Percolating transition in wind-induced water wave

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Abstract: Under strong wind, wave turbulence exhibiting spatiotemporal fluctuations over a wide range of scales can be excited on the water surface. The turbulent water surface waves take space to develop through the interplay between turbulent wind and the fluctuating water surface. Through direct visualization, rich spatiotemporal dynamical behaviors of nonlinear waves can be explored. However, the transition scenario to strong wave turbulence and the associated waveform dynamics, especially for the percolating transition remain elusive. In this work, through wavelet transform, hot and cold turbulent sites (HTSs and CTSs, respectively) in the 1+1D spatiotemporal (*y*-*t*) space (*y*, normal to a wave propagating direction) are identified. It is found that, with increasing fetch (*x*, the distance from the wind entrance), the transition from the weakly to the strongly turbulent state is associated with the sporadic emergence of HTSs in the form of power-law size distributed clusters, followed by the formation of a large HTSs cluster percolating through the entire *y*-*t* space. The scaling exponents around the critical point are also similar to those found in the theoretical predictions and experimental studies of directed percolating transition to hydrodynamic turbulence

Keywords: percolation, wave turbulence, wind-induced water surface wave

1. Introduction

In nonlinear extended systems, increasing external drive can lead order to disorder transition. The transition from ordered plane wave to the wave turbulence in nonlinear wave system and from the laminar to the hydrodynamic turbulence in pipe flows are good examples. Conventionally, the transition is mainly characterized by the broadening of peaks, scaling behaviors of power spectra, and non-Gaussian wave height distributions.

The recent studies demonstrate the transition follows the scenario of the sporadic emergence of the turbulent sites from the ordered background in the spatiotemporal space in the form of clusters with various sizes. As one of the turbulent clusters percolates through the entire space, the system becomes fully turbulent [1-3]. This scenario is the same as the observation in the non-equilibrium systems, such as forest fire and epidemic spreading, governed by the percolating theory [4]. Nevertheless, beyond the order-disorder transition, whether the transition from the weakly turbulent state to the strongly turbulent also follows the similar scenario remain elusive.

In this study, we use wind-driven water surface wave system as a platform to study the above issue of the transition from weak to strong wave turbulence. It is found that the weak to strong wave turbulence transition starts with a small fraction of hot turbulent sites (HTSs), region of localized high energy turbulent waves, at small *x*, accompanied with the intermittent emergence and decaying of HTSs in the form of clusters with scale-free size distributions. Further increasing *x* leads to the formation of a large HTS cluster percolating through the entire 1+1D spatiotemporal space, leading to strong wave turbulence. The transition is associated with the continuous increase of the HTS fraction, scale-free HTS cluster size distribution, and the suppression of quiescent gaps between HTS clusters, which is similar to the percolating turbulent transition found in hydrodynamic flows and dust acoustic waves.

Wind-induced water wave, a widely observed wave system in nature which attracts much attention, is excited through the interplay of the wind and the water surface at their interface. Its proper spatiotemporal scales make it a good platform for optical large-area waveform imaging.

2. Technical works

The experiment is conducted in a wind-driven water wave system. The air flow sequentially passes through the honeycomb rectifier, the contraction section, the wind tunnel of the water tank, the diffuser, and a smooth upward bending section. Four parallel high-power counter-rotating exhausting fans at the outlet are used to generate uniform wind at the inlet of the wind tunnel. The averaged wind speed U_W can be adjusted by controlling the exhaust fan speed. The working fluid is 10% milk/water solution. The wave height $h(x, y, t)$ is obtained by using diffusive light photography.

The waveform with typical wavelength ≈ 2 cm and tilted wave fronts of the wind-induced water surface wave at U_W = 8.0 m/s in the *x*-*y* plane is shown in the figure 1(a). Figure 1(b) shows the corresponding transversely averaged power spectra at different fetches (*x*). In general, the averaged wave amplitude downstream is larger than the one upstream, but large amplitude waves with similar length scale can also be intermittently excited upstream, as shown in Fig. 1(a) marked by the red arrows. Moreover, the energy spanning across wide range of scales at every fetch manifest that the wave is turbulent and the frequency bump centered at around f = 16 Hz gradually becomes higher as the fetch increasing. Therefore, the spatially-increased amplitude of the turbulent wave can be viewed as the competition of high- and low- energy waves. Through the wavelet transform, the instantaneous energy $E(t)$ at every position in the *x*-*y* space is extracted. By setting an energy threshold, TS can be classified into CTSs (cold TSs) and HTSs (hot TSs) according to whether the local E is below or above the threshold.

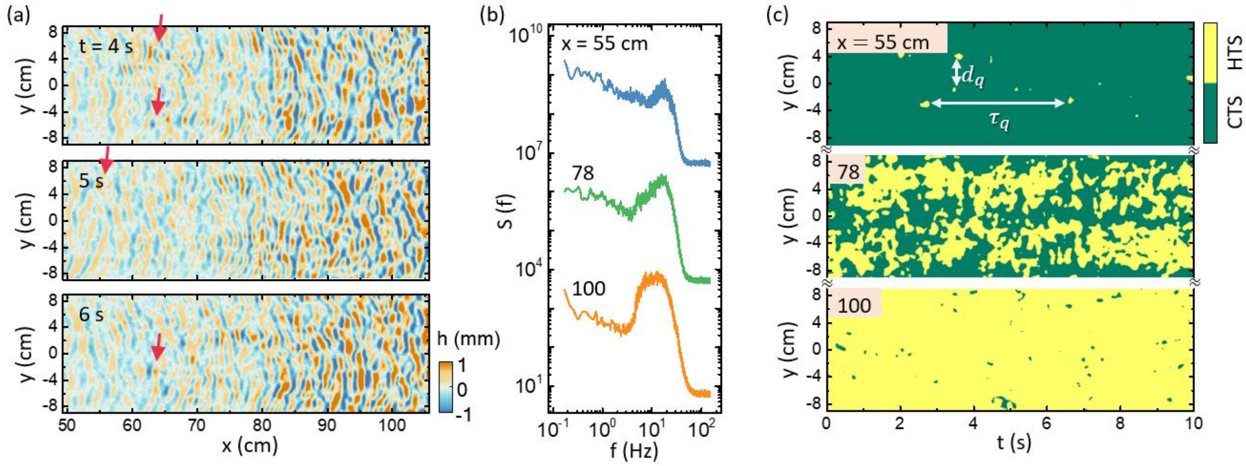

Fig. 1. (a) Color-coded plots of wave height showing the spatial waveform in the *x*-*y* space. The wind is blowing from left to right (along the *x*-axis). (b) Transversely average frequency power spectra at x = 55, 78, 100 cm (brown, red, and blue line, respectively). (c) Distributions of HTSs (yellow) and CTSs (green) in the *y*-*t* space at different fetches (x).

How do these HTSs and CTSs distribute spatiotemporally? The distributions of TSs in the *y*-*t* space are depicted in figure 1(c) for x = 55, 78, 100 cm, color-coded HTSs (yellow) and CTSs (green) with E > and < 1, respectively. Both HTSs and CTSs appear in the form of clusters with various sizes. For small x, TSs are mainly CTSs with low energy and sporadic HTSs. With increasing x, the cluster size of the HTSs gradually increases, eventually a HTS cluster percolating through the *y*-*t* space, indicating the transition from weak wave turbulence to strong wave turbulence. Similar to the previous studies in wave turbulence transition [1-3], we use the volume fraction of HTSs in the *y*-*t* space to characterize the transition from weak to strong wave turbulence. Distributions of cluster size, the quiescent time, and the quiescent distance are also discussed and their scaling exponents near the critical fetch are compared with those in 1+1D directed percolation.

3. Conclusions

In conclusion, using wind-induced 2+1D water surface waves and wavelet analysis, the HTSs and CTSs space are identified according to the instantaneous energy above or below the energy threshold. It is found that the transition from weak wave turbulence with sporadic emergence of small-size HTSs to strong wave turbulence with HTS cluster percolating through the spatiotemporal space (*y*-*t*), follows a smooth and rapid growth of the fraction of HTSs. These local CTSs and HTSs competitions, clustering, and percolating transition behavior are similar to the percolating phase transition of many nonlinear systems.

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