demands changes that are relatively large. If interval uncertainty sets are used, they will be better to meet the change of outpatient appointments demand. The probable interval uncertain sets are as follows:

$$D_t = \left\{ D_t \mid D_t \in (D_t^0 - D_t, D_t^0 + D_t) \right\}, \quad 0 \leq t \leq T$$

It can be seen that the demand of outpatient appointments “ $D_t$ ” ranges in the set  $(D_t^0 - D_t, D_t^0 + D_t)$ . Letter “ $D_t^0$ ”, which represents the average demand of outpatient appointments at the time “ $t$ ”, can be set according to different population densities and the levels of economic development. By using the fluctuation range to replace the demand distribution, this set can overcome the dependence on the assumptions and estimates of the traditional optimization model.

As for the uncertainty of demand, we can draw lessons from Reference Melvyn Sim (2004). In other words, we can use a parameter  $\delta$  to constrain the uncertainty of sets.  $\delta$  may not be an integer. On the constraints, there are maximum  $\lfloor \delta \rfloor$  of sets can change, and one of them will change into  $(\delta - \lfloor \delta \rfloor) \tilde{D}_t$ . The function of  $\delta$  is to adjust the robustness and the optimal solution. In this paper,  $\delta_\alpha \in [0, J_\alpha]$ ,  $\alpha \in [0, 3]$

For each constraint of Eq. (6) and Eq. (7), a control set  $\delta_0$  is needed, and the constraints should corresponds with it as follows:

$$L = \max_{\substack{t \in S_0 \\ \left\{ \begin{array}{l} s_0 \cup k_\alpha | s_0 \in J_0 \\ |s_0| = J_0 \\ k_\alpha \in J_\alpha / S_0 \end{array} \right\}}} \left\{ \sum_{t \in S_0} \{ [n(1-q)F - Y_t] (1-m_t) \pi_t \} D_t + (\delta_0 - \lfloor \delta_0 \rfloor) D_{k_\alpha} \{ (1-m_t) [n(1-q)F - Y_t] \pi_{k_\alpha} \} \right\} \quad (9)$$

$$\sum_t \pi_t D_t^0 + \max_{\substack{t \in S_\alpha \\ \left\{ \begin{array}{l} s_\alpha \cup k_\alpha | s_\alpha \in J_\alpha \\ |s_\alpha| = \lfloor J_\alpha \rfloor \\ k_\alpha \in J_\alpha / S_\alpha \end{array} \right\}}} \left\{ \sum_{t \in S_\alpha} \pi_t D_t + (\delta_\alpha - \lfloor \delta_\alpha \rfloor) \pi_{k_\alpha} D_{k_\alpha} \right\} \leq \sum_t V_t \quad (10)$$

The methods of dealing with the Reference Melvyn Sim (2004), Eq. (9) can be equivalent to the linear programming as follows:

$$\min f(V, X, D) = -nFR + \gamma$$

s.t.

$$\sum_t \{ [n(1-q)F - Y_t] \pi_t \} D_t^0 + \delta_0 z_0 + \sum_t \xi_t^0 \leq \gamma$$

$$z_0 + \xi_t^0 \geq \{ [n(1-q)F - Y_t] \pi_t \} D_t$$

$$\sum_t \pi_t D_t^0 + \delta_1 z_1 + \sum_t \xi_t^1 \leq \sum_t V_t$$

$$z_1 + \xi_t^1 \geq \pi_t D_t$$

$$\sum_t (1-n)(1-q) \pi_t D_t^0 + \delta_2 z_2 + \sum_t \xi_t^2 \leq (1-n)R$$

$$z_2 + \xi_t^2 \geq (1-n)(1-q) \pi_t D_t$$

$$\sum_t \{ V_t \} \leq kR$$

$$n(1-q) \sum_t \pi_t (D_t^0 - D_t) \geq nR$$

$$V_t \leq D_t^0 + D_t$$

$$z_0, z_1, z_2 \geq 0; \xi_t^0, \xi_t^1, \xi_t^2 \geq 0$$

Parameter  $\delta$  can effectively adjust the robustness of the solution. With the increase of  $\delta$ , the value of the objective function is also increasing, and the robustness of the solution is also enhanced.

#### 4. Calculation

According to data of outpatient appointments for 2006-2015, which is from a scaled professional hospital, its outpatient rate is 93% each year at the same period. The in-time outpatient rate is about 93%, which means that about 7% of them can't get treatment timely due to some reasons. It brings some difficulties to the hospital for matching medical resources effectively. For this reason, overbooking strategy can be used to meet patients' requirements. Meanwhile, it can also bring benefits to hospitals.

Pursuant to recorded data from some hospitals we can get the following figures via statistical analyses. For example, its appointment deliberate default cost is 120 Yuan, the capacity of treatment is 400 persons, and the no show rate “ $q$ ” is equivalent to 7%. The cost of patients paying for appointment is determined by appointment opening time.

What they consider is two factors: reserving the capacity of treatment for outpatient and adjusting doctors' time in advance for appointment, which rains influence on their works and hospitals' proceeds. To optimize the relationship between services and patients' needs, we promote patients seeing doctors through the reservation system as soon as possible in normal situations in case of holding up illness and losses. Considering the emergency nature of patients' conditions, we adjust cost procedures for patients who are in emergent conditions after appointment system releasing 10 days. The network appointment system is opened 10 days in advance, allowing the patients to make an appointment and to book the check items through the Internet. You could defray the fees which is equivalent to the day once you reserved an appointment. Compared with other costs in other periods, the

former is economical enough. More detailed prices can be seen from chart one. The average of the invalid visits to doctors is determined by population density. We can see it from chart two. The average of their needs is swaying in reality. So we set some figures critical to calculating figures, such as 2%, 5%, 10%. To simplify our calculation, we assume that each restrictive prerequisite on variation is equal, that is to say,  $\delta_0 = \delta_i = \delta$ . The opening time of appointment is earlier than one who seeing to doctors 10 days. The patients need to log on 10 days in advance if they want to make appointments through the network appointment system to get treatments and to book the check items, namely  $T = 10d$ . The ratio between maximum reservation and the capacity of outpatient is  $k = 105\%$ .

Table 1: Price of outpatient appointment

$t$	1	2	3	4	5	6	7	8	9	10
$Y_t$	30	35	35	45	60	75	85	70	55	35

Tab.2 The quantity demanded and disturbance variable (2%)

$t$	1	2	3	4	5	6	7	8	9	10
$D_t^0$	18	35	48	57	68	56	49	47	25	36
$D_t$	2	4	5	6	7	6	5	4	3	4

Tab.3 The optimal limit under different control level and disturbance variable (2%)

	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$	$V_8$	$V_9$	$V_{10}$	$f$
$\delta = 0$	0	34	50	60	68	63	46	21	31	41	33425
$\delta = 3$	15	23	45	61	69	62	54	41	4	35	32216
$\delta = 6$	15	35	50	59	72	0	56	50	31	42	32124
$\delta = 9$	15	35	25	62	72	63	56	50	31	0	29873

Tab.4 Revenue compared under different control levels

Disturbance variable	control levels	$f$	$\frac{f}{f_0}$
2%	$\delta = 0$	33425	1
	$\delta = 3$	32216	0.9638
	$\delta = 6$	32024	0.9580
	$\delta = 9$	29873	0.8937
5%	$\delta = 0$	31325	1
	$\delta = 3$	30332	0.9683

	$\delta = 6$	30014	0.9581
	$\delta = 9$	25783	0.8231
10%	$\delta = 0$	30124	1
	$\delta = 3$	28732	0.9538
	$\delta = 6$	28330	0.94045
	$\delta = 9$	12973	0.43065

There is a certain error in the prediction of the patients' demand for outpatient treatment. Under this circumstance, if the error of prediction does not change, namely, the disturbance level of the outpatients appointment demands is fixed, and the robustness of the maximization algorithm for hospital's outpatient revenue mentioned in this paper will increase. That is to say, the higher the level of control, the less amount of overbooking in outpatients appointment occurs. When the no-show cases are the same as the overbooking (the worst case), the optimal solution of the hospital in the case of overbooking can be obtained, which means the increase of control level will lead to the conservatism improvement of the optimal solution. At the same time, it can be seen from Tab 4 that with the decrease of the number of overbooking outpatients appointment (the increase of the control level), the income of outpatients will decrease. For the hospital outpatient system without an overbooking appointment, namely, the control level is zero, the hospital's outpatients profit is the lowest when a no-show case happens.

According to the average level of historical records in this period, the overbooking outpatient appointments, namely, the control capacity can be setted. If a higher disturbance level of appointment occurs, which means the number of patients is increasing or decreasing suddenly, it will have an influence on hospital profit. From Tab 4, we can see that the greater wave of the demand of outpatient appointments shows, the greater impact it will have on the benefit of hospitals. (Except for economic benefit, the relative income can also be seen as a part of total benefit). And there is a decline tendency of hospital's outpatient profit.

Based on economic environment, population density, competitiveness and other comprehensive analyses, we can predict the disturbance level of patients' needs on outpatient. According to our conservative preference, we should select corresponding Overbooking Strategy to ensure its outpatient services rationally, optimizing its proceeds.

## 5. Conclusion

This essay analyses the phenomenon of treatment appointment. The patients' needs on outpatient are uncertain. Based on the robust optimization, we not only study the Overbooking Strategy on outpatient appointment, but also overcome the uncertain problems which cannot be solved in old ways, which is determined by requirement distribution and random optimization. We make use of building the model of overbooking to control the amount of outpatient uncertainties efficiently. Pursuant to the research, we render optimize its profits at rational range, which providing a visionary perspective on studying its optimizing proceeds for health community.

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## REFERENCES

- Aaron R, Wendell G, Ann M. Revenue management for outpatient appointments: joint capacity control and overbooking with class-dependent no-shows. *Flexible Services and Manufacturing*, 2012, 24: 516-548.
- Adan I, Bekkers J, Dellaert N, Vissers J, Yu X. Patient mix optimization and stochastic resource requirements: a case study in cardiothoracic surgery planning. *Health Care Management Science* 2009, 12: 129-41.
- Bailey N. A study of queues and appointment systems in hospital outpatient departments with special reference to waiting-time [J]. *Journal of the Royal Statistical Society*, 1952 14,185-199.
- Chatwin, R. E. (1993). *Optimal airline overbooking*. Palo Alto, CA, Stanford University.
- Chatwin, R. E. (1999). "Continuous-time airline overbooking with time-dependent fares and refunds." *Transportation Science* 33: 805-819.
- Chatwin, R. E. (1999). "Multiperiod airline overbooking with a single fare class." *Operations Research* 46: 805-819.
- Defife JA, CZ Conklin, JM Smith, J Poole. *Psychotherapy*

appointment no-shows: Rates and reasons.  
Psychotherapy: Theory Research, Practice, Training,  
2010, 47(3): 413–417.

Karaesmen, I. and G. J. Van Ryzin (2004). "Coordinating  
Overbooking and Capacity Control Decisions on a  
Network." Submitted.

Moore CG, Wilson-Witherspoon P, Proust [J]. Time and  
money: effects of no-shows at a family practice  
residency clinic. Family Medicine 2001, 33(7):522–527.

Melvyn Sim. Robust optimization [M]. Cambridge:  
Massachusetts Institutor of Technology Sloan school of  
Management, 2004 .

McGill, J. I. and G. J. van Ryzin (1999). "Revenue  
Management: Research Overview and Prospects."  
Transportation Science 33(2): 233-256.

Van Berkel P, Blake J. A comprehensive simulation for wait  
time reduction and capacity planning applied in general  
surgery. Health Care Management Science 2007, 10:  
373–85.